DISTRIBUTED NONCOHERENT TRANSMIT BEAMFORMING FOR DENSE SMALL CELL NETWORKS

Quang-Doanh Vu, Le-Nam Tran†, and Markku Juntti**

* Centre for Wireless Communications, University of Oulu, P.O.Box 4500, FI-90014, Finland;
Email: {doanh.vu, markku.juntti}@oulu.fi.
† School of Electrical and Electronic Engineering, University College Dublin, Ireland. Email: nam.tran@ucd.ie.

ABSTRACT

Beamforming design for downlink coordinated multi-point (CoMP) transmission of dense small cell networks is considered. The goal is to maximize the weighted sum rate of the system subject to constraints on maximum transmit power at macro cell and small cell base stations (BSs). Here we focus on the noncoherent joint transmission technique, since it does not require tight network synchronization, and thus is more practically appealing. The considered optimization problem is intractable, thus we adopt the inner approximation (IA) to efficiently derive a locally optimal solution. Then, we develop a distributed implementation of the proposed IA based algorithm, based on the alternating direction method of multipliers (ADMM). More explicitly, in the distributed algorithm, the convex approximate subproblems obtained by the IA principles are solved by the ADMM procedure. As the result, the beamforming vectors are computed locally at the BSs. Numerical results are provided to confirm the validity of the proposed algorithms.

Index Terms— Dense small cell networks, noncoherent joint transmission, beamforming design, distributed algorithm, inner approximation, alternating direction method of multipliers.

1. INTRODUCTION

The development of wireless communications has risen the challenge of tremendous mobile data demand for the upcoming mobile networks. It is foreseen that the total mobile traffic will be a tenfold increase by 2023 compared to 2017 [1]. In addition, the future mobile networks are going to introduce various service categories such as enhanced mobile broadband, massive machine type communications (MTC), and ultra-reliable low-latency communications (URLLC) to support the diverse communications requirements [2]. Many technologies have been developed in order to meet these challenges, and dense small cell deployment is considered as one of the promising solutions [3]. With densification of low-cost base stations (BSs), not only the existing spectrum is exploited efficiently by the spatial reuse, but also the energy efficiency is enhanced due to the short-range wireless transmission [4]. Additionally, the proximity of the cells to the users can support low latency services as well as guarantee quality of experience (QoE) [5].

However, since the cells become closer, efficiently managing inter-cell interference is one of the keys to a successful implementation [6]. To this end, coordinated multi-point (CoMP) transmission is widely considered in small cell networks where the users receive data from multiple BSs, i.e., joint transmission (JT) [7, 8]. In this regard, the coherent JT would be the best technique improving network capacity. However, the requirement of strict network synchronization accuracy makes it difficult and high-cost to practically deploy. Therefore, the *noncoherent JT* has been received growing attention since it requires less strict network synchronization accuracy compared to coherent counterpart [8–11].[1] In particular, beamforming designs for minimizing the power consumption subject to users' minimum data rate were investigated in [8, 10]. The problem of load balancing was investigated in [9]. Very recently, beamforming design for the problem of weighted sum rate (WSR) under the limited fronthaul capacity for cloud radio access network was considered in [11]. Commonly, the prior works mentioned above mainly focus on centralized approaches, and the distributed algorithms are not available, while distributed architecture is more preferred in dense small cell networks [3].

In this paper, we study the beamforming design problem in the dense small cell networks where the BSs cooperate to serve a set of users following the noncoherent JT protocol. The target is to maximize the WSR under the constraints of maximum transmit power at BSs. The WSR problem is considered since it is general enough to include other performance measures such as spectral efficiency and guarantee the quality of services for the users (via appropriate weights) as special cases [12]. Assuming that the beamforming computation is performed at the BSs due to the distributed architecture of the small cell networks, we propose an efficient distributed algorithm where the beamforming vectors are determined locally (at BSs). Toward the goal, we first equivalently transform the nonconvex WSR problem to which the inner approximation (IA) framework is applied to achieve an efficient solution [13, 14]. Then, motivated by the appreciated success of the alternating direction method of multipliers (ADMM) in designing distributed algorithms reported in recent publications [15–18], we also rely on ADMM to decompose the convex approximated problems obtained by the IA-based method into subproblems which can be solved locally at the BSs. More explicitly, at each ADMM iteration, macro BS solves a second order cone program (SOCP) while each of the small cell BSs solves a quadratically constrained quadratic program (QCQP).

