FEEDBACK-CONTROLLED CHANNEL ESTIMATION WITH LOW-RESOLUTION ADCS IN MULTIUSER MIMO SYSTEMS

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ABSTRACT

The nonlinear nature of the quantization operation performed by the ADCs imposes great challenges to the systems employing coarse quantization. Systems operating in moderate to high SNR regimes experience a pronounced capacity loss and also impairments in the quality of the channel estimates, compared to the infinite precision case. This work proposes a multiuser MIMO channel estimation algorithm that operates with low-resolution receiving ADCs. The strategy is based on an coarse quantizer followed by an EM-ML estimator, that takes as input the observation vector subtracted by an offset value produced by a first rough channel estimate. The approach performed by this feedback-controlled double-stage estimator aims to make the quantizer thresholds based on the low-SNR Cramér-Rao lower bound approximation valid in the high SNR region. Numerical analysis reveals that reliable estimation is achieved in coarse quantization for a wide range of SNR with a very small and constant gap in relation to infinite precision quantized estimation.

***Index Terms*— ** Quantization, MIMO, multiuser, channel estimation, EM algorithm, Cramér-Rao Lower Bound

1. INTRODUCTION

Massive Multiple-input multiple-output (MIMO) systems in wireless communications promise to meet the ever growing demands for higher throughput and reliable wireless links of the next generation wireless communications systems as 5G cellular system. However, the large number of antennas poses new challenges for system design and hardware implementation as the energy consumption and circuit complexity increases accordingly. For example, the energy consumption of an analog to digital converter (ADC) grows exponentially as a function of the quantization resolution. An additional motivation for reducing the ADC resolution is to limit the amount of data that has to be transferred over the link that connects the RF components and the baseband-processing unit. Alleviating this capacity bottleneck is of particular importance in a cloud radio access network (C-RAN) architecture [1], where the baseband processing is migrated from the base stations (BS) to a centralized unit, which may be placed at a significant distance from the BS antenna array. Therefore using low resolutions ADC is a cost-effective solution for massive MIMO systems when array size becomes very large or when the sampling rate becomes very high. To address these challenges, recent works considers the use of low-resolution ADCs for massive MIMO systems in order to reduce circuit complexity and save energy. However the use of low-resolution ADCs generates significant nonlinear distortion and, thus further challenges are imposed on the signal processing techniques in order to provide reliable data transmission, requiring the development of detection and estimation algorithms that operate on coarsely quantized signals.

Recent works on the estimation of unknown parameters based on quantized data can be found in [2–5]. In [6], a closed-form solution for the maximum likelihood channel estimate with one-bit quantized data is presented. In [2], a more general setting is studied, where the Expectation Maximization (EM) based algorithm [7] is proposed to solve the Maximum a Posteriori Probability (MAP) channel estimation problem when few-bits quantized data is employed. The use of Cramér-Rao lower bound (CRLB) to set the thresholds of a fixed quantizer have been shown to produce mean square error (MSE) curves that are non monotonic with the SNR; moreover, there is an optimal SNR that results in the minimum MSE value. Besides, the gap to the ideal (infinite resolution) ADC in terms of the estimation performance is relatively small at the low SNR region and this gap degrades when the SNR is above the optimal value, which means that the noise may be favorable at certain level. This phenomenon was observed in several works on quantized data [2, 6, 8] and poses a challenge on channel estimation in high SNR region using fixed quantization thresholds.

It was shown in [9, 10] that the estimation of a constant parameter based on quantized noisy measurements achieves a constant loss (w.r.t to CRLB) in relation to the unquantized one when an offset given by the feedback from the output is used at the input of the quantizer. For constant input offset, binary quantization, and Gaussian noise, the best choice for the offset has been shown to be the parameter itself, and in this case the constant loss is $\frac{\pi}{2}$. For few-bits quantized data an EM algorithm is presented for MAP parameter estimation in noisy measurements using a known feedback-control given by the previous estimates. However, the choice of the quantizer thresholds that minimize the constant loss w.r.t to CRLB of the unquantized data is not shown for few-bits quantized data.

Thus, in this work a feedback-controlled EM approach is used to implement a double-stage Maximum Likelihood (ML) channel estimation algorithm with optimal fixed thresholds for MIMO systems using few-bits ADC is proposed. In particular, using time-multiplexed pilots [6], the MIMO channel is transformed into a SISO channel (considering independent noise at the receiver) and the estimation problem is reduced to a set of scalar channel estimations. Using this approach, each channel coefficient of the MIMO matrix is estimated independently using the feedback-controlled EM algorithm with a reduced complexity as no channel inversion is required. Here, the EM-ML channel estimate output itself is used as an offset at the input of the quantizer in order to approximate the low-SNR loss factor w.r.t to CRLB of the unquantized estimate that depends only on the quantizer features. This allows the determination of thresholds that minimize and maintains fixed the loss factor throughout a wide SNR range. In order to use the most reliable feedback EM-ML channel estimate in high SNR region, we propose to use dithering to approximate to the SNR region where the MSE is minimum.

