# DECISION FEEDBACK SEMI-BLIND ESTIMATION ALGORITHM FOR SPECULAR OFDM CHANNELS

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## ABSTRACT

This paper deals with semi-blind channel estimation in Single-Input Single-Output (SISO) Orthogonal Frequency Division Multiplexing (OFDM) communications system. The proposed algorithm proceeds in two main stages. The first one addresses the pilot-based Time-Of-Arrival (TOA) estimation using subspace methods and then estimates the channel through its specular model. In the second stage, one considers a decision feedback equalizer that is used to refine the channel parameters estimates. Simulation results show that good performance can be reached with only one OFDM pilot symbol with appropriate windowing using only one iteration. A significant performance improvement as compared to the pilot-based TOA method is observed.

***Index Terms*—** SISO-OFDM, TOA estimation, Channel estimation, Subspace methods.

## 1. INTRODUCTION

Channel estimation is of crucial importance for implementing equalization scheme and coherent symbol detection in most wireless communications systems. Existing channel estimation techniques can be divided into three main classes: (i) blind (using only unknown data) [1], [2] ; (ii) pilot-based [3] and (iii) semi-blind methods using both training sequences and unknown data [4], [5].

Channel identification can be done by estimating the channel parameters, i.e. parametric channel estimation [6], or by estimating directly the channel coefficients [5]. Channel estimation approaches based on the parametric channel modeling require Time-Of-Arrival (TOA) (multipath delays) estimation. Many TOA estimation approaches, based on pilots, have been developed and long established in sensor array processing. Among these methods, one can refer to the subspace based techniques such as the MUltiple SIgnal Classification (MUSIC) algorithm [7], [8], root MUSIC (rootMUSIC) [9], [10], [11] and ESPRIT algorithm (Estimation of Signal Parameters via Rotational Invariance Technique) [12]. A new spectral search algorithm using the Partial Relaxation (PR) technique is proposed in [13], [14] and considered here in our work. The objective of this paper is to propose an efficient pilot-based and semi-blind channel estimation algorithms for SISO-OFDM system based on TOA estimation.

Firstly, the TOAs are estimated using only one OFDM pilot. The latter is used to generate a group of sub vectors, with an appropriate windowing, to which one can apply subspace methods to estimate the TOA. Secondly, one can incorporate the unknown data on the channel estimation process. The semi-blind TOA estimation is done using a Decision Feedback process [15], where a first estimate of the transmitted data is used with the existing pilot to enhance the TOA estimation performance.

## 2. SISO-OFDM SYSTEM COMMUNICATIONS MODEL

We consider OFDM signal transmission. Each OFDM symbol is composed of $K$ samples and is extended in time domain by the insertion of its last $L$ samples in its front considered as a Cyclic Prefix (CP). The CP duration is assumed to be greater than or equal to delay spread. The received signal considered in baseband, after removing the CP, is given in the time domain by the following equation:

$$y(t) = x(t) * \sum_{p=1}^{N} \bar{h}_i \mathrm{sinc}(t - \tau_i) + v(t), \tag{1}$$

where $x(t)$ is the transmitted signal, $N$ the number of multipaths, $\bar{h}_i$ and $\tau_i$ are respectively the complex gain and the time delay (TOA) of the $i$-th path.

After sampling the received OFDM signal (using the sampling rate $T_s$) and taking its $K$-point FFT, the received signal can be written as:

$$\mathbf{y} = \mathbf{x} \odot \left(\mathbf{A}(\tau)\bar{\mathbf{h}}\right) + \mathbf{v}, \tag{2}$$

where $\mathbf{y}$ (respectively $\mathbf{x}$) is the $K \times 1$ vector for each received (respectively transmitted) OFDM symbol. The $\odot$ symbol denotes the element-wise multiplication and $\bar{\mathbf{h}}$ the channel complex gain vector defined as $\bar{\mathbf{h}} = [\bar{h}_1, \cdots, \bar{h}_N]^T$. The matrix $\mathbf{A}(\tau) \in \mathbb{C}^{K\times N}$ is given by:

$$\mathbf{A}(\tau) = \begin{pmatrix} 1 & \cdots & 1 \\ e^{-\frac{2\pi i \tau_1}{KT_s}} & \cdots & e^{-\frac{2\pi i \tau_N}{KT_s}} \\ \vdots & \ddots & \vdots \\ e^{-\frac{2\pi i (K-1)\tau_1}{KT_s}} & \cdots & e^{-\frac{2\pi i (K-1)\tau_N}{KT_s}} \end{pmatrix}, \quad (3)$$

and $\mathbf{v}$ is an additive white Circular Gaussian noise satisfying $E\left[\mathbf{v}(k)\mathbf{v}(i)^H\right] = \sigma_{\mathbf{v}}^2 \mathbf{I}_K \delta_{ki}$ where $(.)^H$ is the Hermitian operator; $\sigma_{\mathbf{v}}^2$ the noise variance; $\mathbf{I}_K$ the identity matrix of size $K \times K$ and $\delta_{ki}$ the Kronecker delta.

