MULTICAST BEAMFORMING USING SEMIDEFINITE RELAXATION AND BOUNDED PERTURBATION RESILIENCE

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ABSTRACT

Semidefinite relaxation followed by randomization is a well-known approach for approximating a solution to the NP-hard max-min fair multicast beamforming problem. While providing a good approximation to the optimal solution, this approach commonly involves the use of computationally demanding interior point methods. In this study, we propose a solution based on superiorization of bounded perturbation resilient iterative operators that scales to systems with a large number of antennas. We show that this method outperforms the randomization techniques in many cases, while using only computationally simple operations.

***Index Terms*—** Multicast beamforming, Semidefinite relaxation, Superiorization, Perturbation resilience

1. INTRODUCTION

Important applications in wireless networks involve multicast communication, which can be defined as the transmission of identical information to multiple users. Therefore, physical layer multicasting schemes have been extensively investigated in the last two decades. In particular, the authors of [1] show that the performance of multicast transmission can be greatly improved by exploiting channel state information (CSI) at the transmitter. They formulate a max-min-fair multicast beamforming problem that aims at maximizing the lowest signal-to-noise ratio (SNR) among a group of users, subject to a unit power constraint on the beamforming vector. While this formulation of the multicast beamforming problem is known to be NP-hard, the optimal solution can be approximated by semidefinite relaxation (SDR) with subsequent randomization [1]. This technique, however, has two major drawbacks. Firstly, solving the semidefinite program with standard interior point methods or first order methods can be both computationally complex and time consuming. Secondly, although performance bounds are provided in [2], the worst case approximation gap increases with the number of users.

To address the drawbacks of the aforementioned methods, we pose the semidefinite relaxation of the multicast beamforming problem as a convex feasibility problem in a real Hilbert space of Hermitian matrices. One of the main advantages of this formulation is that it enables us to apply superiorization techniques [3]. In more detail, we first derive a fixed point algorithm that is known to converge to the solution to the proposed convex feasibility problem. By using recent results in the literature [4], we can show that this algorithm is resilient to bounded perturbations. Therefore, we further improve the proposed fixed point algorithm by adding perturbations that try to steer the iterates towards the unknown solution to the original NP-hard problem. Simulations show that the resulting algorithm produces beamforming vectors with performance close to optimal in some practical scenarios, and it greatly outperforms previous methods based on semidefinite programming and randomization.

This paper is organized as follows. In the remainder of this section, we define notation and system model, state the multicast beamforming problem, and briefly present existing solutions. In Section 2, we propose a fixed point algorithm that approximates the nonconvex beamforming problem using bounded perturbations. Numerical results are provided in Section 3, and we complete the paper with conclusions in Section 4.

1.1. Notation

In the following, lower case letters denote scalars, lower case letters in bold typeface denote column vectors, and upper case letters in bold typeface denote matrices. For any closed convex set C in a Hilbert space $(\mathcal{H}, \langle\cdot,\cdot\rangle)$, we denote by $P_C(\mathbf{X})$ the projection of $\mathbf{X} \in \mathcal{H}$ onto C, and by $(\forall \mathbf{X} \in \mathcal{H})\ T_C^\lambda(\mathbf{X}) := \mathbf{X} + \lambda(P_C(\mathbf{X}) - \mathbf{X})$ the relaxed projection onto C, where $\lambda \in (0,2)$ is a relaxation parameter. We denote by Id the identity operator, and by $\mathbf{I}_N$ and $\mathbf{0}_N$ the $N \times N$-identity matrix and the $N \times N$-all-zero matrix, respectively. We write $\mathbf{X} \succcurlyeq \mathbf{0}$ for positive semidefinite matrices $\mathbf{X}$. The spectrum of a matrix $\mathbf{X}$ is denoted by $\sigma(\mathbf{X})$.

