NOVEL LOWER BOUND ON THE PERFORMANCE OF A PARTIAL ZERO FORCING RECEIVER IN A MIMO CELLULAR NETWORK

Rei Richter, Sagi Ezri and Itsik Bergel

Faculty of Engineering
Bar-Ilan University
Email: itsik.bergel@biu.ac.il

ABSTRACT

We derive and study a novel lower bound on the performance of a partial zero forcing (PZF) receiver in the uplink of a cellular network, where the mobile locations are modeled as a homogeneous Poisson Point Process (HPPP). The PZF is a suboptimal receiver. Yet, it is easy to analyze and in many cases is close to optimal. Furthermore, the analysis of the PZF gives more insight on the behavior of the network than the optimal MMSE receiver (for example, we study the optimal distance within which interference should be suppressed). Unlike the existing analysis for the optimal MMSE receiver, our novel bound holds also in the presence of thermal noise and for finite number of antennas. This bound is easy to evaluate and proved to be asymptotically tight. Comparing to the asymptotic result for MMSE, we also give the exact SINR loss of PZF compared to MMSE.

Index Terms— 5G, MIMO, MMSE, ZF

1. INTRODUCTION

Massive multiple-input multiple-output (MIMO) cellular networks use a large number of antennas at the base-stations (BSs) to increase the spatial diversity. This increased spatial diversity can increase the network performance and improve its reliability.

Massive MIMO networks were studied extensively in recent years (see for example [1]–[4] and references therein). Yet, the overall network performance has not been fully understood so far. To study the network performance, an analysis must assume a statistical model for the nodes location.

The most popular model for analytical studies of two-dimensional networks is the Homogenous Poisson Point Process (HPPP). The HPPP has originally gained popularity for the modeling of ad-hoc wireless networks (e.g., [5]–[8]), and later was also used for the modeling of mobiles and even BSs in cellular networks (e.g., [9]–[16]). This work is based on modeling of the mobile locations as HPPP, which is a natural extension to the intuitive assumption that the mobiles are uniformly distributed.

Jindal et al. [17] used the HPPP model to study the signal to noise plus interference ratio (SINR) in the uplink of a cellular network that utilizes a PZF receiver. They showed that the SINR scales as $(L/\lambda)^{\alpha/2}$, where L is the number of BS antennas, λ is the mobile density and α is the path-loss exponent. This scaling allowed them to prove that the network performance can scale linearly with the number of antennas (by keeping the user SINR fixed while increasing the mobile density). This work focused on the asymptotic scaling, and hence could not characterize the effect of thermal noise.

The asymptotic performance of the optimal MMSE receiver was derived by Govindasamy et al. [18] using large matrix theory. Their result indeed matched the scaling of Jindal et al., and also gave the exact multiplier of the SINR. But, again, the asymptotic analysis can only characterize the perfromance for infinite number of antennas and without the effect of the noise.

In this work we extend the work of Jindal et al. on the performance of PZF, and derive a lower bound on the user rate that holds for finite number of antennas and also captures the effect of the thermal noise.

The novel lower bound can characterize the network performance in any scenario, and is proved to be asymptotically tight. While the PZF receiver is sub-optimal, the new lower bound has two distinct advantages over the MMSE analysis: it enables the analysis of the performances in the presence of thermal noise and it bounds the performance for any number of antennas. Furthermore, the bound derivation allows a better understanding of the balance between interference cancellation and desired signal gain.

2. SYSTEM MODEL

We consider the uplink of a cellular network with single antenna mobiles and L antennas per BS. The mobile locations are modeled as an independent homogeneous Poisson point processes (HPPPs), with a density of λ mobiles per unit area.

We analyze the performance of a typical mobile, termed the probe mobile. Without loss of generality, the probe mobile is located at the origin, and is labeled as mobile 0. The transmission of the probe mobile is considered as the desired signal, while the transmissions of all other mobiles are considered as interference. The probe mobile is associated with its nearest BS. As this BS is the only one that processes the desired signal, we can focus on it and ignore all other BSs. Thus, the analysis hereon will only consider this single BS. For convenience, we also index all interfering mobiles according to their distance from the BS. Thus, we have $r_i \leq r_{i+1}$ for $i \geq 1$ where r_i denotes the distance between the BS and the i-th mobile. Note that r_0 is the distance of the desired (probe) mobile, which does not obey this indexing rule (i.e., r_0 may be larger then other distances in the network).

