OPTIMUM SAMPLING FOR PACKET ASSISTED ROUND TRIP TIME MEASUREMENT

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ABSTRACT

The paper presents a sampling method that dramatically improves the measurement accuracy of the round trip time (RTT) measurement in digital communication systems. For state of the art methods, the accuracy of time measurements in discrete time systems and low noise scenarios is limited to the sampling period T. If the measurement is repeated m-times, the upper bound of the quantization mean square error (MSE) is $\sim \frac{1}{m}$. The proposed sampling scheme reduces the upper bound of the MSE to $\sim \frac{1}{m^2}$. This exceptional feature is enabled by applying a prime relation between sampling rate and bit rate of the communication system.

Index Terms— round trip time, measurement accuracy, localization, WSN, synchronization, internet of things, industrial wireless sensor network, IWSN, sampling

1. INTRODUCTION AND RELATED WORK

RTT measurements, obtained in a two-way message exchange [1], are the basic building block for precise delay-compensated synchronization [2, 3], time-based ranging [4] and localization [5], or security applications such as distance bounding protocols [6]. Moreover, RTT measurements are used for RFID-tag identification [7] and for ranging in radar applications [8].

In conventional systems, in which the sampling frequency is chosen as a multiple to the symbol rate (and hence of the packet rate), the MSE of the RTT from a single measurement is bound by the sampling period T with $\frac{T^2}{12}$, if the jitter of the RTT is sufficiently small. Based on standard estimation theory, by repeating the measurements m-times at m random instants of time, the MSE can be reduced by $\frac{1}{m}$. If there is no strict random sampling, this reduction does not hold, which is usually the case if the sampling rate is an integer multiple of the bit rate. Therefore, the standard methods can only be substantially improved by increasing the sample rate.

In this work we propose to use a different sampling scheme that guarantees reduction of the MSE with $\frac{1}{m^2}$. Based on the results given in the paper, the sampling frequency has to be chosen with a fractional prime relation to the bit rate. This sampling method allows to reduce the MSE substantially without the need to increase the sampling rate. Additionally, we derive a tight upper bound of the MSE independent of the value of the RTT, which is not possible with conventional systems.

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Obviously, the new method supports multi node ranging algorithms focusing on RTT methods and in combination with ranging. According to [9] it is possible to avoid the lack of scalability resulting from pairwise sequentially ranging. Also in wired network topology the round trip time plays a major role in synchronizing nodes on a pairwise base [3]. The presented method is even more relevant for underwater environments using an asymmetrical round trip path to achieve node localization on large-scale. In those applications, energy-efficient and accurate RTT estimates without prior time synchronization are demanded [10]. Also, it has to be mentioned that RTT measurements in 5G systems [11] are profiting immediately from the proposed method, because the sampling period is limited to coarse time steps. In those cases the measurement jitter is very small compared to the sampling time and the proposed method is improving the measurement accuracy.

In the next sections we show that it is possible to reduce the MSE without increasing the sampling rate of the measurement system. In Sec. 2 the description of the standard estimator is given, including the quantization error and the MSE of the jitter. It is followed by Sec. 3, where the proposed sampling method is described and the upper bounds on its MSE is derived. In Sec. 4 simulations are given which confirm the theoretically derived bounds.

2. NON PERIODIC RTT MEASUREMENT

The RTT is the time of flight,i.e., the time a signal takes to be sent out by the transmitter and to return from a passive RFID-tag, radar target, other reflecting device or an active receiver and transmitter combination re-sending the signal. In order to increase the RTT estimation accuracy, m measurement pulses are conducted with a measurement period of NT, where N is an integer number and T is the sampling clock period. Thereby, the jth pulse deviates from the nominal pulse period NT/m by a random phase $\tau_j \in [0, NT/m - G[$, referred to as non periodic measurement (NPM). G represents a guard time to guarantee that consecutive pulses do not overlap.