1.2. System Model, Problem Definiton, and Existing Solutions

Following the system model in [1], we assume that a network has a transmitter with N antennas and a multicast group $\mathcal{K} = \{1, \ldots, K\}$ of K users equipped with single receive antennas. In this multicast setting, the transmitter sends the same information $x \in \mathbb{C}$ to all users. The receive signal for the kth user can be written as $y_k = \mathbf{w}^H \mathbf{h}_k x + n_k$, where $\mathbf{w} \in \mathbb{C}^N$ is a beamforming vector, $\mathbf{h}_k \in \mathbb{C}^N$ is a realization of a complex Gaussian random vector representing the channel to user k, and $n_k \in \mathbb{C}$ — drawn from the distribution $\mathcal{CN}(0, \sigma_k^2)$ — is the noise at the receiver. If γ_k denotes the SNR-requirement of user k, the multicast beamforming problem with QoS-constraints[1] can be formally posed as [1, 5]:

$$\begin{aligned} \underset{\mathbf{w} \in \mathbb{C}^N}{\text{minimize}} \quad & \|\mathbf{w}\|_2^2 \\ \text{s.t.} \quad & (\forall k \in \mathcal{K})\ |\mathbf{w}^H \mathbf{h}_k|^2 \geq \gamma_k. \end{aligned} \tag{1}$$

Problem (1) is a quadratically constrained quadratic program with nonconvex constraints, and this problem is known to be NP-hard [1]. Therefore, previous studies [1, 5] have proposed alternative formulations based on semidefinite programming that give rise

[1] It has been shown in [1] that the max-min fair and QoS constrained multicast beamforming problem formulations are equivalent up to scaling if all γ_k are identical.

to more tractable optimization problems. One of the main ideas of these reformulations is to exploit the equality $\mathrm{trace}(\mathbf{v}\mathbf{v}^H) = \mathbf{v}^H\mathbf{v}$, which is valid for any vector $\mathbf{v} \in \mathbb{C}^N$, to obtain the following optimization problem from (1):

$$\begin{aligned} \underset{\mathbf{X}\in\mathbb{C}^{N\times N}}{\text{minimize}} \quad & \mathrm{trace}(\mathbf{X}) \\ \text{s.t.} \quad & (\forall k \in \mathcal{K})\ \mathrm{trace}(\mathbf{X}\mathbf{Q}_k) \geq \gamma_k \\ & \mathbf{X}^H = \mathbf{X},\ \mathbf{X} \succcurlyeq \mathbf{0} \\ & \mathrm{rank}(\mathbf{X}) = 1, \end{aligned} \tag{2}$$

where $\mathbf{Q}_k = \mathbf{h}_k\mathbf{h}_k^H$. Note that, if $\mathbf{w}_\star$ is a solution to (1), then $\mathbf{X}_\star = \mathbf{w}_\star\mathbf{w}_\star^H$ is a solution to (2), and the converse is also valid.

Problem (2) is still intractable in general because of the nonconvex rank-1 constraint. However, it is in a form that enables us to obtain a convex relaxation by simply dropping the rank constraint:

$$\begin{aligned} \underset{\mathbf{X}\in\mathbb{C}^{N\times N}}{\text{minimize}} \quad & \mathrm{trace}(\mathbf{X}) \\ \text{s.t.} \quad & (\forall k \in \mathcal{K})\ \mathrm{trace}(\mathbf{X}\mathbf{Q}_k) \geq \gamma_k \\ & \mathbf{X}^H = \mathbf{X},\ \mathbf{X} \succcurlyeq \mathbf{0}, \end{aligned} \tag{3}$$

which is a convex semidefinite program that can in principle be solved with off-the-shelf methods. One of the main challenges of the above formulation is to recover a beamforming vector from a solution $\tilde{\mathbf{X}}$ to (3). A simple technique is to use as the beamforming vector $\mathbf{w}$ the largest principal component of a solution $\tilde{\mathbf{X}}$ to Problem (3), and to scale this vector to satisfy all constraints in (1). However, this approach produces beamforming vectors that might be highly suboptimal in the sense of (1). To mitigate this problem, the authors of [1] propose randomization techniques to generate a set $\{\mathbf{w}_l\}$ of candidate beamforming vectors from $\tilde{\mathbf{X}}$. After these candidate vectors are scaled such that they satisfy all QoS requirements, the one with lowest power is selected. We refer readers to [1] for detailed information on these randomization techniques.

2. PROPOSED ALGORITHM

The approach described in the previous section has two potential drawbacks that may limit its applicability. First, Problem (3) is typically solved with standard interior point methods, and these methods can suffer from numerical issues in large-scale problems. Second, the performance of the randomization techniques typically decreases with the number of constraints [2], whereby sampling a satisfactory beamforming vector may become prohibitively time-consuming.