Notation: Bold lower and upper case letters represent vectors and matrices, respectively; $||\cdot||_2$ represents the ℓ_2 norm; $|\cdot|$ represents the absolute value; $\mathbb{C}^{x\times y}$ represents the space of complex

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[1] We note that the term 'noncoherent' used in this work refers to the signal procesing coordination amongs BSs. It is distinguished from which refering to the tranmission without knowledge of channel state information at both sides of tranmitter and receiver.

matrices of dimensions given in superscript; $\mathcal{CN}(0,a)$ denotes a complex Gaussian random variable with zero mean and variance a; $\mathrm{Re}\{\cdot\}$ represents real part of the argument. $\mathbf{X}^{\mathrm{T}}$ and $\mathbf{X}^{\mathrm{H}}$ stand for the transpose and the Hermitian transpose of $\mathbf{X}$, respectively.

2. SYSTEM MODEL

We consider a region covered by a macro cell BS and a set of K small cell BSs. Let us denote by $\mathcal{K}=\{1,2,...,K+1\}$ the set of all BSs where $\{1\}$ refers to the macro BS and the rest the small cell BSs. BS k is equipped with $M_k > 1$ antennas. The BSs simultaneously serve a set of N single-antenna users, denoted by $\mathcal{N}=\{1,2,...,N\}$ under the same frequency band. Let B_k and U_i denote BS k and user i, respectively. Herein, we assume that the BSs collaborate using nonconherent JT. That means a user receives data from multiple BSs, and the information for a specific user is coded independently at individual BSs [8]. Particularly, let s_{ik} and $\mathbf{v}_{ik} \in \mathbb{C}^{M_k\times 1}$ be the normalized symbol and the beamforming vector at B_k for U_i, respectively. Let $\mathbf{h}_{ik} \in \mathbb{C}^{1\times M_k}$ (row vector) denote the channel between B_k and U_i. The signal received at U_i under the assumption of flat channels is given by

$$r_i = \sum_{k\in\mathcal{K}} \mathbf{h}_{ik}\mathbf{v}_{ik}s_{ik} + \sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{N}\setminus\{i\}} \mathbf{h}_{ik}\mathbf{v}_{jk}s_{jk} + z_i \quad (1)$$

where $z_i \sim \mathcal{CN}(0,\sigma_i^2)$ is the additive white Gaussian noise. The first and second sum in the right side of (1) are the desired signal and the interference, respectively. In this paper, users are assumed to deploy successive interference cancellation technique to detect its own signal and treat signal of other users as noise, the *effective* signal-to-interference-plus-noise ratio (SINR) at U_i can be written as [8][2]

$$\gamma_i = \frac{\sum_{k\in\mathcal{K}}|\mathbf{h}_{ik}\mathbf{v}_{ik}|^2}{\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{N}\setminus\{i\}}|\mathbf{h}_{ik}\mathbf{v}_{jk}|^2+\sigma_i^2}. \quad (2)$$

We note that $\{\gamma_i\}_{i\in\mathcal{N}}$ are achieved without phase synchronization between BSs. We also remark that $\{\gamma_i\}_{i\in\mathcal{N}}$ are the aggregated instantaneous SINR, i.e., the total information received at U_i is $\log(1+\gamma_i)$ [8] (the derivation of γ_i can also be found in [11]).

We aim at designing beamforming vectors $\{\mathbf{v}_{ik}\}_{i,k}$ so that the WSR is maximized under the constraints of transmit power budget at the BSs. Mathematically, the problem of interest reads

$$\underset{\{\mathbf{v}_{ik}\}}{\text{maximize}} \sum_{i\in\mathcal{N}} w_i\log(1+\gamma_i) \quad (3a)$$

$$\text{subject to } \sum_{i\in\mathcal{N}}\mathbf{v}_{ik}^{\mathrm{H}}\mathbf{v}_{ik} \le P_k, \forall k\in\mathcal{K} \quad (3b)$$

where $w_i > 0$ represents the priority of U_i, and P_k is the maximum total transmit power available to B_k. Note that the SINR in (2) is nonconvex with $\{\mathbf{v}_{ik}\}$, which makes problem (3) mathematically intractable [8].