In a noiseless system, the RTT is constant of duration D. Due to channel imperfections, jitter adds on the RTT which is modeled as follows: Consider the signal $x(t)$ that is equal to 1 between transmission and reception of a pulse in a noiseless system, i.e., $x(t) = 1$ for $t \in [j\,NT/m + \tau_j, j\,NT/m + \tau_j + D]$, $j = 1, \ldots, m$, and $x(t) = 0$ otherwise. To account for jitter on the RTT, we consider

$$r(t) = x(t) + n(t)\,, \tag{1}$$

where $n(t)$ is a rectangular signal with non-zero amplitude at rising and falling edges of $x(t)$, as depicted in Fig. 1. This yields a duration of the jth RTT of

$$d(j) = D + w_+[j] + w_-[j]\,,$$

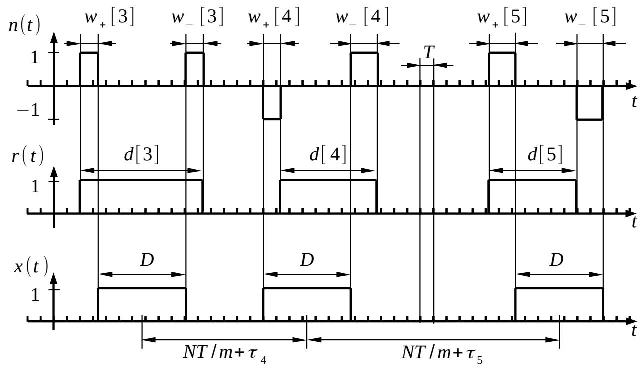

Fig. 1. Three individual pulse triggered RTT measurements corrupted by noise.

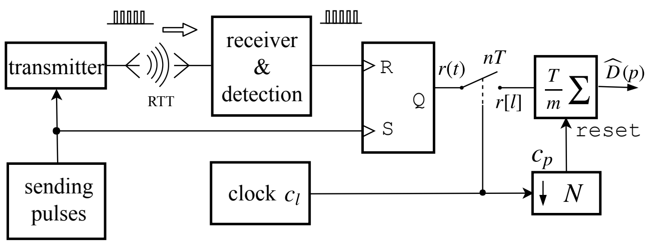

Fig. 2. RTT measurement with randomly sent pulses

where $w_+[j]$ and $w_-[j]$ are the jitter noise parts at the positive respectively negative edge. According to [5], the probability density of the jitter length is a Gaussian distribution $w_+[j], w_-[j] \sim \mathcal{N}(0, \sigma)$.

For estimation, $r(t)$ is sampled with the sample clock c_l with period T at discrete time steps $r[l] = r(lT)$. The pth averaged estimate over m RTT duration's is

$$\widehat{D}(p) = \frac{T}{m}\sum_{i=0}^{N-1} r[i+pN] = \frac{T}{m}\sum_{i=0}^{N-1} x[i+pN] + \frac{T}{m}\sum_{i=0}^{N-1} n[i+pN]\,. \tag{2}$$

This estimator is summarized in Fig. 2.

For the averaging process in (2), all m pulses and jitter contributions are treated as one sum. Nevertheless, there exist individual pulses and we analyze them individually before averaging is applied. Additionally, the time discrete integration introduces a quantization error $d_q[j]$. Considering all noise contributions, a measured value of the jth RTT is given by

$$d[j] = d(j) + d_q[j] = D + d_q[j] + w_+[j] + w_-[j]\,. \tag{3}$$

Hence, the measurement error is

$$d_e[j] = d[j] - D = d_q[j] + w_+[j] + w_-[j]\,. \tag{4}$$

As commonly known, according to the Cramer Rao bound [12], the MSE of the estimator in (2) with the identical estimation error (4) yields for m independent and uncorrelated measurements is

$$\overline{e^2} = \frac{1}{m}E\left\{d_q^2[j]\right\} + \frac{1}{m}E\left\{w_+^2[j]\right\} + \frac{1}{m}E\left\{w_-^2[j]\right\}\,. \tag{5}$$

To characterize the quantization error $E\left\{d_q^2[j]\right\}$ of the noise free RTT, we consider the fractional part D_f of the jth measurement of the sampled version of $x(t)$ (see Fig. 3), given by

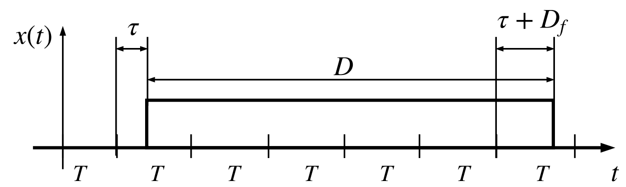

Fig. 3. Timing diagram of a random RTT measurement with discrete time alignment.