The proposed few-bits MU-MIMO estimation algorithm, comprised by the characteristics aforementioned, achieves estimation performance that maintains constant loss compared to the unquantized estimator for any SNR, resulting in improved performance compared to other strategies proposed up to date.

The following notation is used: $\mathcal{N}(a, b)$ denotes a Gaussian distributed random variable, with mean a and variance b. The indicator function $\mathbb{I}_{\mathcal{A}}(c)$ assumes the value 1 if $c \in \mathcal{A}$, and 0 otherwise. The cumulative distribution function of the standard normal distribution is denoted by Φ. Finally, $\mathbf{1}_d$ corresponds to the vector with unit entries and length d, and $\mathrm{circshift}(\mathbf{A})$ performs the circular shift operation on the columns of the matrix $\mathbf{A}$.

2. SYSTEM MODEL

This work considers the pilot-aided channel estimation phase of a multiuser MIMO communication system. In this scenario, a BS equipped with $\tilde{M}$ antennas receive training symbols from K single-antenna users located within a cell. The signals are simultaneously transmitted by the users over successive transmission periods, and after traversing the wireless channel, considered a Rayleigh-flat block-fading channel, are received by the BS and then quantized with two low-resolution ADCs on each receive antenna (totaling $M = 2\tilde{M}$ quantizers), for subsequent processing. The complex data vector that cascades the information collected over L transmission periods, after the conversion from complex to real domain of the quantities involved, $\mathbf{y} \in \mathbb{R}^{ML\times 1}$, is given by:

$$\mathbf{y} = \mathcal{X}\mathbf{h} + \mathbf{n}, \tag{1}$$

where $\mathcal{X} \in \mathbb{R}^{ML\times MK}$ contains the information transmitted in the training sequence, $\mathbf{h} \in \mathbb{R}^{MK\times 1}$ denotes the wireless channel coefficients, and $\mathbf{n} \in \mathbb{R}^{ML\times 1}$ corresponds to the the additive noise vector, with entries that follow a circularly symmetric white Gaussian probability density function (p.d.f.) with zero mean and variance σ^2.

The ADC is represented by a finite-precision quantizer $\mathcal{Q}$ that operates on the received vector, such that $\mathbf{r} = \mathcal{Q}(\mathbf{y})$. The entries of the quantized data vector belong to the alphabet $\mathcal{A}$, which elements, r_j, corresponds to the labels of a uniform symmetric mid-riser type quantizer [11]:

$$r_j \in \left\{ \left(-\frac{2^b}{2} - \frac{1}{2} + k \right) \Delta;\ k = 1, \ldots, 2^b \right\}, \tag{2}$$

where Δ is the quantizer step size and b is the number of quantization bits. Here the lower and upper quantization thresholds are:

$$r_j^{\text{lo}} = \begin{cases} r_j - \frac{\Delta}{2}, & \text{for } r_j \geq -\frac{\Delta}{2}\left(2^b - 2\right) \\ -\infty, & \text{otherwise} \end{cases} \tag{3}$$

and

$$r_j^{\text{up}} = \begin{cases} r_j + \frac{\Delta}{2}, & \text{for } r_j \leq \frac{\Delta}{2}\left(2^b - 2\right) \\ +\infty, & \text{otherwise.} \end{cases} \tag{4}$$

3. DECOUPLED CHANNEL ESTIMATION

3.1. ML Estimation and EM Algorithm

The maximum a posteriori (MAP) estimate of the channel vetor $\mathbf{h}$, denoted by $\hat{\mathbf{h}}$, is given by:

$$\hat{\mathbf{h}} = \underset{\mathbf{h}\in\mathbb{R}^{MK}}{\arg\max}\, \mathcal{L}(\mathbf{h}). \tag{5}$$