3. PROPOSED DISTRIBUTED ALGORITHMS

3.1. Efficient Solution for (3) via Inner Approximation

We adopt the IA framework to develop efficient solution to (3), which was inspired by our earlier work in [19]. To do so, we first reveal the hidden convexity in (3) by transforming the problem into the following equivalent form

$$\underset{\{\mathbf{v}_{ik}\},\{\mu_i\}}{\text{maximize}} \sum_{i\in\mathcal{N}} w_i\log(1+\mu_i) \quad (4a)$$

$$\text{subject to } \frac{\sum_{k\in\mathcal{K}}|\mathbf{h}_{ik}\mathbf{v}_{ik}|^2}{\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{N}\setminus\{i\}}|\mathbf{h}_{ik}\mathbf{v}_{jk}|^2+\sigma_i^2} \ge \mu_i, \forall i\in\mathcal{N}, \quad (4b)$$

$$\sum_{i\in\mathcal{N}}\mathbf{v}_{ik}^{\mathrm{H}}\mathbf{v}_{ik} \le P_k, \forall k\in\mathcal{K} \quad (4c)$$

where $\{\mu_i \ge 0\}$ are newly introduced variables. Problem (4) is equivalent to (3) in the sense of the optimal set, which is justified by the fact that, the constraints in (4b) are active at the optimality. Now, the nonconvex parts lie in (4b), which can be equivalently rewritten as

$$(4\text{b}) \Leftrightarrow \begin{cases} \sum_{k\in\mathcal{K}}|\mathbf{h}_{ik}\mathbf{v}_{ik}|^2/u_i \ge \mu_i \\ \sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{N}\setminus\{i\}}|\mathbf{h}_{ik}\mathbf{v}_{jk}|^2+\sigma_i^2 \le u_i \end{cases} \quad (5)$$

where $\{u_i > 0\}$ are slack variables. Note that the quadratic-over-linear function is convex with the involved variables. In light of the IA approach, we use a first order approximation as a convex lower bound to derive an approximate convex problem. More explicitly, let $(\{\mathbf{v}_i^{(t)}\},\{\mu_i^{(t)}\},\{u_i^{(t)}\})$ be a feasible point, then the approximate problem is

$$\underset{\{\mathbf{v}_{ik}\},\{\mu_i\},\{u_i\}}{\text{maximize}} \sum_{i\in\mathcal{N}} w_i\log(1+\mu_i) \quad (6a)$$

$$\text{subject to } \sum_{k\in\mathcal{K}}(\mathrm{Re}\{\mathbf{g}_{ik}^{(t)}\mathbf{v}_{ik}\} - A_{ik}^{(t)}u_i) \ge \mu_i, \forall i\in\mathcal{N}, \quad (6b)$$

$$\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{N}\setminus\{i\}}|\mathbf{h}_{ik}\mathbf{v}_{jk}|^2+\sigma_i^2 \le u_i, \forall i\in\mathcal{N}, \quad (6c)$$

$$\sum_{i\in\mathcal{N}}\mathbf{v}_{ik}^{\mathrm{H}}\mathbf{v}_{ik} \le P_k, \forall k\in\mathcal{K} \quad (6d)$$

where $\mathbf{g}_{ik}^{(t)} = (2/u_i^{(t)})(\mathbf{v}_{ik}^{(t)})^{\mathrm{H}}\mathbf{h}_{ik}^{\mathrm{H}}\mathbf{h}_{ik}$ and $A_{ik}^{(t)} = (|\mathbf{h}_{ik}\mathbf{v}_{ik}^{(t)}|/u_i^{(t)})^2$.

3.1.1. SOCP-Based Approximation

Since each w_i is generally different, problem (6) containing a mix of exponential cones and second order cones SOCs is treated as a generic convex program. For computational benefit, we provide an SOCP approximation by using a lower bound of the logarithm function given as $\log(1+\mu_i) \ge \log(1+\mu_i^{(t)}) + 2 - 2\sqrt{(1+\mu_i^{(t)})/(1+\mu_i)}$. The validity of the bound according to the IA principles is justified in [20, Sec. III-E]. With the bound and by introducing new variables $\{\delta_i\}$ and $\{\pi_i\}$, we arrive at the following SOCP approximation

$$\underset{\substack{\{\mathbf{v}_{ik}\},\{\mu_i\},\{u_i\}\\ \{\delta_i\},\{\pi_i\}}}{\text{minimize}} \sum_{i\in\mathcal{N}} \tilde{w}_i^{(t)}\pi_i \quad (7a)$$

$$\text{subject to } \| [2,(\pi_i-\delta_i)] \|_2 \le (\pi_i+\delta_i),$$
$$1+\mu_i \ge \delta_i^2, \delta_i \ge 1, \forall i\in\mathcal{N}, \quad (7b)$$

$$(6\text{b}), (6\text{c}), (6\text{d}) \quad (7c)$$

where $\tilde{w}_i^{(t)} = w_i\sqrt{1+\mu_i^{(t)}}$. Successively solving (7) and updating $(\{\mathbf{v}_i^{(t)}\},\{\mu_i^{(t)}\},\{u_i^{(t)}\})$ by the optimal solution of (7), we obtain the sequence $\{\sum_{i\in\mathcal{N}} w_i\log(1+\mu_i^{(t)})\}_{t=1}^{\infty}$ which is nondecreasing and converges, since the feasible set of (4) is compact and nonempty [14, Corollary 2.3].