$$D_f = \frac{D}{T} - \left\lfloor \frac{D}{T} \right\rfloor. \tag{6}$$

Assuming that $\tau = \mod(\tau_j, T)$ is uniformly distributed in $[0, T[$, we have as MSE of the quantization error [13]

$$E\left\{d_q^2[j]\right\} = \overline{d_q^2} = \frac{1}{T}\int_{\tau=0}^{T} d_q^2(t+\tau)d\tau \tag{7}$$

with $d_q = D_f$ for $\tau \in [0, T - D_f[$ and $d_q = T - D_f$ for $\tau \in [T - D_f, T[$. Therefore the MSE is

$$\begin{aligned} E\left\{d_q^2[j]\right\} = \overline{d_q^2} &= \frac{1}{T}\int_{\tau=0}^{T-D_f} D_f^2 d\tau + \frac{1}{T}\int_{\tau=T-D_f}^{T} (T-D_f)^2 d\tau \\ &= D_f T - D_f^2. \end{aligned} \tag{8}$$

The additive jitter is assumed identically distributed at the positive and negative edge and the MSE comes to

$$E\left\{w_-^2[j]\right\} = E\left\{w_+^2[j]\right\} = E\left\{w^2[j]\right\}.$$

Obviously, the jitter is also quantized which results in additive quantization noise with an MSE of $E\{w_q^2[j]\}$ (which is neglected in most orther works), i.e.,

$$E\left\{w^2[j]\right\} = \sigma^2 + E\{w_q^2[j]\}\,. \tag{9}$$

In Fig. 4 the probability density functions of different jitter noise processes are plotted to illustrate its influence on quantization. Within the interval of $\pm\frac{T}{2}$ the jitter is not altering the quantization of $d[j]$ given in (8). If the absolute value of the jitter is larger than $\frac{T}{2}$, a quantization error occurs with an error amplitude T. Hence, we have to evaluate the probability P_e of false decisions based on the quantized jitter.

The PDF of the quantized RTT has two parameters, the amplitude x of the random phase and the jitter amplitude ϕ. The PDF of the jitter amplitude is

$$p_w(\phi) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\phi - x)^2}{2\sigma^2}} \tag{10}$$

with ϕ as jitter amplitude and x as uniformly distributed random variable to consider all different realizations of the sampled RTT. The PDF of x is a uniform distribution with $p_x(v) = \frac{1}{T}$ for $v \in [0, T[$ and $p_x(v) = 0$ otherwise. As jitter and quantization are statistically independent, the joint PDF is

$$p_{w,v}(v, \phi) = p_w(\phi)p_x(v) = \frac{1}{T}\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\phi - v)^2}{2\sigma^2}}. \tag{11}$$

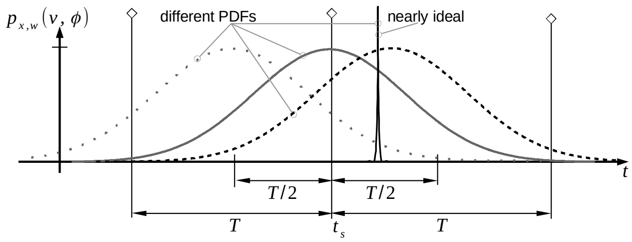

Fig. 4. PDF of four different jitter measurements with $\sigma < T$.

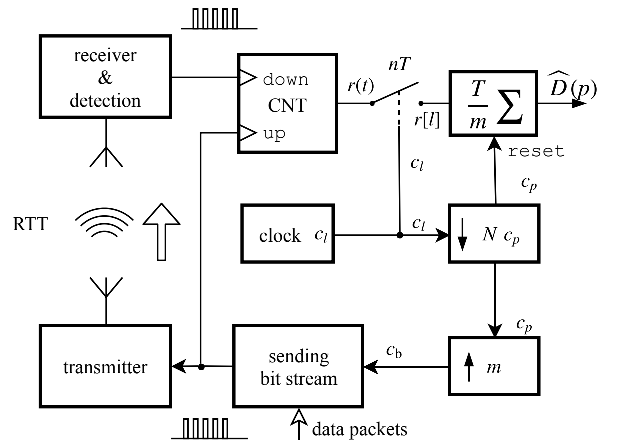

Fig. 5. Packet assisted RTT estimator

Thus, the error amplitude can be either $+T$ or $-T$ and the error probability for both cases is given by