Denote $\mathbf{h}$ the vector of the channel transfer function per subcarrier, defined as $\mathbf{h} = \mathbf{A}(\tau)\bar{\mathbf{h}}$. Equation (2) then becomes:

$$\mathbf{y} = \mathbf{x} \odot \mathbf{h} + \mathbf{v}. \quad (4)$$

## 3. PROPOSED CHANNEL ESTIMATION

This section concerns the proposed Decision Feedback (DF) semi-blind channel estimation algorithm. This algorithm is based on the concept of the decision feedback equalizer technique described in [15]. It is composed of two main stages summarized in Fig. 1. The first one, described in Section 3.1, provides a coarse estimate of the channel parameters that are used for its first stage equalization. The decision of this stage is then feeded back to the second one, developed in Section 3.2, to improve the channel estimation performance.

### 3.1. First stage: pilot-based TOA estimation

To identify the channel, the first stage focuses on the estimation issue of the time delays, i.e. $\tau_i$ due to multipaths, exploiting the known training sequences. The latter, also referred to as pilots, are organized according to a block-type pilot arrangement where $N_p$ ODFM symbols are dedicated to pilots and $N_d$ OFDM symbols are reserved for data [5]. The known training sequences are then used by the receiver to estimate the TOA.

Consider $\oslash$ the element wise division. Each element $\mathbf{y}_i$ of the received signal corresponding to the $i$-th OFDM symbol, is divided by the $i$-th OFDM pilot vector $\mathbf{x}_i$. An average is then performed on the $N_p$ division results as follows:

$$\mathbf{z} = \frac{1}{N_p}\sum_{i=1}^{N_p} \mathbf{y}_i \oslash \mathbf{x}_i = \mathbf{A}(\tau)\bar{\mathbf{h}} + \tilde{\mathbf{v}}. \quad (5)$$

$\tilde{\mathbf{v}}$ being the resulting average noise term. To apply the subspace methods, one needs a 'sufficient' number (larger than $N$) of data vectors satisfying the parametric model in (5). For that $N_G$ symbols, i.e. $N_G = K - G + 1$, are built from $\mathbf{z}$ using a shift windowing of size $N < G < K$. As proposed in [16], these shifted symbols are concatenated in one matrix $\mathbf{Z} = [\mathbf{z}_1, \cdots, \mathbf{z}_{N_G}] \in \mathbb{C}^{G\times N_G}$ given by:

$$\mathbf{Z} = [\mathbf{A}_1(\tau)\bar{\mathbf{h}}, \cdots, \mathbf{A}_{N_G}(\tau)\bar{\mathbf{h}}] + \tilde{\mathbf{V}}, \quad (6)$$

where $\tilde{\mathbf{V}}$ corresponds to the resulting shifted noise term. One observes that each matrix $\mathbf{A}_g(\tau) \in \mathbb{C}^{G\times N}$ is equal to $\mathbf{A}_1(\tau)$ multiplied by a diagonal matrix $\mathbf{D}^g \in \mathbb{C}^{N\times N}$. The latter is given by:

$$\mathbf{D}^g = \mathrm{diag}\left\{e^{-\frac{2\pi i (g-1)\tau_1}{KT_s}} \cdots e^{-\frac{2\pi i (g-1)\tau_N}{KT_s}}\right\}. \quad (7)$$

with $g = 1, \cdots, N_G$. Therefore, equation (6) is rewritten as:

$$\mathbf{Z} = \mathbf{A}_1(\tau)\mathbf{S} + \tilde{\mathbf{V}} \text{ with } \mathbf{S} = [\mathbf{D}^1\bar{\mathbf{h}}, \cdots, \mathbf{D}^{N_G}\bar{\mathbf{h}}]. \quad (8)$$