Considering that $p_{\mathbf{r},\mathbf{h}}(\mathbf{r},\mathbf{h})$ is the joint probability density function (p.d.f.) of $\mathbf{r}$ and $\mathbf{h}$, then $\mathcal{L}(\mathbf{h}) = \ln p_{\mathbf{r},\mathbf{h}}(\mathbf{r},\mathbf{h})$ denotes the log-likelihood function. However, a closed-form solution for this maximization is not available. An alternative way is to employ the EM algorithm, which by exploiting the statistics of the hidden vector $\mathbf{y}$, offers effective ways to iteratively perform this maximization. Recognizing that the conditional p.d.f. of $\mathbf{y}$ given $\mathbf{r}$ is distributed as $p_{\mathbf{y}|\mathbf{h}} = \mathcal{N}\left(\mathcal{X}\mathbf{h}, \sigma^2\mathbf{I}_{ML}\right)$ and that the quantizer can be represented by the conditional probability mass function given by $\mathbb{I}_{D(\mathbf{r})}(\mathbf{y})$, where $D(\mathbf{r}) = \left(\mathbf{y} \in \mathbb{R}^{ML} \middle| r_i^{\text{lo}} \leq y_i \leq r_i^{\text{up}}; \forall i \in \{1,\ldots,ML\}\right)$. As a result, the joint p.d.f. of $\mathbf{r}$, $\mathbf{y}$ and $\mathbf{h}$ is given by:

$$\begin{aligned} &p_{\mathbf{r},\mathbf{y},\mathbf{h}}(\mathbf{r},\mathbf{y},\mathbf{h}) = p_{\mathbf{r}|\mathbf{y},\mathbf{h}}(\mathbf{r}|\mathbf{y},\mathbf{h})p_{\mathbf{y}|\mathbf{h}}(\mathbf{y}|\mathbf{h})p_{\mathbf{h}}(\mathbf{h}) \\ &= \mathbb{I}_{D(\mathbf{r})}(\mathbf{y}) \frac{1}{\left(2\pi^{\frac{1}{2}}\sigma\right)^{KM}} \exp\left(-\frac{\|\mathbf{y} - \mathcal{X}\mathbf{h}\|_2^2}{2\sigma^2} \right) p_{\mathbf{h}}(\mathbf{h}). \end{aligned} \tag{6}$$

The EM algorithm computes in the lth iteration the expectation of the MAP log-likelihood function

$$q\left(\mathbf{h}, \hat{\mathbf{h}}^{(l)}\right) = \mathbb{E}_{\mathbf{y}|\mathbf{r},\hat{\mathbf{h}}^{(l)}}\left[\ln p_{\mathbf{r},\mathbf{y},\mathbf{h}}(\mathbf{r},\mathbf{y},\mathbf{h})\right], \tag{7}$$

were $\hat{\mathbf{h}}^{(l)}$ represents the current estimate of $\mathbf{h}$. The maximization of this expectation with respect to $\mathbf{h}$ results in the new estimate, $\hat{\mathbf{h}}^{(l+1)}$, to be used in the next iteration of the algorithm:

$$\hat{\mathbf{h}}^{(l+1)} = \arg\max_{\mathbf{h}\in\mathbb{R}^{MK}} q\left(\mathbf{h}, \hat{\mathbf{h}}^{(l)}\right). \tag{8}$$

3.2. Decoupled EM-based ML Estimation

The proper choice of the training signal can lead to significant simplification of the estimation algorithm aforementioned. A training signal composed by pilot symbols temporally multiplexed, described as

$$\mathcal{X} = \sqrt{P_T}\begin{bmatrix}\mathbf{I}_{MK} & \mathbf{I}_{MK} & \cdots\end{bmatrix}^{\mathrm{T}}, \tag{9}$$

was considered, where the power of the pilot symbols is denoted by P_T, and $\mathcal{X} \in \mathbb{R}^{ML\times MK}$ is composed by the concatenation of $n_T = \frac{L}{K}$ identity matrices. By this choice of construction, $n_T \in \mathbb{N}^*$ is a requirement and corresponds to the number of pilot symbols associated to each h_i of $\mathbf{h}$. By considering this choice of training sequence and admitting independent noise vector components, the transformation of the MIMO channel estimation problem into a series of MK parallel SISO channels is straightforward. In order to accomplish this, the permutation matrix, $\mathbf{P}$ conveniently reorders the entries of $\mathbf{y}$. First, consider $\mathbf{P}_{h_1}$, with dimension $MKn_T \times n_T$, that operates to group the elements of $\mathbf{y}$ associated to h_1:

$$\mathbf{P}_{h_1} = \begin{bmatrix}\mathbf{e}_1^{(MK)} & \mathbf{e}_2^{(MK)} & \ldots & \mathbf{e}_{n_T}^{(MK)}\end{bmatrix}, \tag{10}$$

where $\mathbf{e}_k^{(MK)}, k \in \{1, 2, \ldots, n_T\}$ is a unit vector of length MKn_T with 1 in position kMK and 0 elsewhere. Subsequently, defining $\mathbf{P}_{h_i} = \text{circshift}(\mathbf{P}_{h_1}, i), i \in \{1, 2, ..., MK\}$, the $MKn_T \times MKn_T$ full permutation matrix $\mathbf{P}$ is given by:

$$\mathbf{P} = \begin{bmatrix}\mathbf{P}_{h_1} & \mathbf{P}_{h_2} & \ldots & \mathbf{P}_{h_{MK}}\end{bmatrix}^{\mathrm{T}}. \tag{11}$$