$$P_{e_r}(v) = \int\limits_{t_s+\frac{T}{2}}^{\infty} p_{w,x}(\phi, v)d\phi = \frac{1}{T}\frac{1}{\sigma\sqrt{2\pi}} \int\limits_{t_s+\frac{T}{2}}^{\infty} e^{-\frac{(\phi-v)^2}{2\sigma^2}} d\phi \tag{12}$$

with a quantizer threshold at $\pm\frac{T}{2}$. Furthermore, all existing realizations with a relevant amplitude variation between $-\frac{T}{2}$ and $\frac{T}{2}$ add up to the error probability of

$$\begin{aligned} P_e(\sigma, T) =& \int\limits_{t_s-\frac{T}{2}}^{t_s+\frac{T}{2}} P_{e_r}(v)dv = \int\limits_{t_s-\frac{T}{2}}^{t_s+\frac{T}{2}} \int\limits_{t_s+\frac{T}{2}}^{\infty} p_{w,x}(\phi, v)d\phi dv \\ =& \frac{1}{T} \int\limits_{t_s-\frac{T}{2}}^{t_s+\frac{T}{2}} \frac{1}{\sigma\sqrt{2\pi}} \int\limits_{t_s+\frac{T}{2}}^{\infty} e^{-\frac{(\phi-v)^2}{2\sigma^2}} d\phi dv. \end{aligned} \tag{13}$$

After proceeding the integration, the error probability is

$$P_e(\sigma, T) = \frac{1}{2} + \frac{\sigma}{\sqrt{2\pi}T} - \frac{\sigma e^{-\frac{T^2}{2\sigma^2}}}{\sqrt{2\pi}T} - \frac{1}{2}\mathrm{erf}\left(\frac{T}{\sqrt{2}\sigma}\right). \tag{14}$$

With (14) we find that the quantization MSE in (9) is

$$E\{w_q^2[j]\} = P_e(\sigma, T)T^2\,. \tag{15}$$

Plugging (8), (9) and (15) in (5) yields the closed form expression of the MSE with

$$\overline{e^2} = \frac{1}{m}\left(D_f T - D_f^2 + 2\sigma^2 + 2P_e(\sigma, T)T^2\right) \tag{16}$$

$$\begin{aligned} =& \frac{1}{m}\left(D_f T - D_f^2 + 2\sigma^2\right) + \\ & \frac{2T^2}{m}\left(\frac{1}{2} + \frac{\sigma(1 - e^{-\frac{T^2}{2\sigma^2}})}{\sqrt{2\pi}T} - \frac{1}{2}\mathrm{erf}\left(\frac{T}{\sqrt{2}\sigma}\right)\right) \end{aligned} \tag{17}$$

which is a tight bound for NPM.