To estimate the TOAs, subspace techniques such as MUSIC [7, 8], rootMUSIC [9, 10, 11], ESPRIT [12], and the DOA estimation method using Partial Relaxation (PR) [13] are exploited and compared in the sequel. The received OFDM symbols are assumed to be i.i.d. and uncorrelated with the channel noise. An estimate of the covariance matrix $\hat{\mathbf{R}}$ of the processed signal $\mathbf{z}$ is given by:

$$\hat{\mathbf{R}} = \frac{1}{N_G}\mathbf{Z}\mathbf{Z}^H. \quad (9)$$

Based on the subspace approach, using eigenvalue decomposition (EVD), the covariance matrix is decomposed:

$$\hat{\mathbf{R}} = \hat{\mathbf{U}}\hat{\mathbf{\Lambda}}\hat{\mathbf{U}}^H = \left[\hat{\mathbf{U}}_s \middle| \hat{\mathbf{U}}_n\right] \begin{bmatrix} \hat{\mathbf{\Lambda}}_s & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{\Lambda}}_n \end{bmatrix} \begin{bmatrix} \hat{\mathbf{U}}_s^H \\ \hat{\mathbf{U}}_n^H \end{bmatrix}, \quad (10)$$

where the diagonal matrix $\hat{\mathbf{\Lambda}}_s$, of size $N \times N$, contains the largest eigenvalues $(\hat{\lambda}_1, \cdots, \hat{\lambda}_N)$; and $\hat{\mathbf{U}}_s \in \mathbb{C}^{G\times N}$ represents the signal subspace containing the corresponding principal eigenvectors of $\hat{\mathbf{R}}$. Similarly, the noise subspace $\hat{\mathbf{U}}_n \in \mathbb{C}^{G\times(G-N)}$ is associated with the $(G-N)$ smallest eigenvalues $\hat{\mathbf{\Lambda}}_n \in \mathbb{C}^{(G-N)\times(G-N)}$.

**Remark**: Note that instead of averaging the OFDM symbols in (5) followed by the windowing of vector $\mathbf{z}$, one can apply first the windowing on each OFDM symbol and average the results through the sample estimate covariance matrix in (9). The latter approach is more expensive but allows us to slightly improve the estimation accuracy of the TOA parameters.

The standard subspace method (MUSIC algorithm) exploits the orthogonality of the noise and signal subspaces to estimate the TOAs according to [7, 8]: $\min_\tau \|\hat{\mathbf{U}}_n^H \mathbf{a}(\tau)\|^2$ where $\mathbf{a}(\tau) = [1, e^{-\frac{2\pi i \tau_1}{KT_s}}, \cdots, e^{-\frac{2\pi i (G-1)\tau_1}{KT_s}}]^T$. To avoid this complex non-linear optimization problem, a simplified subspace approach using polynomial rooting (rootMUSIC) has been proposed in the literature [9, 10, 11]. On the other hand, to improve the estimation accuracy, one should minimize $\|\hat{\mathbf{U}}_n^H \mathbf{A}(\tau)\|^2$ which requires a joint estimation of all TOA parameters. This generally associated with a high computational complexity, and hence an alternative solution is the one given in [13], [14] using partial relaxation.

Once the TOA $\boldsymbol{\tau}$ is estimated using only pilots ($\hat{\boldsymbol{\tau}}_{\mathrm{OP}}$), the least-squares estimate of the complex gain vector $\bar{\mathbf{h}}$ and the global channel $\mathbf{h}$, using equation (5), is deduced as follows:

$$\hat{\bar{\mathbf{h}}} = \mathbf{A}(\hat{\boldsymbol{\tau}}_{\mathrm{OP}})^{\sharp}\mathbf{z}, \qquad \hat{\mathbf{h}}_{\mathrm{OP}} = \mathbf{A}(\hat{\boldsymbol{\tau}}_{\mathrm{OP}})\hat{\bar{\mathbf{h}}}, \tag{11}$$

where $(.)^{\sharp}$ denotes the pseudo inverse matrix. Once estimating the channel ($\hat{\mathbf{h}}_{\mathrm{OP}}$), a linear equalizer is performed and a hard decision is applied to obtain a first estimate of the transmitted signals ($\hat{\mathbf{X}}_d$). The latter, concatenated to the pilots, are exploited by the second stage as a new training sequence:

$$\mathbf{X}_p = [\mathbf{X}_p \;\; \hat{\mathbf{X}}_d] \in \mathbb{C}^{K\times(N_p+N_d)}. \tag{12}$$

### 3.2. Second stage: DF semi-blind channel estimation

The first stage feeds back the estimated data (equation (12)) to the second stage. This data is now considered as pilots and is then used to re-estimate the TOAs and channel (i.e. $\hat{\boldsymbol{\tau}}_{SB}$, $\hat{\mathbf{h}}_{SB}$).