The permutation performs a fixed reordering pattern that operates on the unquantized $\mathbf{y}$ to produce $\mathbf{y}^d = \mathbf{P}\mathbf{y}$ or, interchangeably, on the observation $\mathbf{r}$, resulting in $\mathbf{r}^d$, which has the form:

$$\mathbf{r}^d = \begin{bmatrix}\mathbf{r}_1^d & \ldots & \mathbf{r}_{MK}^d\end{bmatrix}^{\mathrm{T}} = \mathbf{P}\mathbf{r} = \mathcal{Q}(\mathbf{P}\mathcal{X}\mathbf{h} + \mathbf{P}\mathbf{n}), \tag{12}$$

where the quantized data, $\mathbf{r}_i^d \in \mathcal{A}^{n_T}$, depends only on the ith channel component, as given by:

$$\mathbf{r}_i^d = \mathcal{Q}(\mathbf{P}_{h_i}\mathcal{X}\mathbf{h} + \mathbf{P}_{h_i}\mathbf{n}) = \mathcal{Q}\left(\sqrt{P_T}\mathbf{h}_i + \mathbf{P}_{h_i}\mathbf{n}\right) \tag{13}$$

where $\mathbf{h}_i = h_i\mathbf{1}_{n_T}$ and $\mathbf{1}_{n_T}$ is the vector of length n_T with unit entries.

Using (6), EM algorithm can be further developed. In the $(l+1)$th iteration, the estimate of $\mathbf{h}$ is calculated as:

$$\begin{aligned} \hat{\mathbf{h}}^{(l+1)} &= \underset{\mathbf{h}\in\mathbb{R}^{MK}}{\arg\min} \|\mathcal{X}\mathbf{h} - \hat{\mathbf{y}}\|^2 - \sigma^2 \ln p_{\mathbf{h}}(\mathbf{h}) \\ &= \underset{\mathbf{h}\in\mathbb{R}^{MK}}{\arg\min} \sqrt{P_T}\sum_i \left\| h_i\mathbf{1}_{n_T} - \hat{\mathbf{y}}_i^d \right\|^2 - \sigma^2 \sum_i \ln p_{h_i}(h_i) \end{aligned} \tag{14}$$

where $\hat{\mathbf{y}}_i^d = \mathbb{E}_{\mathbf{y}_i^d|\mathbf{r}_i^d,\hat{h}_i^{(l)}}\left[\mathbf{y}_i^d\right]$.

Choosing an uninformative p.d.f. of h_i, only the first summation is taken into consideration in (14). Thus, the determination of h_i corresponds to an ML problem, in which in the first step, $\hat{\mathbf{y}}_i^d$ is calculated as:

$$\begin{aligned} \hat{\mathbf{y}}_i^d &= \sqrt{P_T}\hat{\mathbf{h}}_i^{(l)} \\ &- \frac{\sigma}{\sqrt{2\pi}} \cdot \frac{e^{-\frac{\left(\mathbf{r}_i^{\text{up}} - \sqrt{P_T}\hat{\mathbf{h}}_i^{(l)}\right)^2}{2\sigma^2}} - e^{-\frac{\left(\mathbf{r}_i^{\text{lo}} - \sqrt{P_T}\hat{\mathbf{h}}_i^{(l)}\right)^2}{2\sigma^2}}}{\Phi\left(\frac{\mathbf{r}_i^{\text{up}} - \sqrt{P_T}\hat{\mathbf{h}}_i^{(l)}}{\sigma}\right) - \Phi\left(\frac{\mathbf{r}_i^{\text{lo}} - \sqrt{P_T}\hat{\mathbf{h}}_i^{(l)}}{\sigma}\right)}, \end{aligned} \tag{15}$$

where $\mathbf{h}_i^{(l)} = \mathbf{1}_{n_T} h_i^{(l)}$, and $(\mathbf{r}_i^{up}, \mathbf{r}_i^{lo}) \in \mathbb{R}^{n_T}$ contain elements that are, respectively, the upper bounds and lower bounds of the segments in which the entries of $\mathbf{r}_i^d$ belong. In the following, the ML estimate is calculated as:

$$\hat{h}_i^{(l+1)} = \left(P_T \mathbf{1}_{n_T}^{\mathrm{T}} \mathbf{1}_{n_T}\right)^{-1} \sqrt{P_T} \mathbf{1}_{n_T}^{\mathrm{T}} \hat{\mathbf{y}}_i^d$$
$$= \frac{1}{n_T \sqrt{P_T}} \mathrm{tr}\left[\mathrm{Diag}\left(\hat{\mathbf{y}}_i^d\right)\right], \quad i = 1, 2, \ldots, MK \qquad (16)$$