3. PERIODIC RTT MEASUREMENT

In contrast to the previous NPM method, where m individual random phase pulses are sent, we introduce a method using m periodic pulses in this section. The pulses are grouped in a data packet of length m in which is repeatedly sent at a specific packet rate c_p. In standard digital communication systems each pulse in the packet represents one bit or symbol. The proposed method can be applied when the receiver is able to detect the reception of each single symbol in the packet. In Fig. 5, this periodic RTT measurement is depicted. It shows a system which sends out data packets with a length of m-bits. Each of these bits has to be separately detectable with identifiable edges at each bit. To be more general, the m individual pulses don't have to be necessarily information bits, any marker or other transmit pulse can be used if the time of flight is not mutually influenced. When the pulses are sent out, the a counter is incremented, triggered by the rising edge of each bit. Subsequently, the individual bits are received after RTT. Each rising edge of the received bit stream decrements the counter in Fig. 5. Transmitter, receiver and counter are implemented in the time continuous domain and therefore sampling has to be performed at the counter output $r(t)$. The sampling of the continuous-time measurement signal $r(t)$ is performed at $t = lT$ with the sampling clock rate c_l and therefore $r[l] = r(lT)$. We consider $c_l > c_b$ throughout the section. The m bits are sent at a rate c_b, which is lower than the sampling rate c_l. Moreover, c_l and c_b are coupled by a prime relation. To achieve this prime relation, the clock c_l is reduced by a prime number N to the packet rate c_p. Based on this setup we use periodic prime relation measurement (PPM) as abbreviation. In order to generate the bit rate c_b, the intermediate clock c_p is interpolated by a non prime factor m, which is the length of the packet. Usually m is a power of two, originated by period doubling. The summation in Fig. 5 is calculating the duration of all RTT within one measurement period N by summing up and multiplying the result with $\frac{T}{m}$. The summation is restarted after N sample intervals at a rate of c_p, i.e., for each packet. With this rate, estimates $\widehat{D}$ of the RTT are calculated by (2) where p is the packet counter. As in NPM, the sampled signal is modeled with additive noise $r[l] = x[l] + n[l]$. In the following we use (2) with $\frac{T}{m} = \frac{P}{N}$ for the estimation of PPM by

$$\widehat{D}(N, P, T) = P\frac{1}{N}\sum_{l=0}^{N-1} r[l] = \frac{P}{N}\sum_{l=0}^{N-1} x[l] + \frac{P}{N}\sum_{l=0}^{N-1} n[l].$$

Following [14], we use the DC component resulting from a Fourier transform to describe the sum over $x[l]$ in the previous equation and we write

$$\widehat{D}(N,P,T) = \frac{P}{N}X(0) + \frac{P}{N}\sum_{l=0}^{N-1} n[l]. \qquad (18)$$

The estimation is corrupted by jitter (3) and aliasing [14]. Therefore, the MSE of the RTT estimator is derived as function of D, P, T and N. Hence, it is given as the expectation of the squared error

$$\begin{aligned}\overline{e^2(D,N,P,T)} &= E\{|D - \widehat{D}(N,P,T)|^2\} \qquad (19)\\ &= E\left\{\left|D - \frac{P}{N}X(0)\right|^2\right\} + E\left\{\left|\frac{P}{N}\sum_{l=0}^{N-1} n[l]\right|^2\right\}\\ &= \overline{e_D^2(D,N,P,T)} + \overline{e_n^2(D,N,P,T)}.\end{aligned}$$

Because $n[l]$ are independent random variables similar to those of (2) we can reuse the previous results for the error contribution of the jitter. In [14] the error analysis of the duration estimation is performed and we can apply these results to the PPM as

$$\overline{e_D^2(D,N,P,T)} = \sum_{k=-\infty\setminus 0}^{\infty} \frac{P^2}{N^2}\sin^2\left(\frac{\pi k D}{P}\right)\frac{\sin^2(\pi N k \frac{T}{P})}{(k\pi)^2 \sin^2(\pi k \frac{T}{P})}. \qquad (20)$$

With $m = \frac{NT}{P}$ and $m \in \mathbb{N}$ (20) yields

$$\overline{e_D^2(D,N,P,T)} = \sum_{k=-\infty\setminus 0}^{\infty} \frac{P^2}{N^2}\sin^2\left(\frac{\pi k D}{P}\right)\frac{\sin^2(\pi m k)}{(k\pi)^2 \sin^2(\pi k \frac{m}{N})}, \qquad (21)$$

where the numerator $\sin(\pi k m)$ is zero for all k. Obviously, this holds for all terms of the sum, if the denominator is not zero, hence N has to be a prime number. If $k \in \mathbb{Z} \setminus 0$ and N is a finite number, still infinite contributions at multiples of N exist, where a fraction of terms in (21) result in a division zero by zero. This was analyzed further in [14]. There, we have shown that $\overline{e_D^2}$ can be rewritten as

$$\overline{e_D^2(m,T,D)} = \sum_{i=1}^{\infty} \frac{2T^2}{(i\,m\pi)^2}\sin^2\left(\frac{\pi i\, mD}{T}\right). \qquad (22)$$