To address these drawbacks, we formulate the semidefinite relaxation of the QoS-constrained beamforming problem as a convex feasibility problem in a real Hilbert space in Section 2.1. In Section 2.2, we make use of fixed point algorithms to solve the feasibility problem, and in Section 2.3 we exploit their bounded perturbation resilience property to try to enforce a rank-1 solution, as required in the original problem formulation in (2). Section 2.4 considers some practical aspects of the proposed method.

2.1. Semidefinite Relaxation as a Convex Feasibility Problem

To recast Problem (3) as a convex feasibility problem, we first need to define an appropriate Hilbert space. To this end, let $\mathcal{H} := \{\mathbf{X} \in \mathbb{C}^{N\times N} |\ \mathbf{X} = \mathbf{X}^H\}$ be the *real* Hilbert space of Hermitian matrices with the inner product

$$\langle \mathbf{X}, \mathbf{Y} \rangle := \mathrm{Re}\left\{\mathrm{trace}\left(\mathbf{X}\mathbf{Y}\right)\right\} \tag{4}$$

inducing the standard Frobenius norm

$$||\mathbf{X}|| = \sqrt{\langle \mathbf{X}, \mathbf{X} \rangle} = \sqrt{\mathrm{trace}\left(\mathbf{X}\mathbf{X}\right)}. \tag{5}$$

Note that $\mathcal{H}$ satisfies all axioms of a real inner product space because $\langle \mathbf{X}, \mathbf{Y} \rangle = \langle \mathbf{Y}, \mathbf{X} \rangle$ and scalar multiplication is restricted to real scalars, whereby $(\forall \mathbf{X}, \mathbf{Y} \in \mathcal{H})$ $(\forall \alpha \in \mathbb{R})$ $\langle \alpha\mathbf{X}, \mathbf{Y} \rangle = \alpha \langle \mathbf{X}, \mathbf{Y} \rangle$. Positive definiteness is clear from the fact that this particular inner product induces the Frobenius norm.

Now, consider the following convex feasibility problem:

$$\text{Find } \mathbf{X} \in \mathcal{H} \text{ such that } \mathbf{X} \in C_\star = \bigcap_{k=1}^{K} C_k \cap B_P \cap C_+, \tag{6}$$

where

$$C_+ := \left\{\mathbf{X} \in \mathcal{H} |\ (\forall \mathbf{v} \in \mathbb{C}^N)\ \mathbf{v}^H\mathbf{X}\mathbf{v} \geq 0\right\} \tag{7}$$

is the positive semidefinite cone ($\mathbf{X} \in C_+ \Leftrightarrow \mathbf{X} \succcurlyeq \mathbf{0}$),

$$C_k := \left\{\mathbf{X} \in \mathcal{H} |\ \langle \mathbf{X}, \mathbf{Q}_k \rangle \geq \gamma_k\right\}, \tag{8}$$

is the half-space representing the QoS-constraints of user $k \in \mathcal{K}$, and

$$B_P := \left\{\mathbf{X} \in \mathcal{H} |\ \langle \mathbf{X}, \mathbf{I}_N \rangle \leq P\right\} \tag{9}$$

is the half-space containing matrices whose trace is upper bounded by a given design parameter $P > 0$. If this parameter is chosen to be the optimal objective value $P = \mathrm{trace}(\tilde{\mathbf{X}})$ of the problem in (3), we can verify that $\tilde{\mathbf{X}}$ solves (6) if and only if $\tilde{\mathbf{X}}$ solves (3). Further, let $P \geq \mathrm{trace}(\mathbf{X}_\star) = \|\mathbf{w}_\star\|_2^2$, where $\mathbf{X}_\star = \mathbf{w}_\star\mathbf{w}_\star^H$ is a solution to Problem (2). In this case, any solutions to Problem (2) solves Problem (6), while the converse does not hold in general.

2.2. Algorithmic Solution to the Feasibility Problem

The advantage of working with the convex feasibility formulation in (6) is that this problem can be solved by a plethora of computationally simple projection methods. In particular, it can be solved with the projections onto convex sets (POCS) algorithm[2] given by

$$\mathbf{X}_{n+1} = T_\star(\mathbf{X}_n) := P_{C_+} T_{B_P}^{\lambda} T_{C_K}^{\lambda} \ldots T_{C_1}^{\lambda}(\mathbf{X}_n), \tag{10}$$

with relaxed projections T_C^λ as defined in Section 1.1.