The thresholds are chosen to minimize the estimation error. The lower bound for the estimation error of an unbiased estimator, given by the CRLB is related to the Fisher information matrix, $\mathbf{J}(\mathbf{h})$, as $\mathbb{E}_{\mathbf{h}}\left[(\mathbf{h} - \hat{\mathbf{h}})(\mathbf{h} - \hat{\mathbf{h}})^{\mathrm{T}}\right] \succeq (\mathbf{J}(\mathbf{h}))^{-1}$. In turn, $\mathbf{J}(\mathbf{h})$ is given by:

$$\mathbf{J}(\mathbf{h}) = \mathbb{E}_{\mathbf{r}|\mathbf{h}}\left[\nabla_{\mathbf{h}} \ln p_{\mathbf{r},\mathbf{h}}(\mathbf{r}, \mathbf{h}) \nabla_{\mathbf{h}}^{\mathrm{T}} \ln p_{\mathbf{r},\mathbf{h}}(\mathbf{r}, \mathbf{h})\right]. \qquad (17)$$

The ith diagonal element of $\mathbf{J}(\mathbf{h})$, denoted $\mathbf{J}_i(h_i)$, is calculated as:

$$\mathbf{J}_i(h_i) = \frac{n_T P_T}{2\pi\sigma^2} \sum_{j=1}^{2^b} \frac{\left[e^{-\frac{\left(r_j^{\mathrm{up}} - \sqrt{P_T} h_i\right)^2}{2\sigma^2}} - e^{-\frac{\left(r_j^{\mathrm{lo}} - \sqrt{P_T} h_i\right)^2}{2\sigma^2}}\right]^2}{\Phi\left(\frac{r_j^{\mathrm{up}} - \sqrt{P_T} h_i}{\sigma}\right) - \Phi\left(\frac{r_j^{\mathrm{lo}} - \sqrt{P_T} h_i}{\sigma}\right)}. \qquad (18)$$

In the low SNR regime ($\sigma \gg \sqrt{P_T}|h_i|$), (18) is approximated by:

$$\mathbf{J}_i = \frac{n_T P_T}{2\pi\sigma^2} \sum_{j=1}^{2^b} \frac{\left[e^{-\frac{\left(r_j^{\mathrm{up}}\right)^2}{2\sigma^2}} - e^{-\frac{\left(r_j^{\mathrm{lo}}\right)^2}{2\sigma^2}}\right]^2}{\Phi\left(\frac{r_j^{\mathrm{up}}}{\sigma}\right) - \Phi\left(\frac{r_j^{\mathrm{lo}}}{\sigma}\right)}. \qquad (19)$$

From this approximation, the lower bound for the minimum estimation error in low SNR is solely dependent of the quantizer characteristics.

In Fig. 1 the normalized mean square error (NMSE) of the EM-ML channel estimation is shown, considering 2 and 3-bit quantizers with fixed step size Δ_{opt} chosen to maximize (19), as a function of $\mathrm{SNR} = \frac{P_T \mathbb{E}\left[h_i^2\right]}{\sigma^2}$. The channel matrix to be estimated is comprised by entries drawn from a Gaussian distribution with zero mean and unit variance. The NMSE is calculated as

$$\mathrm{NMSE} = \frac{1}{KM} \frac{\left\|\mathbf{h} - \hat{\mathbf{h}}\right\|^2}{\left\|\mathbf{h}\right\|^2}. \qquad (20)$$

The curves therein are non-monotonic and, moreover, achieve a minimum normalized mean estimation error for a certain SNR value and then experience a steep increase of the estimation error for greater SNR values, diverging from the estimator based on unquantized data.

4. FEEDBACK-CONTROLLED ESTIMATION WITH DITHERING

As shown in Fig. 1, the approximation (19) used to determine the quantizer step size employed in the EM-ML estimator resulted in a small gap in terms of NMSE to the estimation based on unquantized data in low SNR regime. On the other hand, the use of (19) degraded the estimator performance for high SNR values. As seen in (18), in this SNR region the step size that would maximize the Fisher information matrix (here referred as the optimum step size) is channel dependent.