Because $|\sin()|$ is always less than one, an upper bound for (22) is given with $|\sin()| = 1$ which implies that the argument of $\sin()$ is $\pi \frac{D}{T} i m = \frac{\pi}{2} + v\pi = \pi\frac{1}{2}(1+2v)$. It holds if $\frac{D}{T} = \frac{1}{2}$ and i is an odd number with $v \in \mathbb{N}$. Finally, we derive an upper bound for (22) with odd contributions of i by

$$\overline{e_D^2(m,T,D)} \le \frac{2T^2}{(m\pi)^2}\sum_{v=1}^{\infty}\frac{1}{(1+2v)^2} = \frac{T^2}{4m^2} \qquad (23)$$

which yields the error bound if N is a prime number

$$\overline{e^2} \le \frac{T^2}{4m^2} + \frac{2\sigma^2}{m} + \frac{2T^2}{m}\left(\frac{1}{2} + \frac{\sigma(1-e^{-\frac{T^2}{2\sigma^2}})}{\sqrt{2\pi}T} - \frac{1}{2}\mathrm{erf}\left(\frac{T}{\sqrt{2}\sigma}\right)\right). \qquad (24)$$

When (17) and (24) are compared, it can be clearly seen that the quantization error decays $\sim \frac{1}{m^2}$ instead of $\sim \frac{1}{m}$ if a periodic measurement with N being a prime number (PPM) is used.

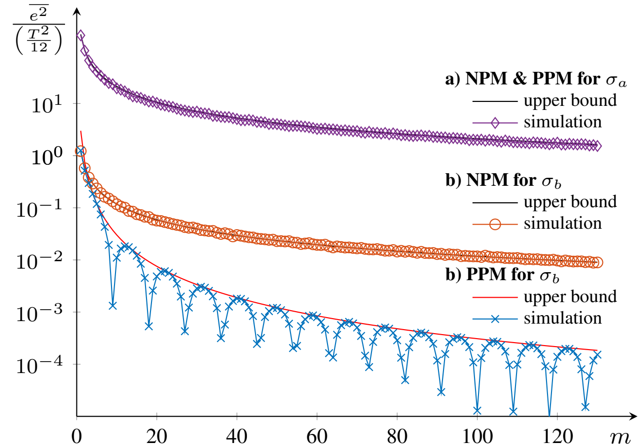

Fig. 6. Comparison of MSE normalized to $T^2/12$ for the two presented methods with repeated measurements under different noise conditions.

4. SIMULATION RESULTS

Monte Carlo simulations are used to compare NPM and PPM for RTT estimation with respect to the MSE . The simulation setup covers two different jitter levels. Each setup was performed with 5000 random phases and the RTT was kept constant with $D = 7.11011056786\,T$.

a) The root mean square error (RMS) of the jitter is ten times the quantization error, $\sigma_a = 10T/\sqrt{12}$. In this case NPM and PPM perform similar. The jitter is dominating the resulting MSE and it decays with $\frac{1}{m}$ according to (17) respectively (24), c.f. Fig. 6.

b) Both estimators are used in a simulation where the jitter is very low compared to the quantization noise, $\sigma_b = 10^{-3}T/\sqrt{12}$. In this case, the difference between estimators NPM and PPM are remarkable because the quantization noise is limited due to (24). The upper bound of the periodic estimator PPM is decaying with $\frac{1}{m^2}$ and the simulated results are sometimes even better because some values of m are optimal to estimate individual values of RTT. A comprehensive study on the optimal selection of m will be part of further research.

5. CONCLUSION

In this work we proposed a sampling scheme to significantly increase the measurement accuracy of RTT measurements in quantized systems. For profound analysis, we derived closed form expressions of the achievable MSE and conducted numerical evaluations. Most notably, for m repeated measurements in low jitter conditions, the proposed scheme has a scaling behavior of the MSE of $\frac{1}{m^2}$ instead of $\frac{1}{m}$ for conventional systems. Thereby, conventional systems in which the sampling clock and the pulse repetition rate have a integer relation, require a pulse transmission at random phase in order to achieve its best scaling behavior of $\frac{1}{m}$. The here proposed method in which the sampling clock is chosen as fractional prime relation to the pulse repetition, always outperforms conventional systems and additionally supports periodic pulse transmission as in packet-based communication systems. Hence, it is well suited to improve many existing RTT measurement systems.

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