Three DF approches are derived according to the involved TOA estimation algorithm, namely: the MUSIC algorithm (i.e MUSIC-DF), the rootMUSIC algorithm (rootMUSIC-DF) algorithm and the PR algorithm (PR-DF).

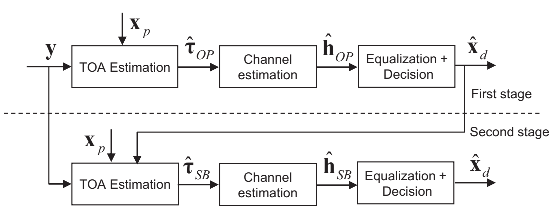



**Fig. 1**: DF semi-blind TOA estimation approach.

## 4. SIMULATION RESULTS

This section discusses the performance of the proposed DF semi-blind channel estimation algorithm. The pilot sequences correspond to those specified in the IEEE 802.11n standard [17]. The parameters of simulations are summarized in the next table. The estimation performance is measured in terms of the Normalized Root Mean Square Error (NRMSE), given by:

$$\mathrm{NRMSE} = \sqrt{\frac{1}{N_{mc}}\sum_{i=1}^{N_{mc}}\frac{\left\|\hat{\boldsymbol{\theta}}^{(i)}-\boldsymbol{\theta}\right\|^2}{\|\boldsymbol{\theta}\|^2}}, \tag{13}$$

where $N_{mc} = 100$ represents the number of Monte Carlo realizations and $\boldsymbol{\theta}$ represents the parameter under performance analysis ($\boldsymbol{\tau}$ or $\mathbf{h}$).

| **Parameters** | **Specifications** |
|---|---|
| Channel model | IEEE 802.11n |
| Frequency sampling | $\frac{1}{T_s} = 20MHz$ |
| Number of multipaths | $N$ = 4 |
| Time Of Arrivals | $\tau = [2\ 6\ 10\ 15]T_s$ |
| Number of paths | $N$ = 4 |
| Number of pilot OFDM symbols | $N_p$ |
| Number of data OFDM symbols | $N_d$ |
| Pilot signal power | $P_{x_p}$ = 23 dBm |
| Data signal power | $P_{x_d}$ = 20 dBm |
| Number of sub-carriers | $K$ = 512 |
| Cyclic prefix | $L$ = 64 |
| Size of the partitioned symbol | $G$ = 128 |
| Number of equivalent symbols | $N_G$ = 385 |

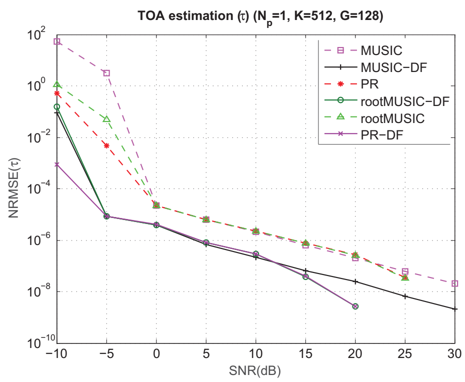
**Table 1**: Simulation parameters.

Fig. 2 illustrates the channel estimation performance versus the Signal to Noise Ratio (SNR). Fig. 2a compares the performance between MUSIC, Root-MUSIC and PR estimators when using only the first stage with one OFDM pilot symbol and the complete scheme DF semi-blind (i.e. two stages when the data symbols are feeded back).

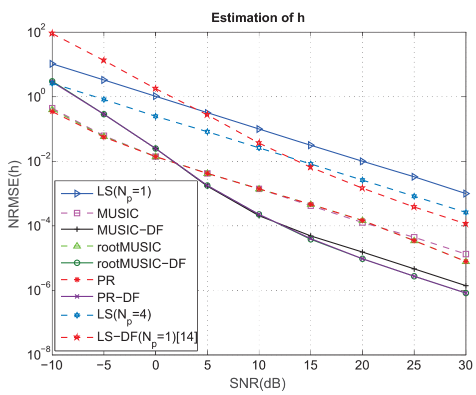
In Fig. 2a, one can observe that using one OFDM pilot leads to a good estimation of TOA. This estimation is enhanced when DF technique (referred to as MUSIC-DF, rootMUSIC-DF and PR-DF) is applied even at low SNR.