In order to overcome this characteristic of quantized systems, a parameter estimation strategy is developed here to address the channel dependency on the quantizer. To accomplish this goal, an offset in $\mathbf{y}_i^d$ is introduced; moreover, making this offset sufficiently close to h_i would make the assumption $\sigma \gg \sqrt{P_T} h_i$ valid and therefore (18) becomes independent of channel power through a wider SNR range. This offset is subtracted from $\mathbf{y}_i^d$ in a feedback branch and the result

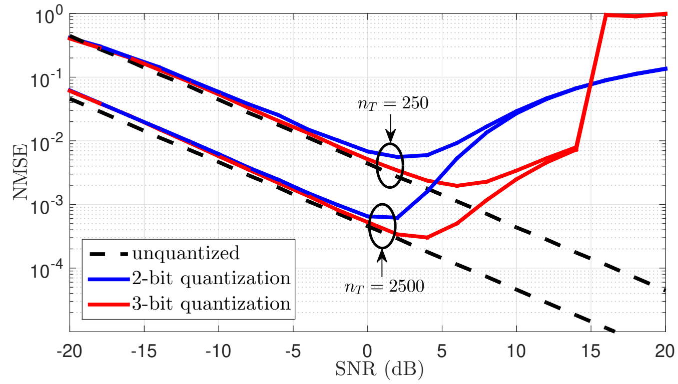

Fig. 1. Normalized mean square error of EM-ML estimator. $M = 4$, $K = 4$, $\Delta_{\mathrm{opt}} = 0.705$ (2-bit), and $\Delta_{\mathrm{opt}} = 0.485$ (3-bit), for $L = 1 \times 10^3$ and $L = 1 \times 10^4$.

is the input of the second step of the algorithm, turning the EM-ML estimator insensitive to the channel parameter for any SNR region. This kind of feedback controlled estimation algorithm was proposed in [10], where the performance loss to the unquantized-data based estimation in terms of the CRLB was shown to be constant.

Here the proposed feedback-controlled estimation algorithm tries to turn the performance loss to the unquantized data constant and also minimum by means of the use of an offset that turns the Fisher information channel independent as depicted in (19) which allows the computation of optimum step size for any SNR region.

The offset α_i is the multiplication the initial estimate of h_i by $\sqrt{P_T}$. This estimate, identified by $\hat{h}_i^{\mathrm{ini}}$, is obtained by an algorithm that can produce a sufficiently close estimate of $\mathbf{h}_i$, turning the optimum quantizer step size constant and independent of the channel parameters. Due to the straighforward implementation and good performance of EM-ML strategy, this approach is considered as the estimation algorithm to produce α_i. EM-ML estimation performance displayed in Fig. 1 shows the existence of an SNR value for which the NMSE is minimum, denoted SNR_α, and a noise variance $\sigma_\alpha^2 = \frac{P_T \mathbb{E}\left[h_i^2\right]}{\mathrm{SNR}_\alpha}$. In order to the algorithm to produce the best possible offset, considering the estimation operating at a given SNR value dithering can be applied to $\mathbf{y}_i^d$ so that the resulting SNR becomes SNR_α. In such case, an addition of noise signal, with p.d.f. given by:

$$p_{\mathbf{w}_i}(\mathbf{w}_i) = \begin{cases} \mathcal{N}\left(0, \sigma_{\mathrm{dith}}^2\right), & \mathrm{SNR} > \mathrm{SNR}_\alpha \\ \mathbb{I}_{\{0\}}(\mathbf{w}_i), & \text{otherwise} \end{cases} \qquad (21)$$

is added to $\mathbf{y}_i^d$ producing at the input of the quantizer $\mathbf{y}_i^{\mathrm{dith}} = \sqrt{P_T} h_i \mathbf{1}_{n_T} + \mathbf{n}_i + \mathbf{w}_i$, and $\sigma_{\mathrm{dith}}^2 = \sigma_{\mathrm{opt}}^2 - \sigma_{\mathrm{cur}}^2$. After the first phase of EM-ML, α_i is the output and the negative feedback branch produces $\mathbf{y}_i^\alpha = \mathbf{y}_i^{\mathrm{dith}} - \alpha_i \mathbf{1}_{n_T}$, which is such that $p_{\mathbf{y}_i^\alpha|\alpha_i}(\mathbf{y}_i^\alpha|\alpha_i)$ is approximately zero-mean, and then adequate to be the input of the second phase of EM-ML algorithm, that will produce an estimate of the residual $\hat{h}_i^\alpha$. Finally, the refined estimate of h_i, $\hat{h}_i$, is obtained as:

$$\hat{h}_i = \hat{h}_i^{\mathrm{ini}} + \hat{h}_i^\alpha. \qquad (22)$$

The proposed feedback-controlled EM-ML estimation algorithm (FC-EM-ML) is summarized in Algorithm 1.