For reference, the projections of $\mathbf{X} \in \mathcal{H}$ onto the half spaces C_k and B_P are given by

$$(\forall k \in \mathcal{K})\ P_{C_k}(\mathbf{X}) = \begin{cases} \mathbf{X}, & \text{if } \mathbf{X} \in C_k \\ \mathbf{X} + \frac{\gamma_k - \langle \mathbf{X}, \mathbf{Q}_k \rangle}{||\mathbf{Q}_k||^2}\mathbf{Q}_k, & \text{otherwise} \end{cases} \tag{11}$$

and

$$P_{B_P}(\mathbf{X}) = \begin{cases} \mathbf{X}, & \text{if } \mathbf{X} \in B_P \\ \mathbf{X} + \frac{P - \langle \mathbf{X}, \mathbf{I}_N \rangle}{N}\mathbf{I}_N, & \text{otherwise} \end{cases}, \tag{12}$$

respectively. Using the eigendecomposition $\mathbf{X} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$, we can write the projection onto the positive semidefinite cone as

$$P_{C_+}(\mathbf{X}) = \mathbf{V}\mathbf{\Lambda}_+\mathbf{V}^H, \tag{13}$$

where $\mathbf{\Lambda}_+ = \mathrm{diag}\left(\max(\lambda_1, 0), \ldots, \max(\lambda_N, 0)\right)$, and $(\forall i \in \{1, \ldots, N\})$ $\lambda_i \in \sigma(\mathbf{X})$ are the (real-valued) eigenvalues of $\mathbf{X} \in \mathcal{H}$, such that $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \ldots, \lambda_N)$.

[2] In general, different relaxation parameters can be used for the particular projections. However, we use the same relaxation parameter for all relaxed projections for simplicity.

According to the fundamental theorem of POCS [6, Thm 2.5-1], the sequence $(\mathbf{X}_n)_{n\in\mathbb{N}}$ produced by the update rule in (10) is guaranteed to converge to a solution to the feasibility problem in (6) for any $\mathbf{X}_0 \in \mathcal{H}$, if a solution exists.

As mentioned before, extracting the dominant principal component from the solution obtained with (10) is likely to result in bad performance compared to the solution to the original problem in (1). It is therefore reasonable to seek a feasible solution that concentrates much of its energy in the largest principal component (i.e., whose largest principal component is as large as possible, while the other principal components are as small as possible). This can be achieved by adding perturbations in each iteration of (10). To this end, we make use of the superiorization methodology [3] to define a perturbed version of the operator $T_\star$ in (10) that preserves the convergence guarantees towards a point in $C_\star$.

2.3. Incorporating the Rank Constraint by Bounded Perturbations

In this subsection, we define a superiorized version of the operator $T_\star$ by adding bounded perturbations in each iteration, with the intent to steer the iterates towards the set of rank-1 matrices. While objective functions used for superiorization are usually convex functions, the distance to this nonconvex set constitutes a nonconvex superiorization objective, whereby this approach does not follow exactly the superiorization methodology in [3]. Nevertheless, we show that $T_\star$ is bounded perturbation resilient, such that its superiorized version

$$\mathbf{X}_{n+1} = T_\star(\mathbf{X}_n + \beta_n \mathbf{Y}_n) \tag{14}$$

is guaranteed to converge to a point in $C_\star$ if $\beta_n\mathbf{Y}_n$ are bounded perturbations; i.e., $(\beta_n)_{n=0}^\infty$ is a sequence of nonnegative real numbers such that $\sum_{n=0}^\infty \beta_n < \infty$, and the sequence $(\mathbf{Y}_n)_{n=0}^\infty$ is bounded. To define the sequence of perturbations, we use the eigenvalue decomposition $\mathbf{X}_n = \mathbf{V}_n\mathbf{\Lambda}_n\mathbf{V}_n^H$ with eigenvalues $\mathbf{\Lambda}_n = \mathrm{diag}(\lambda_1^n, \ldots, \lambda_N^n)$ ordered such that $(\forall i > j)\ \lambda_i^n \geq \lambda_j^n$. Then $(\forall n \in \mathbb{N})$ the matrix