[2] We note that the decoding order has no impact on γ_i [11].

3.2. ADMM-Based Distributed Algorithm

We now develop a decentralized algorithm for the above IA-based solution where the beamforming vectors $\{\mathbf{v}_{ik}\}_i$ are designed locally at B_k. The central idea is to use the ADMM to solve the convex approximation subproblem (7), which eventually leads to a distributed implementation. Towards the goal, we first equivalently rewrite (7) into the form amenable to the ADMM as

$$\underset{\substack{\{\mathbf{v}_{ik}\},\{\mu_i\},\{u_i\},\{\delta_i\},\\ \{\pi_i\},\{\hat{q}_{ik}\},\{\tilde{q}_{ik}\},\{q_{ik}\},\\ \{\hat{y}_{ik}\},\{\tilde{y}_{ik}\},\{y_{ik}\}}}{\text{minimize}} \quad \sum_{i\in\mathcal{N}} \tilde{w}_i^{(t)}\pi_i \tag{8a}$$

subject to

$$\mathrm{Re}\{\mathbf{g}_{i1}^{(t)}\mathbf{v}_{i1}\} - \sum_{k\in\mathcal{K}} A_{ik}^{(t)}u_i + \sum_{k\in\bar{\mathcal{K}}}\tilde{y}_{ik} \geq \mu_i, \forall i\in\mathcal{N}, \tag{8b}$$

$$\mathrm{Re}\{\mathbf{g}_{ik}^{(t)}\mathbf{v}_{ik}\} \geq \hat{y}_{ik}, \forall i\in\mathcal{N}, \forall k\in\bar{\mathcal{K}} \tag{8c}$$

$$\sum_{j\in\mathcal{N}\setminus\{i\}} |\mathbf{h}_{i1}\mathbf{v}_{j1}|^2 + \sum_{k\in\bar{\mathcal{K}}}\tilde{q}_{ik} + \sigma_i^2 \leq u_i, \forall i\in\mathcal{N}, \tag{8d}$$

$$\sum_{j\in\mathcal{N}\setminus\{i\}} |\mathbf{h}_{ik}\mathbf{v}_{jk}|^2 \leq \hat{q}_{ik}, \forall i\in\mathcal{N}, \forall k\in\bar{\mathcal{K}} \tag{8e}$$

$$\hat{q}_{ik} = q_{ik}, \hat{y}_{ik} = y_{ik}, \forall i\in\mathcal{N}, \forall k\in\bar{\mathcal{K}} \tag{8f}$$

$$\tilde{q}_{ik} = q_{ik}, \tilde{y}_{ik} = y_{ik}, \forall i\in\mathcal{N}, \forall k\in\bar{\mathcal{K}} \tag{8g}$$

$$\text{(6d)}, \text{(7b)} \tag{8h}$$

where $\{\hat{q}_{ik}\}$, $\{\tilde{q}_{ik}\}$, $\{q_{ik}\}$, $\{\hat{y}_{ik}\}$, $\{\tilde{y}_{ik}\}$ and $\{y_{ik}\}$ are newly introduced variables for decomposing (6b) and (6c) into constraints which will be handled locally at macro cell BS and small cell BSs; $\bar{\mathcal{K}} = \{2, ..., K+1\}$ is the set of small cell BSs. For presentation convenience, let us denote by $\tilde{\mathbf{x}} \triangleq \{\{\mathbf{v}_{i1}\}_i, \{\mu_i\}_i, \{u_i\}_i, \{\delta_i\}_i, \{\pi_i\}_i, \{\tilde{q}_{ik}\}_{i,k}, \{\tilde{y}_{ik}\}_{i,k}\}$ the local variables at the macro cell BS, and define its local feasible set as

$$\tilde{\mathcal{S}} \triangleq \{\tilde{\mathbf{x}}|\text{(8b)}, \text{(8d)}, \text{(7b)}, \sum_{i\in\mathcal{N}} \mathbf{v}_{i1}^{\mathrm{H}}\mathbf{v}_{i1} \leq P_1\}. \tag{9}$$