Fig. 2b presents the channel estimation performance. The proposed approach performs well compared to the Least Squares (LS) estimator even if the latter uses 4 OFDM pilot symbols instead on 1. Moreover the DF semi-blind approach behaves good even from relatively low SNRs. Note that at very low SNRs (lower than 2dB), the DF approach becomes inefficient due to the ill channel equalization and hence the high decision error rate in that context. In the same plot, we present a comparison between the proposed approaches and the LS-DF algorithm proposed in [15], where we can observe that a significant gain is obtained in favor of the two methods presented in this paper. While considering the Symbol Error Ratio (SER) plots of Fig. 3, one can see also a non-negligible performance gain in favor of the proposed DF-based approach.

At a given SNR=-5dB, Fig. 4 shows the influence of increasing the number of pilot OFDM symbols $N_p$ in the estimation process on the performance of the pilot-based TOA (i.e. first stage). Indeed the TOA estimation performance is improved when the number of pilot OFDM symbols $N_p$ is increased. Note that using few pilots ($N_p < 3$) PR gives better performance than the two other subspace methods and from $N_p = 4$ the three estimators have the same behavior.




(a)




(b)

**Fig. 2**: NRMSE versus $SNR$ when $N_p$=1 and $N_d$=8: (a) TOA ($\boldsymbol{\tau}$) estimation ; (b) global channel estimation ($\mathbf{h}$).

Fig. 5 illustrates the impact of increasing the number of data OFDM symbols in the DF semi-blind channel estimation, on the performance of the TOA estimation compared to pilot-based TOA estimation (i.e. only the first stage) represented by horizontal lines. As can be seen, only very few data symbols are needed to achieve most of the semi-blind performance gain.

## 5. CONCLUSION

This paper addressed two channel estimation approaches using TOA pilot-based and semi-blind estimation. The first one exploits pilot symbols with a subspace estimation method and the second employed semi-blind approach using a decision feedback (DF). Simulation results showed that good performance can be reached using only one OFDM pilot symbol with appropriate windowing. Extension to the MIMO case is under consideration.

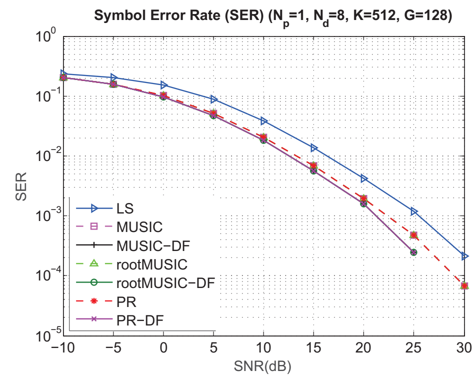
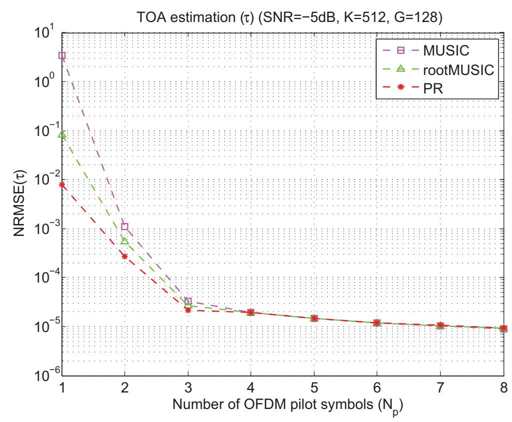



**Fig. 3**: SER versus $SNR$ when $N_p$=1 and $N_d$=8.




(a)

**Fig. 4**: TOA estimation performance versus the number of pilot OFDM symbols $N_p$ for $SNR = -5dB$.

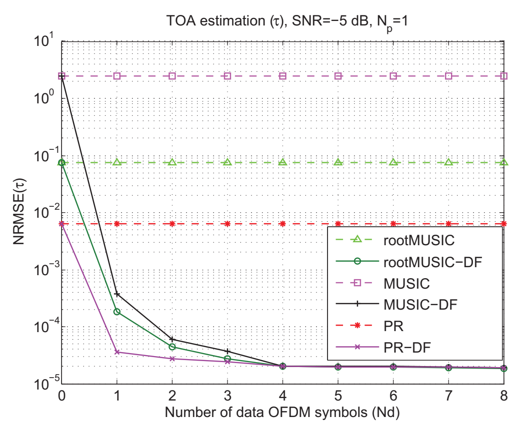



**Fig. 5**: TOA estimation performance versus the number data OFDM symbols $N_d$ when $N_p$=1 and $SNR$=-5 dB.

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