$$\mathbf{Y}_n = -\mathbf{V}_n \mathrm{diag}(0, \lambda_2^n, \ldots, \lambda_N^n)\mathbf{V}_n^H. \tag{15}$$

subtracts all principal components of $\mathbf{X}_n$ except for the largest. In [4, Thm. 3.1], the authors have proved the bounded perturbation resilience of α-averaged mappings with nonempty fix-point set in finite-dimensional real Hilbert spaces. This result equally applies to mappings operating on the *real* Hilbert space of complex matrices $(\mathcal{H}, \langle\cdot,\cdot\rangle)$ defined above because of the isomorphism between any two Hilbert spaces of the same dimension. In particular, $(\mathcal{H}, \langle\cdot,\cdot\rangle)$ is isometrically isomorphic to the real Euclidean Hilbert space $\mathbb{R}^{N(N+1)}$ endowed with the standard inner product, as shown below.

Proposition 1 *$\mathcal{H}$ is isometrically isomorphic to a real Euclidean space $\mathbb{R}^J$ with standard inner product $\langle \mathbf{x}, \mathbf{y}\rangle_{\mathbb{R}^J} = \mathbf{x}^T\mathbf{y}$, where $J = N(N+1)$. The bijection between the two spaces is given by*

$$\phi(\mathbf{X}) = \begin{bmatrix} \mathrm{Re}\{\overline{\mathrm{vec}}(\mathbf{X})\} \\ \mathrm{Im}\{\overline{\mathrm{vec}}(\mathbf{X})\} \end{bmatrix}, \tag{16}$$

where $\overline{\mathrm{vec}}(\mathbf{X})$ is a function which extracts the upper triangular part of $\mathbf{X}$ (including the diagonal entries) into an $\frac{N(N+1)}{2}$-dimensional complex vector.

Proof: *$\mathcal{H}$ is said to be isometrically isomorphic to $\mathbb{R}^J$ if there exists a bijective linear mapping $\phi : \mathcal{H} \to \mathbb{R}^J$ such that $(\forall \mathbf{X}, \mathbf{Y} \in \mathcal{H})\ \langle \mathbf{X}, \mathbf{Y}\rangle_{\mathcal{H}} = \langle \phi(\mathbf{X}), \phi(\mathbf{Y})\rangle_{\mathbb{R}^J}$. Using the definition in* (4)*, we obtain*

$$\begin{aligned}\langle \mathbf{X}, \mathbf{Y}\rangle_{\mathcal{H}} &= \mathrm{Re}\left\{\overline{\mathrm{vec}}(\mathbf{X})^T\overline{\mathrm{vec}}(\mathbf{Y})\right\} \\ &= \mathrm{Re}\{\overline{\mathrm{vec}}(\mathbf{X})\}^T\mathrm{Re}\{\overline{\mathrm{vec}}(\mathbf{Y})\} + \mathrm{Im}\{\overline{\mathrm{vec}}(\mathbf{X})\}^T\mathrm{Im}\{\overline{\mathrm{vec}}(\mathbf{Y})\} \\ &= \langle \phi(\mathbf{X}), \phi(\mathbf{Y})\rangle_{\mathbb{R}^J}.\end{aligned} \tag{17}$$

To show convergence properties of the iterates in (14), we need the following known result, and we include the proof for completeness.

Remark 1 *[7, Example 17.12(a)] The operator $T_\star$ in* (10) *is α-averaged.*
Proof: *By [8, Definition 4.23], for any nonempty subset $D \subseteq \mathcal{H}$, $T : D \to \mathcal{H}$ is α-averaged if there exist $\alpha \in (0, 1)$ and a nonexpansive operator $R : D \to \mathcal{H}$ such that $T = (1-\alpha)\mathrm{Id} + \alpha R$.*

Note that, for every nonempty closed convex subset $C \subset \mathcal{H}$, the reflector $R_C = \mathrm{Id} + 2(P_C - \mathrm{Id})$ is nonexpansive. Therefore, $\forall\lambda \in (0, 2)$, the operator

$$T_C^\lambda = \mathrm{Id} + \lambda(P_C - \mathrm{Id}) = \mathrm{Id} + \frac{\lambda}{2}(R_C - \mathrm{Id}) \tag{18}$$

is $\lambda/2$-averaged. Further (see e.g. [4]), the composite of finitely many averaged mappings remains averaged.