Similarly, let us denote by $\hat{\mathbf{x}}_k \triangleq \{\{\mathbf{v}_{ik}\}_i, \{\hat{q}_{ik}\}_i, \{\tilde{y}_{ik}\}_i\}$ the local variables at small cell BS B_k, $k\in\bar{\mathcal{K}}$, and define its local feasible set as

$$\hat{\mathcal{S}}_k = \{\hat{\mathbf{x}}_k|\mathrm{Re}\{\mathbf{g}_{ik}^{(t)}\mathbf{v}_{ik}\} \geq \hat{y}_{ik}, \forall i\in\mathcal{N}, \\ \sum_{j\in\mathcal{N}\setminus\{i\}} |\mathbf{h}_{ik}\mathbf{v}_{jk}|^2 \leq \hat{q}_{ik}, \forall i\in\mathcal{N}, \sum_{i\in\mathcal{N}} \mathbf{v}_{ik}^{\mathrm{H}}\mathbf{v}_{ik} \leq P_k\}. \tag{10}$$

With these notations, we can rewrite (8) as

$$\underset{\substack{\{\tilde{\mathbf{x}}\in\tilde{\mathcal{S}}\},\{\hat{\mathbf{x}}_k\in\hat{\mathcal{S}}_k\}\\ \{q_{ik}\},\{y_{ik}\}}}{\text{minimize}} \quad \sum_{i\in\mathcal{N}} \tilde{w}_i^{(t)}\pi_i \tag{11a}$$

$$\text{subject to } \tilde{\boldsymbol{\pi}} = \tilde{\boldsymbol{\phi}}, \hat{\boldsymbol{\pi}}_k = \hat{\boldsymbol{\phi}}_k, \forall k\in\bar{\mathcal{K}} \tag{11b}$$

where $\tilde{\boldsymbol{\pi}} \triangleq \{\{\tilde{q}_{ik}\}_{i,k}, \{\tilde{y}_{ik}\}_{i,k}\}$, $\hat{\boldsymbol{\pi}}_k \triangleq \{\{\hat{q}_{ik}\}_i, \{\tilde{y}_{ik}\}_i\}$; $\tilde{\boldsymbol{\phi}}$ and $\hat{\boldsymbol{\phi}}_k$ are the rearranged vectors from the same set of global variables $(\{q_{ik}\}_{i,k}, \{y_{ik}\}_{i,k})$. The augmented Lagrangian function of (11) is

$$\mathfrak{L}(\{\tilde{\mathbf{x}}\}, \{\hat{\mathbf{x}}_k\}, \{q_{ik}\}, \{y_{ik}\}; \{\boldsymbol{\xi}\}, \{\boldsymbol{\rho}_k\}) = \\ \Big(\sum_{i\in\mathcal{N}} \tilde{w}_i^{(t)}\pi_i + \boldsymbol{\xi}^{\mathrm{T}}(\tilde{\boldsymbol{\pi}} - \tilde{\boldsymbol{\phi}}) + \frac{m}{2}||\tilde{\boldsymbol{\pi}} - \tilde{\boldsymbol{\phi}}||_2^2\Big) \\ + \sum_{k\in\bar{\mathcal{K}}} \Big(\boldsymbol{\rho}_k^{\mathrm{T}}(\hat{\boldsymbol{\pi}}_k - \hat{\boldsymbol{\phi}}_k) + \frac{m}{2}||\hat{\boldsymbol{\pi}}_k - \hat{\boldsymbol{\phi}}_k||_2^2\Big) \tag{12}$$

where $\boldsymbol{\xi}$ and $\{\boldsymbol{\rho}_k\}_{k\in\bar{\mathcal{K}}}$ are the Lagrangian multipliers and m is the penalty parameter. In what follows, we present the variable update at iteration $(j+1)$ of the ADMM.

3.2.1. Update Local Variables

Let $\boldsymbol{\xi}^{(j)}$, $\tilde{\boldsymbol{\phi}}^{(j)}$, and $\{\boldsymbol{\rho}_k^{(j)}\}$ be the values obtained at iteration j. Macro cell BS B_1 updates its local variables $\tilde{\mathbf{x}}$ by solving the following SOCP

$$\underset{\tilde{\mathbf{x}}\in\tilde{\mathcal{S}}}{\text{minimize}} \ (\boldsymbol{\xi}^{(j)})^{\mathrm{T}}(\tilde{\boldsymbol{\pi}} - \tilde{\boldsymbol{\phi}}^{(j)}) + \frac{m}{2}||\tilde{\boldsymbol{\pi}} - \tilde{\boldsymbol{\phi}}^{(j)}||_2^2 + \sum_{i\in\mathcal{N}} \tilde{w}_i^{(t)}\pi_i. \tag{13}$$