5. NUMERICAL RESULTS

In order to evaluate the proposed algorithm shown in the previous section, the uplink of a multiuser MIMO system is considered where the BS is equipped with $\tilde{M} = 64$ transmit antennas receiving time-multiplexed pilots symbols from $K = 64$ single-antenna users. The

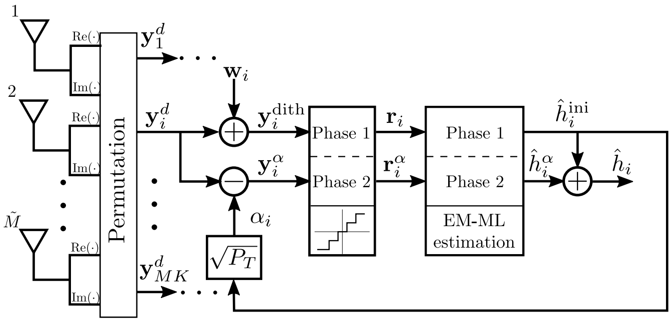

Fig. 2. Block diagram of the proposed estimation strategy.

Algorithm 1 FC-EM-ML estimation algorithm

Input: $P_T, \sigma^2, \text{SNR}_\alpha, n_T, \hat{\mathbf{h}}^{(0)}, \mathbf{P}$

Output: $\hat{\mathbf{h}}$

procedure DECOUPLEDESTIMATION

  Reorder $\mathbf{y}$ to obtain $\mathbf{y}_i^d$, as given by $\mathbf{P}$.

  Determine thresholds $\{\mathbf{r}_j^{\text{lo}}, \mathbf{r}_j^{\text{up}}\}_{j=1}^{2^b}$ that minimize (19).

  if SNR $>$ SNR$_\alpha$ **then** ▷ Dithering

    $\mathbf{w}_i \sim \mathcal{N}(0, \sigma^2_{\text{dith}})$

  else

    $\mathbf{w}_i = 0$

  end if

  $\mathbf{y}_i^{\text{dith}} = \mathbf{y}_i^d + \mathbf{w}_i$

  $\mathbf{r_i} = \mathcal{Q}(\mathbf{y}_i^{\text{dith}})$ ▷ Phase 1

  $\hat{h}_i^{\text{ini}} \leftarrow$ EXPMAX$(\mathbf{r}_i^{up}, \mathbf{r}_i^{lo}, \sigma^2, P_T, n_T, \hat{h}_i^{(0)})$

  $\mathbf{y}_i^{\alpha} = \mathbf{y}_i^d - \sqrt{P_T}\hat{h}_i^{\text{ini}}\mathbf{1}_{n_T}$

  $\mathbf{r}_\mathbf{i}^{\alpha} = \mathcal{Q}(\mathbf{y}_i^{\alpha})$ ▷ Phase 2

  $\hat{h}_i^{\alpha} \leftarrow$ EXPMAX$(\mathbf{r}_{\alpha,i}^{up}, \mathbf{r}_{\alpha,i}^{lo}, \sigma^2, P_T, n_T, 0)$

  $\hat{h}_i = \hat{h}_i^{\text{ini}} + \hat{h}_i^{\alpha}$

  return $\hat{h}_i$

end procedure

function EXPMAX$(\mathbf{r}_i^{up}, \mathbf{r}_i^{lo}, \sigma^2, P_T, n_T, \hat{h}_i^{(l=0)})$

  $\epsilon = \infty$

  while $\epsilon \geq \epsilon_{\max}$ **do**

    Calculate $\hat{\mathbf{y}}_i^d$ as given in (15). ▷ Expectation step

    $\hat{h}_i^{(l+1)} = \frac{1}{n_T\sqrt{P_T}}\text{tr}\left[\text{Diag}\left(\hat{\mathbf{y}}_i^d\right)\right]$ ▷ Maximiz. step

    $\epsilon = |\hat{h}_i^{(l+1)} - \hat{h}_i^{(l)}|$

    $l = l + 1$

  end while

  return $\hat{h}_i^{(l+1)}$

end function

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matrix $\mathbf{H}$ to be estimated models as a Rayleigh block fading channel, and comprises elements drawn from a real zero-mean and unit-variance Gaussian distribution. The estimation strategy was evaluated under different configurations, for 2-bit and 3-bit quantizers and varying the number of transmitted pilots: $n_T = 250$ and $n_T = 2{,}500$, resulting in $L = 16{,}000$ and $L = 160{,}000$ time-multiplexed pilot transmissions, respectively. The numerical results displayed herein are the outcome of 1,000 Monte Carlo runs, and the channel estimation performance is measured in terms of the NMSE.

