ADMM-BASED BEAMFORMING OPTIMIZATION FOR PHYSICAL LAYER SECURITY IN A FULL-DUPLEX RELAY SYSTEM

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ABSTRACT

Although beamforming optimization problems in full-duplex communication systems can be optimally solved with the semidefinite relaxation (SDR) approach, its computational complexity increases rapidly when the problem size increases. In order to circumvent this issue, in this paper, we propose an alternating direction of multiplier method (ADMM) which minimizes the augmented Lagrangian of the dual of the SDR and handles the inequality constraints with the use of slack variables. The proposed ADMM is then applied for optimizing the relay beamformer to maximize the secrecy rate. Simulation results show that the proposed ADMM performs as good as the SDR approach.

Index Terms—Alternating direction of multiplier method, augmented Lagrangian, full-duplex, physical layer security

1. INTRODUCTION

Beamforming optimization problems in various wireless systems, such as multi-user downlink [1], multiple-input-multiple-output (MIMO) relay-based multipoint-to-multipoint communications [2], full-duplex (FD) operation-based energy harvesting (EH) [3], [4], non-orthogonal multiple access (NOMA) [5], and physical layer security systems [6] can be approximated as semidefinite relaxation (SDR) problems [7] which can be accurately solved with the off-the-shelf optimization toolbox (e.g. CVX [8]). However, the SDR solution is based on interior point (IP) methods whose complexity increases drastically when the problem size increases. In order to deal with this issue, several first order methods have been proposed in the literature, for example, low-rank factorization method [9], the block coordinate descent method [10], the dual ascent approach [11], and the eigenvalue saddle point transformation [12]. A common to all of these methods is that they employ simple mathematical operations per iteration, such as matrix-vector multiplications, vector dot products, and eigenvalue-eigenvector computations [13].

Another type of first-order method is the alternating direction method of multipliers (ADMM), in which the optimization variables are first partitioned into several blocks, and then the *augmented Lagrangian function* is minimized with respect to each block by keeping all other blocks fixed at each iteration. The application of ADMM has been studied in various optimization problems, such as non-linear convex optimization [14], l_1-norm minimization problems of compressive sensing [15], and variational inequality problems [16]. Depending on how the positive semidefinite matrix constraints are handled, first-order methods based on augmented Lagrangian function have also been applied to solve the semidefinite programming (SDP) problems [17], [18].

In [19], ADMM is applied within a dual augmented Lagrangian framework, i.e., the augmented Lagrangian function for the dual of the SDP problem is minimized. More specifically, at each iteration, this method first minimizes the dual augmented Lagrangian function with respect to the dual variables associated with the linear equality constraints, and then with respect to the variables associated with the inequality constraints while fixing the other variables. After this, the primal variables are updated. The advantage of this approach is that it can handle SDPs with linear inequality constraints as well as positivity constraint on each element of the positive semidefinite matrix. This approach, which has been tested for some specific problems (e.g., frequency assignment), leads to an ADMM with three or more separable blocks of variables, for which theoretical convergence is not known. On the other hand, a consensus-ADMM method has been proposed in [20] to solve general quadratically constrained quadratic problems (QCQPs) without requiring to approximate them as SDR problems. In this method, however, the updates of one of the blocks of variables of the ADMM requires numerical approach in general.

In this paper, we propose an ADMM-based approach for solving beamforming optimization problems (e.g., in full-duplex communications of [4]- [6]) that can be reformulated as SDR problems and guarantee retrieval of rank-one optimal solutions from the SDR solutions. Motivated from [19], an augmented Lagrangian multiplier function of the dual of the SDR is minimized. However, the inequality constraints are handled by introducing a new set of slack variables. The proposed method is then applied for solving beamforming optimization in a full-duplex relay-based secure communications, wherein the secrecy information rate is maximized.

The remainder of this paper is as follows. In Section 2, the proposed ADMM is presented. Its application for relay beamformer optimization in a physical layer security system is presented in Section 3. Simulation results and conclusions are provided in Section 4 and Section 5, respectively.