Small cell BS B_k updates its local variables $\hat{\mathbf{x}}_k$ by solving the following QCQP

$$\underset{\hat{\mathbf{x}}_k\in\hat{\mathcal{S}}_k}{\text{minimize}} \ (\boldsymbol{\rho}_k^{(j)})^{\mathrm{T}}(\hat{\boldsymbol{\pi}}_k - \hat{\boldsymbol{\phi}}_k^{(j)}) + \frac{m}{2}||\hat{\boldsymbol{\pi}}_k - \hat{\boldsymbol{\phi}}_k^{(j)}||_2^2. \tag{14}$$

3.2.2. Update Global Variables

The global variables $\{q_{ik}\}$ and $\{y_{ik}\}$ are updated via finding the minimum of the following quadratic function extracted from (12)

$$G^{(j)}(\{q_{ik}\}, \{y_{ik}\}) \triangleq \\ \sum_{k\in\bar{\mathcal{K}}}\sum_{i\in\mathcal{N}} \Big([\boldsymbol{\xi}^{(j)}]_{q_{ik}}(\tilde{q}_{ik}^{(j+1)} - q_{ik}) + \frac{m}{2}(\tilde{q}_{ik}^{(j+1)} - q_{ik})^2 \\ + [\boldsymbol{\xi}^{(j)}]_{y_{ik}}(\tilde{y}_{ik}^{(j+1)} - y_{ik}) + \frac{m}{2}(\tilde{y}_{ik}^{(j+1)} - y_{ik})^2 \\ + [\boldsymbol{\rho}_k^{(j)}]_{q_{ik}}(\hat{q}_{ik}^{(j+1)} - q_{ik}) + \frac{m}{2}(\hat{q}_{ik}^{(j+1)} - q_{ik})^2 \\ + [\boldsymbol{\rho}_k^{(j)}]_{y_{ik}}(\hat{y}_{ik}^{(j+1)} - y_{ik}) + \frac{m}{2}(\hat{y}_{ik}^{(j+1)} - y_{ik})^2\Big) \tag{15}$$

where $[\boldsymbol{\xi}^{(j)}]_{q_{ik}}$ is the element in $\boldsymbol{\xi}^{(j)}$ corresponding to constraint $\tilde{q}_{ik} = q_{ik}$; similar definition is applied to $[\boldsymbol{\xi}^{(j)}]_{y_{ik}}$, $[\boldsymbol{\rho}_k^{(k)}]_{q_{ik}}$ and $[\boldsymbol{\rho}_k^{(k)}]_{y_{ik}}$. The closed-form of the minimizer of (15) is given as

$$q_{ik}^{(j+1)} = \frac{([\boldsymbol{\xi}^{(j)}]_{q_{ik}} + m\tilde{q}_{ik}^{(j+1)}) + ([\boldsymbol{\rho}_k^{(j)}]_{q_{ik}} + m\hat{q}_{ik}^{(j+1)})}{2m} \tag{16}$$

$$y_{ik}^{(j+1)} = \frac{([\boldsymbol{\xi}^{(j)}]_{y_{ik}} + m\tilde{y}_{ik}^{(j+1)}) + ([\boldsymbol{\rho}_k^{(j)}]_{y_{ik}} + m\hat{y}_{ik}^{(j+1)})}{2m}. \tag{17}$$

Updating $q_{ik}^{(j+1)}$ and $y_{ik}^{(j+1)}$ can be done at the macro cell BS or small cell BS B_k. We herein suppose that B_1 updates the variables, which acquires the two scalars $\frac{[\boldsymbol{\rho}_k^{(j)}]_{q_{ik}} + m\hat{q}_{ik}^{(j+1)}}{2m}$ and $\frac{[\boldsymbol{\rho}_k^{(j)}]_{y_{ik}} + m\hat{y}_{ik}^{(j+1)}}{2m}$ from B_k.

3.2.3. Update Lagrangian Multipliers

The Lagrangian multipliers are updated as follows

$$\boldsymbol{\xi}^{(j+1)} = \boldsymbol{\xi}^{(j)} + m(\tilde{\boldsymbol{\pi}}^{(j+1)} - \tilde{\boldsymbol{\phi}}^{(j+1)}) \tag{18}$$

$$\boldsymbol{\rho}_k^{(j+1)} = \boldsymbol{\rho}_k^{(j)} + m(\hat{\boldsymbol{\pi}}_k^{(j+1)} - \hat{\boldsymbol{\phi}}_k^{(j+1)}). \tag{19}$$

where $\boldsymbol{\xi}^{(j+1)}$ is determined at B_1 while $\boldsymbol{\rho}_k^{(j+1)}$ is determined at B_k, $k\in\bar{\mathcal{K}}$. For this, B_k requires $q_{ik}^{(j+1)}$ and $y_{ik}^{(j+1)}$ from B_1.