Consequently, according to Proposition 1, Remark 1, and [4, Thm. 3.1], the algorithm in (14) is guaranteed to converge to a point in the solution set $C_\star$ of the feasibility problem in (6), if $C_\star \neq \emptyset$ and $\beta_n\mathbf{Y}_n$ are bounded perturbations. The boundedness of the sequence $(\mathbf{Y}_n)_{n=0}^\infty$ can be easily imposed by replacing $\mathbf{Y}_n$ with $\mathbf{Y}'_n = \min\left(\frac{r}{\|\mathbf{Y}_n\|}, 1\right)\mathbf{Y}_n$ for some $r > 0$. [3]

In order for the feasible set $C_\star$ to contain a solution with rank 1, the desired beamformer power P in the definition of B_P must be at least as large as the optimal objective in Problem (2). This value cannot be computed efficiently, but we show in Section 2.4.1 how this problem can be bypassed.

2.4. Practical Aspects

2.4.1. Dropping the power constraint

Substituting the convex optimization problem in (3) by the feasibility problem in (6) entails the disadvantage that an appropriate value of P for the power constraint has to be known in advance. Unfortunately, determining the optimal value P is difficult. However, our numerical experiments show that severely overestimating the power of the beamforming vector (or even completely discarding the power constraint) affects the performance only marginally[4] when the algorithm is initialized with the all zero matrix $\mathbf{X}_0 = \mathbf{0}_N$.

2.4.2. Computational efficiency

Calculating the perturbations in (15) involves computationally demanding matrix decompositions. However, only the largest eigencomponent is required, which allows the use of fast algorithms (e.g. [9]) for large-scale problems.

[3]For the numerical experiments in Section 3, we did not use this truncation, as we observed that the norm of the perturbations always decreased monotonically over the iterations.

[4]Note that the resulting beamforming vectors were normalized to unit power for comparison. Slackening the power constraint does therefore not improve the worst-case SNR.

For the projection onto C_+ in (13), the entire eigenvalue decomposition is required. Nevertheless, we note that neither the projections onto the QoS-constraints C_k, nor the perturbation may result in points $\mathbf{X} \notin C_+$. Therefore, the projection onto C_+ can be omitted (or included only every few iterations for numerical stability), when the power constraint is dropped.

In addition, the projections onto the half spaces can be computed in an $N(N+1)$ dimensional real Hilbert space according to Proposition 1, so the number of coefficients is roughly halved for large N.

3. SIMULATIONS

In this section, we compare the performance of the algorithm proposed in (14) to the SDR technique with randomization [1]. Throughout this investigation, we set $\gamma_k = 1$, and we assume i.i.d. Rayleigh fading channels $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ and noise with unit variance $\sigma_k^2 = 1$ for all users. Once the beamforming vectors $\mathbf{w}$ are computed with the respective methods, we scale them to unit power. In this way, the solutions correspond to those of the max-min-fair beamforming problem definition [1], which allows us to consider the lowest SNR among all users in the multicast group as a performance measure:

$$\mathrm{SNR}_{\min}(\mathbf{w}) = \min_{k \in \mathcal{K}} \frac{|\mathbf{w}^H \mathbf{h}_k|^2}{\sigma_k^2 \|\mathbf{w}\|_2^2}. \tag{19}$$

The solution $\tilde{\mathbf{X}}$ to the relaxed SDP problem is computed with a standard interior point solver in MATLAB. Subsequently, 10^4 candidate beamforming vectors are generated with each of the randomization techniques "randA", "randB", and "randC" described in [1]. After these vectors have been scaled to unit power, the one with the highest worst-case SNR is selected. An upper-bound on the worst-case SNR can be obtained by normalizing the solution $\tilde{\mathbf{X}}$ to the relaxed problem in (3) to unit power:

$$\left(\forall \mathbf{w} \in \mathbb{C}^N\right) \ \mathrm{SNR}_{\mathrm{SDR}} = \min_{k \in \mathcal{K}} \frac{\mathbf{h}_k^H \tilde{\mathbf{X}} \mathbf{h}_k}{\mathrm{trace}(\tilde{\mathbf{X}})\sigma_k^2} \geq \mathrm{SNR}_{\min}(\mathbf{w}). \tag{20}$$