Regarding the settings of the proposed FC-EM-ML algorithm, the initial guess of first-step EM-ML algorithm is set as $\hat{h}_i^{(0)} = \frac{1}{n_T\sqrt{P_T}}\text{tr}\left[\text{Diag}\left(\mathbf{r}_i\right)\right], \quad i = 1, 2, \ldots, MK$. Also, SNR$_\alpha$ is set to 0 dB for 2-bit quantizers, whereas in the 3-bit quantization case, SNR$_\alpha$ is equal to 4 dB and 2 dB for $n_T = 250$ and $n_T = 2500$, respectively. As in the EM-ML estimator, the fixed step sizes are evaluated to maximize (19), and resulted $\Delta_{\text{opt}} = 0.705$ (2-bit) and $\Delta_{\text{opt}} = 0.485$ (3-bit). Here, the performance of FC-EM-ML is compared with the Bussgang Minimum Mean Square Error (BMMSE) channel estimator [3], the Expectation Maximization Generalized Approximated Message Passing (EM-GAMP) estimator [12], and

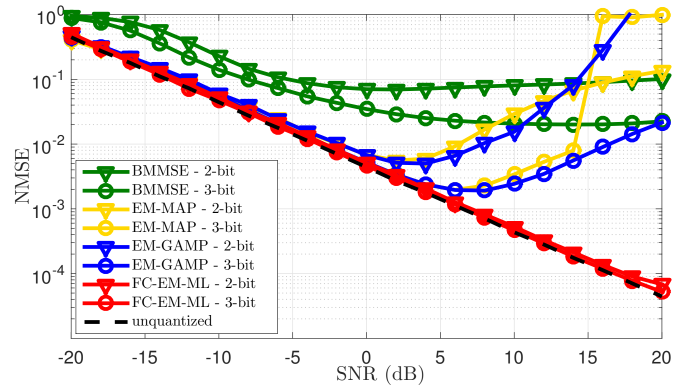

Fig. 3. Normalized mean square error of estimation algorithms. $M = 4$, $K = 4$, $\Delta_{\text{opt}} = 0.705$ (2-bit), and $\Delta_{\text{opt}} = 0.485$ (3-bit), for $L = 16 \times 10^3$.

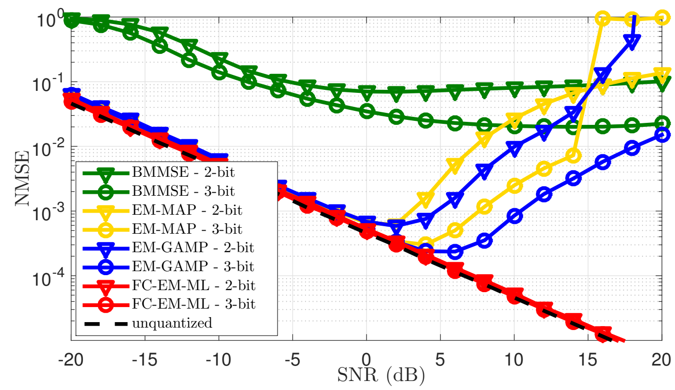

Fig. 4. Normalized mean square error of estimation algorithms. $M = 4$, $K = 4$, $\Delta_{\text{opt}} = 0.705$ (2-bit), and $\Delta_{\text{opt}} = 0.485$ (3-bit), for $L = 16 \times 10^4$.

the EM-MAP estimator. Finally, Figs. 3 and 4 depict the proposed feedback-controlled estimation algorithm performance in terms of NMSE as a function of SNR.

It can be seen that FC-EM-ML outperforms state-of-the-art approaches. Unlike EM-MAP and EM-GAMP estimators, the NMSE curves for both 2-bit and 3-bit quantizers are monotonically decreasing functions of the SNR. Moreover, the curves present slopes similar to those observed for estimation based directly on unquantized observations, for both numbers of pilots. Thus, the proposed algorithm can achieve constant and very small loss to the unquantized signal estimates, even in high SNR.

6. CONCLUSION

In this work, a MIMO channel estimation algorithm for few-bit quantized observations is proposed. The strategy is based on the double execution of EM-MAP algorithm, in which the result of the first stage generates an offset correction of the data to be inputted in the second stage, and then produce the final estimate. This makes the optimum step size that maximize the Fisher information to depend only on the quantizer characteristics, allowing its prior calculation for any SNR. Results show that the gap to the ideal case in terms of NMSE is maintained very small and constant even in the high SNR region.

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