Based on the presented variable updates, we propose a distributed algorithm outlined in Algorithm 1. It includes two stages: the inner stage is the ADMM procedure solving IA subproblems; the outer stage is the IA feasible point update using the values obtained in the inner stage (Step 12). The values obtained at the last iteration of the ADMM at IA iteration t are used for initializing ADMM procedure at IA iteration $t+1$ (Step 13). The initial values for the algorithm (Step 1) will be specified in Section 4.

Algorithm 1 Decentralized procedure solving (3)

Initialization: Set small ϵ_{IA}, $t := 1$ and $j := 1$, choose initial values for $(\{\mathbf{v}_{ik}^{(0)}\}, \{\mu_i^{(0)}\}, \{u_i^{(0)}\})$ and $(\{q_{ik}^{(0)}\}, \{y_{ik}^{(0)}\}; \boldsymbol{\xi}^{(0)}, \{\boldsymbol{\rho}_k^{(0)}\})$.
repeat {Outer stage (SCA procedure)}
  B_k receives $u_i^{(t)}$ from B_1 to form $\mathbf{g}_{ik}^{(t)}$; and B_1 receives scalar $|\mathbf{h}_{ik}\mathbf{v}_{ik}^{(t)}|$ from B_k to form $A_{ik}^{(t)}$, $k \in \bar{\mathcal{K}}$.
  repeat {Inner stage (ADMM procedure)}
    B_1 updates $\tilde{\mathbf{x}}^{(j+1)}$ by solving (13); B_k, $k \in \bar{\mathcal{K}}$, updates $\hat{\mathbf{x}}_k$ using (14)
    B_1 receives $\frac{([\boldsymbol{\rho}_k^{(j)}]_{q_{ik}} + m\hat{q}_{ik}^{(j+1)})}{2m}$ and $\frac{([\boldsymbol{\rho}_k^{(j)}]_{y_{ik}} + m\hat{y}_{ik}^{(j+1)})}{2m}$ from B_k, then updates $q_{ik}^{(j+1)}$ and $y_{ik}^{(j+1)}$ using (16) and (17).
    B_1 updates $\boldsymbol{\xi}^{(j+1)}$ by (18); B_k, $k \in \bar{\mathcal{K}}$, receives $q_{ik}^{(j+1)}$ and $y_{ik}^{(j+1)}$ from B_1 then updates $\boldsymbol{\rho}_k^{(j+1)}$ using (19).
    $j := j + 1$.
  until ADMM convergence
  Obtain $(\{\mathbf{v}_{ik}^*\}, \{\mu_i^*\}, \{u_i^*\}, \{q_{ik}^*\}, \{y_{ik}^*\}; \boldsymbol{\xi}^*, \{\boldsymbol{\rho}_k^*\})$, the solution from the ADMM procedure.
  Update $t := t + 1$, $j := 1$
  Update $(\{\mathbf{v}_{ik}^{(t)}\}, \{\mu_i^{(t)}\}, \{u_i^{(t)}\}) := (\{\mathbf{v}_{ik}^*\}, \{\mu_i^*\}, \{u_i^*\})$
  Update $(\{q_{ik}^{(0)}\}, \{y_{ik}^{(0)}\}; \boldsymbol{\xi}^{(0)}, \{\boldsymbol{\rho}_k^{(0)}\}) := (\{q_{ik}^*\}, \{y_{ik}^*\}; \boldsymbol{\xi}^*, \{\boldsymbol{\rho}_k^*\})$
until $\sum_{i\in\mathcal{N}} w_i(\log(1+\mu_i^{(t+1)}) - \log(1+\mu_i^{(t)})) \le \epsilon_{\text{IA}}$

4. NUMERICAL RESULTS

We consider a circular region with a radius of 500m centered at B_1, and the small cell BSs randomly placed in the annulus with radii 300m and 500m. The channels are simply modeled as $\mathbf{h}_{ik} \sim \mathcal{CN}(0, \ell_{ik}^{-\beta}\mathbf{I})$ where ℓ_{ik} is the distance in meters and β is the path loss exponent which is taken as 4. The noise power density is $N_0 = -139$ dBm/Hz. The operation bandwidth is 1 MHz. The maximum transmission power at the BSs are $P_1 = 43$ dBm and $P_k = 35$ dBm, $\forall k \in \bar{\mathcal{K}}$. Without loss of generality, we take $w_i = 1, \forall i \in \mathcal{N}$.