Notation: Matrices/vectors are denoted by upper/lower case bold face letters; the superscripts $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^{-1}$ stand for transpose, Hermitian, and matrix inverse respectively; the Euclidean/Frobenius norm of the vector/matrix is denoted by $\|\cdot\|$; the trace and positive semidefiniteness of a matrix $\mathbf{X}$ are denoted by $\mathrm{tr}(\mathbf{X})$ and $\mathbf{X} \succeq \mathbf{0}$, respectively. $\mathcal{R}e\{\cdot\}$ denotes the real part, and $\mathcal{R}^{n\times n}$ and $\mathcal{C}^{n\times n}$ denote the real and complex matrices of sizes $n \times n$, respectively.

2. PROPOSED ADMM

Consider the following SDP problem in its standard form

$$\begin{aligned} \min_{\mathbf{X}\succeq 0} \quad & \mathrm{tr}(\mathbf{C}\mathbf{X}) \\ \text{s.t.} \quad & \bar{\mathcal{A}}(\mathbf{X}) = \bar{\mathbf{b}}, \\ & \bar{\mathcal{B}}(\mathbf{X}) \geq \bar{\mathbf{d}}. \end{aligned} \tag{1}$$

where $\mathbf{C} \in \mathcal{C}^{n\times n}$, $\mathbf{X} \in \mathcal{C}^{n\times n}$, $\bar{\mathbf{b}} \in \mathcal{R}^{m\times 1}$, $\bar{\mathbf{d}} \in \mathcal{R}^{q\times 1}$, $\bar{\mathcal{A}}(\mathbf{X}) = [\mathrm{tr}(\mathbf{A}_1\mathbf{X}), \cdots \mathrm{tr}(\mathbf{A}_m\mathbf{X})]^T$, and $\bar{\mathcal{B}}(\mathbf{X}) = [\mathrm{tr}(\mathbf{B}_1\mathbf{X}), \cdots \mathrm{tr}(\mathbf{B}_q\mathbf{X})]^T$. Let $\mathbf{u} = [u_1, \cdots, u_q]^T$ be a $q \times 1$ vector of positive slack variables $\{u_i\}_{i=1}^q$. Then, (1) can be expressed as

$$\begin{aligned} \min_{\mathbf{X}\succeq 0, \{u_i \geq 0\}_{i=1}^q} \quad & \mathrm{tr}(\mathbf{C}\mathbf{X}) \\ \text{s.t.} \quad & \bar{\mathcal{A}}(\mathbf{X}) = \bar{\mathbf{b}}, \\ & \bar{\mathcal{B}}(\mathbf{X}) - \mathbf{u} = \bar{\mathbf{d}}. \end{aligned} \tag{2}$$

The Lagrangian multiplier function for (2) is expressed as

$$\begin{aligned} \mathcal{L}(\bar{\mathbf{y}}, \mathbf{v}, \boldsymbol{\lambda}, \mathbf{S}) \;=\; & \mathrm{tr}\left((\mathbf{C} + \mathcal{A}^*(\bar{\mathbf{y}}) + \mathcal{B}^*(\mathbf{v}) - \mathbf{S})\,\mathbf{X}\right) - \\ & \bar{\mathbf{y}}^T\bar{\mathbf{b}} - \mathbf{v}^T\bar{\mathbf{d}} - (\mathbf{v}+\boldsymbol{\lambda})^T\mathbf{u}, \end{aligned} \tag{3}$$

where $\mathbf{S} \succeq \mathbf{0}$ and $\boldsymbol{\lambda} \geq \mathbf{0}$ are the dual variables associated with the primal constraints $\mathbf{X} \succeq \mathbf{0}$ and $\mathbf{u} \geq \mathbf{0}$, respectively, whereas $\bar{\mathbf{y}} \in \mathcal{R}^{m\times 1}$ and $\mathbf{v} \in \mathcal{R}^{q\times 1}$ are the dual variables associated with the two equality constraints in (2). Moreover, $\mathcal{A}^*(\bar{\mathbf{y}}) = \sum_{i=1}^m \bar{y}_i\mathbf{A}_i$ and $\mathcal{B}^*(\mathbf{v}) = \sum_{i=1}^q v_i\mathbf{B}_i$ with $\bar{\mathbf{y}} \triangleq [\bar{y}_1, \cdots, \bar{y}_m]^T$ and $\mathbf{v} \triangleq [v_1, \cdots, v_q]^T$. The optimal dual and primal variables can be obtained by solving the following optimization problem