For the proposed method in (14), the sequence $(\beta_n)_{n\in\mathbb{N}}$ in (14) was set to $\beta_n = 0.9^{n/500}$ in all simulations, and the relaxation parameter of the relaxed projections in (10) was set to $\lambda = 1.9$. We dropped the power constraint (i.e. $P = \infty$) and initialized with $\mathbf{X}_0 = \mathbf{0}_N$. The algorithm was terminated when a stopping criterion $\|\mathbf{X}_{n+1} - \mathbf{X}_n\| \leq \epsilon$ with $\epsilon = 10^{-6}$ was reached or 1000 iterations were exceeded. The beamforming vector was then obtained by extracting the largest principal component from $\mathbf{X}_n$.

Fig. 1 shows $\mathrm{SNR}_{\min}$ for $K = 30$ users as a function of the number N of transmit antennas. In Fig. 2, $\mathrm{SNR}_{\min}$ is depicted for $N = 30$ transmit antennas as a function of the number K of users. It can be seen that the proposed method outperforms all randomization techniques, and the gains become more pronounced with increasing number of antennas and users. Further, if N is large, the worst-case SNR of the proposed method in Fig. 1 is close to $\mathrm{SNR}_{\mathrm{SDR}}$, so the solutions obtained with the proposed algorithm necessarily close to optimal because $\mathrm{SNR}_{\mathrm{SDR}} \geq \mathrm{SNR}_{\min}(\mathbf{w}_\star)$, where $\mathbf{w}_\star$ is a solution to (1).

4. CONCLUSION

In this paper, we formulated the semidefinite relaxation of the QoS-constrained multicast beamforming problem as a feasibility in a real Hilbert space. We showed that the relaxed projection algorithm in (10) is bounded perturbation resilient, and defined a superiorized version that aims to find a rank-1 solution. Numerical results indicate that this approach can outperform previous methods based on SDR and randomization, especially for large-scale problems. Further, this approach does not introduce the necessity of parameter tuning (i.e. guessing the target beamformer power), as it achieves competitive performance even without a power constraint.

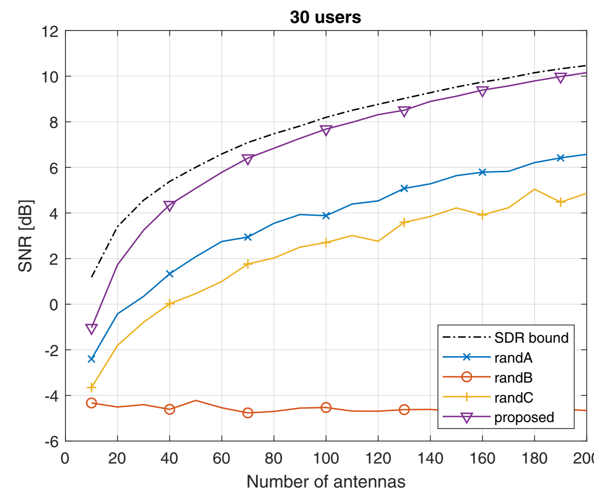

Fig. 1. SNR of the worst link for unit variance noise and unit power beamforming vectors as a function of the number N of transmit antennas (averaged over 30 problem realizations for each N).

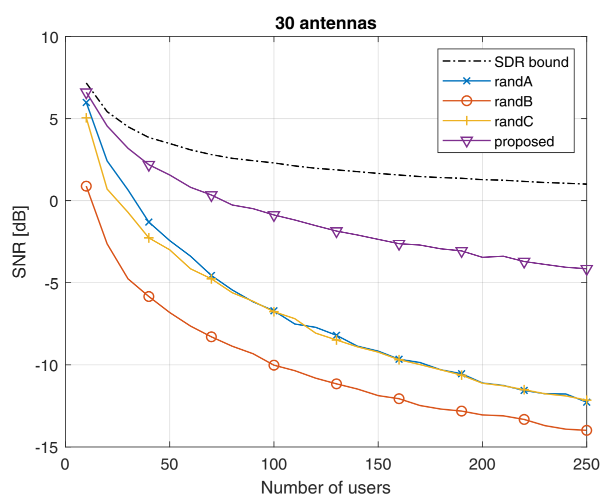

Fig. 2. SNR of the worst link for unit variance noise and unit power beamforming vectors as a function of the number K of users (averaged over 30 problem realizations for each K).

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