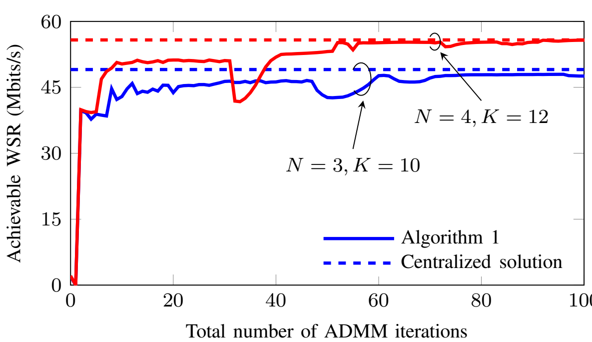

Fig. 1. Achieved weighted sum rate over ADMM iterations with two random channel realizations.

For initial points, we first randomly generate beamforming vectors $\{\mathbf{v}_{ik}^{(0)}\}$ which are scaled (if necessary) so that (3b) is satisfied. Based on $\{\mathbf{v}_{ik}^{(0)}\}$, we determine $\{\mu_i^{(0)}\}$, $\{u_i^{(0)}\}$, $\{q_{ik}^{(0)}\}$, and $\{y_{ik}^{(0)}\}$ by letting(4b), (6c), (8c), and (8e) hold with equality. The initial value for the Lagrangian multipliers are simply taken as $\boldsymbol{\xi}^{(0)} = \mathbf{1}$ and $\boldsymbol{\rho}_k^{(0)} = \mathbf{1}$ for all $k \in \bar{\mathcal{K}}$. The convex programs are solved by the solver MOSEK [21] with the modeling toolbox YALMIP [22].

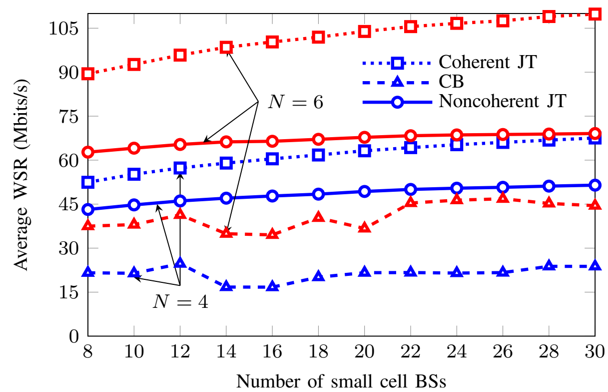

Fig. 2. Average WSR of the considered schemes with different number of small cell BSs. We take the number of users, N, as 4 and 6.

Beside the noncoherent JT, we provide the performance of two other schemes for comparison purpose. The first scheme, labeled ‘coherent JT’, is the coherent joint transmission. The second scheme, labeled ‘CB’, is the coordinated beamforming, i.e., each user only receives data from the nearest BS.

Fig. 1 shows the achieved WSR over ADMM iterations over two random channel realizations. The values are obtained by the caculated beamforming vectors at the corresponding iteration. The ‘centralized solution’ is the solution of (3) achieved via solving approximated problem (7) by the solver. We can see from the figure that on the considered channels, the distributed procedure can achieve a performance close to that of the centralized solution within 100 ADMM iterations.

Fig. 2 plots the average WSR performance of the considered schemes as functions of number of small cell BSs. An interesting result observed from the figure is that CB scheme (with the user association rule of connecting to the nearest BS) might fail to exploit the densification gain. Another result is that the coherent JT clearly is superior to the others. However, as discussed, the requirements of synchronize accuracy make it impractical, and it is not convenient to implement distributively. Although the performances of noncoherent JT is lower than that of coherent JT, it outperforms CB and is capable of exploiting densification gain. The results suggest that the noncoherent JT with the proposed distributed procedure is a promising candidate for dense small cell networks.

5. CONCLUSION

We have investigated downlink noncoherent JT in dense small cell networks. Particularly, we have considered the problem of designing beamforming vectors at the macro cell and small cell BSs for maximizing WSR of the network which is mathematically intractable. In order to overcome the nonconvexity, we has adopted the well-known IA framework, which solves a series of the convex approximate subproblems so that the obtained sequence of objective values is nondecreasing. More importantly, assuming that the beamforming design is carried out among the BSs, rather than a central computing unit as in CRANs, we has developed the distributed algorithms by using ADMM technique to solve the subproblem. The results have demonstrated that the noncoherent JT is a good transmit alternative to the coherent JT in terms of the WSR performance.

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