$$\max_{\{\bar{\mathbf{y}}, \mathbf{v}, \boldsymbol{\lambda}\}} \min_{\{\mathbf{X}, \mathbf{u}\}} \mathcal{L}(\bar{\mathbf{y}}, \mathbf{v}, \boldsymbol{\lambda}, \mathbf{S}) \tag{4}$$

For a given $\mathbf{u}$, the inner minimization w.r.t. $\mathbf{X}$ will be unbounded if $(\mathbf{C} + \mathcal{A}^*(\bar{\mathbf{y}}) + \mathcal{B}^*(\mathbf{v}) - \mathbf{S})$ is not positive semidefinite. As such, $\min_{\{\mathbf{X},\mathbf{u}\}} \mathcal{L}(\bar{\mathbf{y}}, \mathbf{v}, \boldsymbol{\lambda}, \mathbf{S})$ is given by

$$\begin{aligned} \min_{\{\mathbf{u}\}} \quad & -\bar{\mathbf{y}}^T\bar{\mathbf{b}} - \mathbf{v}^T\bar{\mathbf{d}} - (\mathbf{v}+\boldsymbol{\lambda})^T\mathbf{u} \\ \text{s.t.} \quad & \mathbf{C} + \mathcal{A}^*(\bar{\mathbf{y}}) + \mathcal{B}^*(\mathbf{v}) - \mathbf{S} = \mathbf{0}. \end{aligned} \tag{5}$$

The constraint of (5) does not depend on $\mathbf{u}$. For a given $\mathbf{v}$ and $\boldsymbol{\lambda}$, the optimum $\mathbf{u}$ that minimizes the objective function is given by $\mathbf{u} = \max(\mathbf{0}, \mathbf{v}+\boldsymbol{\lambda})$. Substituting this $\mathbf{u}$ into (5), the resulting outer maximization can be expressed as

$$\begin{aligned} \min_{\{\bar{\mathbf{y}}, \mathbf{v}, \boldsymbol{\lambda}, \mathbf{S}\succeq 0\}} \quad & \bar{\mathbf{y}}^T\bar{\mathbf{b}} + \mathbf{v}^T\bar{\mathbf{d}} + (\mathbf{v}+\boldsymbol{\lambda})^T\max(\mathbf{0}, \mathbf{v}+\boldsymbol{\lambda}) \\ \text{s.t.} \quad & \mathbf{C} + \mathcal{A}^*(\bar{\mathbf{y}}) + \mathcal{B}^*(\mathbf{v}) - \mathbf{S} = \mathbf{0}. \end{aligned} \tag{6}$$

Clearly, the optimum $\boldsymbol{\lambda}$ is given by $\boldsymbol{\lambda} = \mathbf{0}$, without loss of generality. Define $\mathbf{b} \triangleq [\bar{\mathbf{b}}^T, \bar{\mathbf{d}}^T]^T$, $\mathbf{y} \triangleq [\bar{\mathbf{y}}^T, \mathbf{v}^T]^T$, $\mathcal{A}(\mathbf{X}) \triangleq [\bar{\mathcal{A}}^T(\mathbf{X}), \bar{\mathcal{B}}^T(\mathbf{X})]^T$, and $\mathcal{A}^*(\mathbf{y}) \triangleq \mathcal{A}^*(\bar{\mathbf{y}}) + \mathcal{B}^*(\mathbf{v})$. Then, the optimization problem (6) can be expressed as

$$\begin{aligned} \min_{\{\mathbf{y}, \mathbf{S}\succeq 0\}} \quad & \mathbf{y}^T\mathbf{b} + \mathbf{y}^T\mathbf{P}_m\mathbf{y} \\ \text{s.t.} \quad & \mathbf{C} + \mathcal{A}^*(\mathbf{y}) - \mathbf{S} = \mathbf{0}, \end{aligned} \tag{7}$$

where $\mathbf{P}_m$ is a $(m+q)\times(m+q)$ matrix of ones and zeros and can be expressed as $\mathbf{P}_m = \begin{bmatrix} \mathbf{0}_{m\times(m+q)} \\ \bar{\mathbf{P}}_m \end{bmatrix}$. Here $\bar{\mathbf{P}}_m$ is a $q\times(m+q)$ matrix whose ith row consists of all-zero elements if $y_i < 0$, where $i = m+1, \cdots, m+q$. This shows that $\mathbf{P}_m$ is a function of $\{y_i\}_{i=m+1}^{m+q}$. However, for some initial $\mathbf{P}_m$, the optimization problem (7) can be solved with the ADMM approach. To this end, the augmented Lagrangian function for this optimization problem can be expressed as

$$\begin{aligned} \mathcal{L}_\mu(\mathbf{X}, \mathbf{y}, \mathbf{S}) = & \mathbf{y}^T\mathbf{b} + \mathbf{y}^T\mathbf{P}_m\mathbf{y} + \mathrm{tr}\left((\mathbf{C} + \mathcal{A}^*(\mathbf{y}) - \mathbf{S})\,\mathbf{X}\right) \\ & + \frac{1}{2\mu}||\mathbf{C} + \mathcal{A}^*(\mathbf{y}) - \mathbf{S}||^2. \end{aligned} \tag{8}$$

In the ADMM, the following optimization problem is solved

$$\min_{\mathbf{X}, \mathbf{y}, \mathbf{S}\succeq 0} \mathcal{L}_\mu(\mathbf{X}, \mathbf{y}, \mathbf{S}). \tag{9}$$

Starting with some $\mathbf{X}^{(k)}$ and $\mathbf{S}^{(k)}$, the joint optimization (9) can be solved by solving the following three sub-problems

$$\mathbf{y}^{(k+1)} = \mathrm{argmin}_{\mathbf{y}}\mathcal{L}_\mu\left(\mathbf{X}^{(k)}, \mathbf{y}, \mathbf{S}^{(k)}\right), \tag{10}$$

$$\mathbf{S}^{(k+1)} = \mathrm{argmin}_{\mathbf{S}\succeq 0}\mathcal{L}_\mu\left(\mathbf{X}^{(k)}, \mathbf{y}^{(k+1)}, \mathbf{S}\right), \tag{11}$$

$$\mathbf{X}^{(k+1)} = \mathbf{X}^{(k)} + \frac{1}{2\mu}\left[\mathbf{C} + \mathcal{A}^*(\mathbf{y}^{(k+1)}) - \mathbf{S}^{(k+1)}\right]. \tag{12}$$

Let us assume $\mathbf{B}_1 \triangleq \mathbf{A}_{m+1}, \cdots \mathbf{B}_q \triangleq \mathbf{A}_{m+q}$ for notational simplicity and define the following matrix

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathrm{tr}(\mathbf{A}_1\mathbf{A}_1^H) & \cdots & \mathcal{R}e\left(\mathrm{tr}(\mathbf{A}_m\mathbf{A}_1^H)\right) \\ \vdots & \vdots & \vdots \\ \mathcal{R}e\left(\mathrm{tr}(\mathbf{A}_1\mathbf{A}_{m+q}^H)\right) & \cdots & \mathrm{tr}(\mathbf{A}_{m+q}\mathbf{A}_{m+q}^H) \end{bmatrix}.$$

Proposition 1. *For a given $\mathbf{S}^{(k)}$ and $\mathbf{X}^{(k)}$, the solution of $\min_{\mathbf{y}}\mathcal{L}_\mu\left(\mathbf{X}^{(k)}, \mathbf{y}, \mathbf{S}^{(k)}\right)$ is given by*

$$\begin{aligned} \mathbf{y}^{(k+1)} \;=\; & (\bar{\mathbf{A}} + 2\mu\mathbf{P}_m)^{-1}\Big\{-\mathcal{A}_R\left((\mathbf{C} - \mathbf{S}^{(k)})^H\right) + \\ & \mu\left(-\mathbf{b} - \mathcal{A}(\mathbf{X}^{(k)})\right)\Big\}, \end{aligned} \tag{13}$$

where

$$\mathcal{A}_R\left((\mathbf{C}-\mathbf{S}^{(k)})^H\right) = \begin{bmatrix} \mathcal{R}e\left(\mathrm{tr}(\mathbf{A}_1\left(\mathbf{C}-\mathbf{S}^{(k)})^H\right)\right) \\ \vdots \\ \mathcal{R}e\left(\mathrm{tr}(\mathbf{A}_{m+q}\left(\mathbf{C}-\mathbf{S}^{(k)})^H\right)\right) \end{bmatrix}.$$

Proof. The proof involves solving the gradient of $\mathcal{L}_\mu(\mathbf{X}^{(k)}, \mathbf{y}, \mathbf{S}^{(k)})$ with respect to $\mathbf{y}$ and is skipped due to space constraint. □

Proposition 2. *For a given $\mathbf{X}^{(k)}$ and $\mathbf{y}^{(k+1)}$, the solution of $\min_{\mathbf{S}\succeq 0}\mathcal{L}_\mu\left(\mathbf{X}^{(k)}, \mathbf{y}^{(k+1)}, \mathbf{S}\right)$ is given by*

$$\mathbf{S}^{(k+1)} = \mathbf{Q}\mathbf{\Lambda}^+_{\mathbf{V}^{(k+1)}}\mathbf{Q}^H \tag{14}$$

where $\mathbf{\Lambda}^+_{\mathbf{V}^{(k+1)}}$ is the diagonal matrix of positive eigenvalues of

$$\mathbf{V}^{(k+1)} = \mathbf{X}^{(k)} + \frac{1}{2\mu}\left(\mathbf{C} + \mathcal{A}^*(\mathbf{y}^{(k+1)})\right)^H, \tag{15}$$

and $\mathbf{Q}$ is the corresponding matrix of eigenvectors.

Proof. The proof follows from solving the gradient of the augmented Lagrangian with respect to $\mathbf{S} \succeq 0$ and is skipped due to space constraint. □

Now the optimization problem (9) is solved with the following iterative algorithm (**Algorithm 1**).

Algorithm 1:

1: Initialize $\mathbf{X}^{(k)}$ and $\mathbf{S}^{(k)}$, the maximum number of iterations (N_{it}) and/or convergence accuracy ϵ, and μ.
2: Obtain $\mathbf{y}^{(k+1)}$ from (13).
3: Obtain $\mathbf{S}^{(k+1)}$ from (14) and (15).
4: Update $\mathbf{X}^{(k+1)}$ using (12)
5: Update μ
6: If $\{y_i^{(k+1)} = 0\}_{i=m+1}^{m+q}$, set ith row of $\bar{\mathbf{P}}_m$ to all-zeros.
7: Go to step 2 until convergence.

In **Algorithm 1**, the following stopping criterion and update procedure for μ is employed [21]. Let $r_{prim}^{(k)} = ||\mathcal{A}(\mathbf{X}^{(k)}) - \mathbf{b}||$ and $r_{dual}^{(k)} = ||\mathbf{C} + \mathcal{A}^*(\mathbf{y}^{(k+1)}) - \mathbf{S}^{(k+1)}||$ be the primal and dual residuals in the kth iteration. The algorithm is considered to be converged if $\max(r_{prim}^{(k)}, r_{dual}^{(k)}) \leq \epsilon$, where ϵ is chosen depending on the application problem. In order to improve the convergence as well as make μ independent of initial choice, we employ the following rule for updating μ [21]:

$$\mu^{(k+1)} = \begin{cases} \tau^u\mu^{(k)} & \text{if } r_{prim}^{(k)} > \rho r_{dual}^{(k)}, \\ \frac{\mu^{(k)}}{\tau^d} & \text{if } r_{dual}^{(k)} > \rho r_{prim}^{(k)}, \\ \mu^{(k)} & \text{otherwise}, \end{cases} \tag{16}$$

where $\rho > 1$, $\tau^u > 1$, and $\tau^d > 1$ are the parameters having typical values as $\rho = 10$ and $\tau^u = \tau^d = 2$.

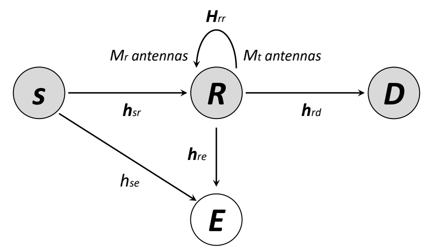

Fig. 1. A FD relay system with an eavesdropper.

3. APPLICATION TO PHYSICAL LAYER SECURITY

In this section, we apply the proposed ADMM to optimize relay beamformer in a communication system consisting of a source S, a FD relay R, a legitimate receiver D, and an eavesdropper E, as shown in Fig. 1. The relay is equipped with M antennas (M_t transmit antennas and $M_r = M - M_t$ receive antennas) and employs decode-and-forward protocol, whereas all other nodes are equipped with a single antenna. It is assumed that the relay estimates the $\mathtt{S}-\mathtt{R}$ and $\mathtt{R}-\mathtt{D}$ channels during the training phase. Similar to most literature, (see e.g. [22] and the references therein) the eavesdropper is assumed to be an active user in the legitimate network, and hence the relay can estimate the $\mathtt{E}-\mathtt{R}$ channel which is used as an estimate for the $\mathtt{R}-\mathtt{E}$ channel by channel reciprocity. We assume that all channels are flat fading and the channel estimates are accurate enough.

The $\mathtt{S}-\mathtt{R}$, $\mathtt{R}-\mathtt{D}$, $\mathtt{S}-\mathtt{E}$, and $\mathtt{R}-\mathtt{E}$ channels are, respectively, represented by $\mathbf{h}_{sr} \in \mathbb{C}^{M_r\times 1}$, $\mathbf{h}_{rd} \in \mathbb{C}^{M_t\times 1}$, $h_{se} \in \mathbb{C}$ and $\mathbf{h}_{re} \in \mathbb{C}^{M_t\times 1}$. The transmit power of the source is denoted by P_s, whereas $\mathbf{H}_{rr} \in \mathbb{C}^{M_r\times M_t}$ denotes the residual loop-interference (LI) channel at the relay. The relay applies a linear beamformer to the received signal to estimate the transmitted signal and then applies a transmit beamformer $\mathbf{w}_t \in \mathcal{C}^{M_t\times 1}$ to the estimated signal. On the other hand, due to full-duplex operation mode and its non-zero processing delay, the $\mathtt{S}-\mathtt{E}$ and $\mathtt{R}-\mathtt{E}$ channels form an inter-symbol interference channel. Assuming a processing delay equivalent to one symbol and a processing length of two blocks, the information secrecy rate is derived in [6]. The objective is to maximize the secrecy rate w.r.t. the relay's receive and transmit beamformers. After substituting the optimum receive beamformer in terms of $\mathbf{w}_t \in \mathcal{C}^{M_t\times 1}$, the secrecy rate maximization problem can be expressed as in (17) (shown on top of the next page) [6], where $c = 1 + \rho_4|h_{se}|^2$, $\tilde{\mathbf{B}} = \rho_5\mathbf{h}_{re}^*\mathbf{h}_{re}^T, \rho_1 = \frac{P_s}{\sigma_r^2}, \rho_2 = \frac{P_r}{\sigma_r^2}, \rho_3 = \frac{P_r}{\sigma_d^2}, \rho_4 = \frac{P_s}{\sigma_e^2}$, and $\rho_5 = \frac{P_r}{\sigma_e^2}$. Here σ_r^2, σ_d^2, and σ_e^2 denote respective noise powers at the relay, destination, and eavesdropper. P_r is the transmit power of the relay. After introducing an auxiliary variable t and applying matrix

$$\max_{||\mathbf{w}_t||=1} \log_2\left\{\left(1+\min\left\{\rho_1 \mathbf{h}_{sr}^H(\rho_2 \mathbf{H}_{rr}\mathbf{w}_t\mathbf{w}_t^H\mathbf{H}_{rr}^H+\mathbf{I})^{-1}\mathbf{h}_{sr}, \rho_3|\mathbf{h}_{rd}^T\mathbf{w}_t|^2\right\}\right) - \frac{1}{2}\log_2\left(c^2+\mathbf{w}_t^H\tilde{\mathbf{B}}\mathbf{w}_t\right)\right\}, \tag{17}$$

inversion, the optimization problem (17) can be expressed as

$$\max_{\{t\geq 1, ||\mathbf{w}_t||=1\}} \quad \frac{t}{(c^2+\mathbf{w}_t^H\tilde{\mathbf{B}}\mathbf{w}_t)^{\frac{1}{2}}}, \tag{18a}$$

$$\text{s.t.} \quad \frac{\mathbf{w}_t^H\mathbf{H}_{rr}^H\mathbf{h}_{sr}\mathbf{h}_{sr}^H\mathbf{H}_{rr}\mathbf{w}_t}{1+\rho_2\mathbf{w}_t^H\mathbf{H}_{rr}^H\mathbf{H}_{rr}\mathbf{w}_t} \leq q(t), \tag{18b}$$

$$\frac{(t-1)}{\rho_3} \leq \mathbf{w}_t^H\mathbf{h}_{rd}^*\mathbf{h}_{rd}^T\mathbf{w}_t, \tag{18c}$$

where $q(t) \triangleq \frac{||\mathbf{h}_{sr}||^2}{\rho_2} - \frac{1}{\rho_1\rho_2}(t-1)$. For a given t, introducing $\mathbf{W}_t \triangleq \mathbf{w}_t\mathbf{w}_t^H$ and relaxing the equality constraint $\mathbf{W}_t = \mathbf{w}_t\mathbf{w}_t^H$ by $\mathbf{W}_t \succeq \mathbf{w}_t\mathbf{w}_t^H$, (18) can be expressed as

$$\min_{\mathbf{W}_t} \operatorname{tr}(\mathbf{W}_t\mathbf{B}), \tag{19a}$$

$$\text{s.t.} \operatorname{tr}\left(\mathbf{W}_t\mathbf{H}_{rr}^H\left[\rho_2 q(t)\mathbf{I}-\mathbf{h}_{sr}\mathbf{h}_{sr}^H\right]\mathbf{H}_{rr}\right) \geq -q(t), \tag{19b}$$

$$\frac{(t-1)}{\rho_3} \leq \operatorname{tr}\left(\mathbf{W}_t\mathbf{h}_{rd}^*\mathbf{h}_{rd}^T\right), \tag{19c}$$

$$\operatorname{tr}(\mathbf{W}_t)=1, \ \mathbf{W}_t \succeq \mathbf{0}, \tag{19d}$$

where $\mathbf{B} = \mathbf{h}_{re}^*\mathbf{h}_{re}^T$. In this problem, rank-one optimum solution can be always recovered from the SDR solution [4]. Thus, comparing (19) with the standard SDP (1), we obtain

$$\begin{aligned}
&\mathbf{X}=\mathbf{W}_t, \ \mathbf{C}=\mathbf{B}, \ \mathbf{A}_1=\mathbf{I}, \mathcal{A}^*(\bar{\mathbf{y}})=\bar{y}_1\mathbf{I}, \ \bar{\mathbf{b}}=[1],\\
&\mathbf{B}_1=\mathbf{H}_{rr}^H\left[\rho_2 q(t)\mathbf{I}-\mathbf{h}_{sr}\mathbf{h}_{sr}^H\right]\mathbf{H}_{rr}, \ \mathbf{B}_2=\mathbf{h}_{rd}^*\mathbf{h}_{rd}^T,\\
&\mathcal{B}^*(\mathbf{v})=v_1\mathbf{H}_{rr}^H\left[\rho_2 q(t)\mathbf{I}-\mathbf{h}_{sr}\mathbf{h}_{sr}^H\right]\mathbf{H}_{rr}+v_2\mathbf{h}_{rd}^*\mathbf{h}_{rd}^T,\\
&\bar{\mathbf{d}}=\left[-q(t), \frac{t-1}{\rho_3}\right]^T.
\end{aligned} \tag{20}$$

A line-search w.r.t. t is then performed to solve the joint optimization [6].

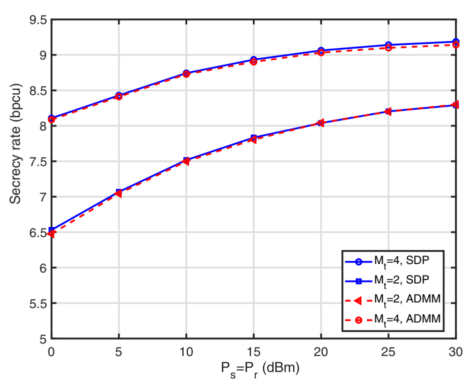

Fig. 2. Comparison between the proposed ADMM and SDR.

4. SIMULATION RESULTS

In this section, we compare the performance of the proposed ADMM with the SDR approach for the considered application problem. Throughout all simulations of the ADMM method, we choose $\rho = 10$, $\tau^u = \tau^d = 2$, set initial value of μ to 10, N_{it} to 2000 and ϵ to 5×10^{-3}. The $\mathtt{S}-\mathtt{R}$, $\mathtt{R}-\mathtt{D}$, $\mathtt{R}-\mathtt{E}$, and $\mathtt{S}-\mathtt{E}$ channel distances are set to 40m, 40m, 50m, and 200m, respectively. We set noise powers at the relay, destination, and eavesdropper to -80 dBm, the pathloss exponent to 3, the variance of the residual loop-interference to 30 dBm, and choose $M = 8$ and $P_s = P_r$.

The performance of the proposed ADMM and SDR methods is shown in Fig. 2 for different M_t. It can be observed from Fig. 2 that the performance of the proposed ADMM is similar to that of the SDR method. Note that we skip all channel realizations for which CVX finds the SDR problem infeasible, since the corresponding $\mathbf{w}_t$ is not available.

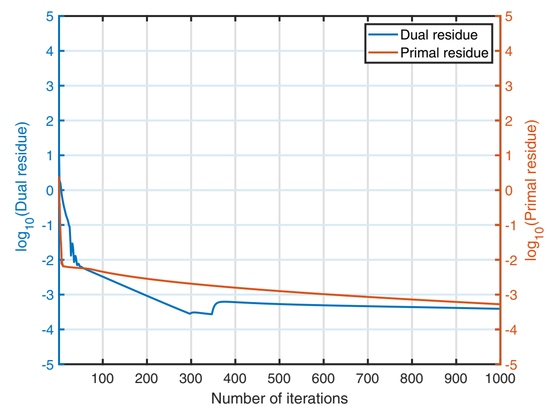

Fig. 3. Convergence for the proposed ADMM.

In Fig. 3, the convergence behavior of the ADMM algorithm is shown, where we take $M_t = 6$, $\epsilon = 5\times 10^{-3}$, and $P_s = 0$ dBm. As the number of iterations increase, both primal and dual residues decrease and both converge to a value less than 5×10^{-3} in about 200 iterations.

5. CONCLUSION

In this paper, we proposed an alternating direction of multipliers method (ADMM) to solve beamforming optimization problems that can be reformulated as semidefinite relaxation (SDR) problems and guarantee rank-one solutions. The proposed ADMM minimizes the augmented Lagrangian function of the dual of the SDR and handles inequality constraints through slack variables. The algorithm is then applied to solve the relay beamforming optimization problem, whereein the objective is to maximize the secrecy rate. Simulation results show that the proposed ADMM provides performance similar to that of the standard SDR method. Future works include theoretical analysis of the convergence of the proposed ADMM.

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