A received symbol at the BS is given by the $L \times 1$ vector:

$$\mathbf{y} = \mathbf{h}_0 x_0 + \sum_{i=1}^{\infty} \mathbf{h}_i x_i + \mathbf{n} \tag{1}$$

where x_i is the symbol transmitted from the i-th mobile with zero-mean and unit variance, $\mathbf{n}$ is the thermal white-noise, with i.i.d. complex normal elements of zero mean and variance of σ^2 ($\mathbf{n} \sim CN(0, \sigma^2 I)$), and $\mathbf{h}_i$ is the channel vector of the i-th mobile. The channel vector is given by:

$$\mathbf{h}_i = r_i^{-\alpha/2} \mathbf{g}_i \tag{2}$$

where $\alpha > 2$ is the path-loss exponent and $\mathbf{g_i} \sim \mathcal{CN}(0, I)$ is the vector of the rayleigh fading gains.

Allowing the use of matrices with infinite dimensions, the channel can be described by the $L \times \infty$ matrix

$$\mathbf{H} = [\mathbf{h}_0, \mathbf{h}_1, \mathbf{h}_2, \ldots]. \tag{3}$$

and the received signal is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{4}$$

where $\mathbf{x} = [x_0, x_1, x_2, \ldots]^T$ is the vector of the symbols transmitted by all mobiles.

Assuming a linear receiver with a weight vector $\mathbf{w}$, the signal to noise plus interference ratio (SINR) for the reception of the desired signal is:

$$\eta = \frac{|\mathbf{w}^H \mathbf{h}_0|^2}{\sum\limits_{j \neq 0} |\mathbf{w}^H \mathbf{h}_j|^2 + \left\|\mathbf{w}^H\right\|^2 \sigma^2}. \tag{5}$$

The receive weight vector that maximizes the SINR is the minimal mean square error (MMSE) receiver:

$$\mathbf{w}_{\mathrm{MMSE}} = \left(\mathbf{H}\mathbf{H}^H + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{h}_0. \tag{6}$$

Note that for the HPPP, the matrix $\mathbf{H}\mathbf{H}^H$ is well defined, even though the matrix $\mathbf{H}$ has infinite dimensions.

The PZF receivers tries to balance between desired signal gain and interference mitigation by choosing a cancellation radius, D. Denoting the number of mobiles within this radius by K, the PZF receive weight vector maximizes the signal to noise ratio (SNR) subject to zeroing the interference to mobiles $1, \ldots, K$. Thus, we note that $r_K \leq D \leq r_{K+1}$ and that K has Poisson distribution with parameter $\lambda \pi D^2$. The PZF weight vector is given by:

$$\mathbf{w}_{\mathrm{PZF}} = \mathbf{H}_K \left(\mathbf{H}_K^H \mathbf{H}_K\right)^{-1} \mathbf{e} \tag{7}$$

where $\mathbf{H}_K \triangleq [\mathbf{h}_0, \mathbf{h}_1, \ldots, \mathbf{h}_K]$ and $\mathbf{e} = [1, , 0, \ldots, 0]^T$.

In the next section we derive a novel lower bound on the average spectral efficiency of the PZF receiver. This bound allows the characterization of the network throughput and the evaluation of the cancellation radius that achieves the optimal balance between interference and noise suppression. For convenience, we assume a bandwidth of 1Hz, and hence we use the terms rate and spectral efficiency interchangeably. The probe user rate is:

$$R = \log_2 \left(1 + \frac{|\mathbf{w}^H \mathbf{h}_0|^2}{\sum\limits_{j > 0} |\mathbf{w}^H \mathbf{h}_j|^2 + \left\|\mathbf{w}^H\right\|^2 \sigma^2} \right) \tag{8}$$

and the average user rate is:

$$\bar{R} = E\left[R\right] \tag{9}$$

where the expectation is taken with respect to the channel fading and with respect to the mobile locations based on the HPPP model.

3. PERFORMANCE ANALYSIS

3.1. PZF lower bound

Theorem 1. *For a cancellation radius, D, the average rate of a user with a PZF receiver, given its distance to the receiver is lower bounded by:*

$$E[R | r_0 = r] \geq R_{\mathrm{LB}}(r; D) \tag{10}$$

where,

$$R_{\mathrm{LB}}(r; D) = E\left[\log_2 \left(1 + \frac{r^{-\alpha} \left(L - K - 1\right)_+}{\frac{2\pi\lambda}{\alpha - 2} D^{2-\alpha} + \sigma^2}\right)\right] \tag{11}$$

K is a Poisson random variable with a mean of $\lambda \pi D^2$, and $(x)_+ = \max\{x, 0\}$. This bound is tight when $\lambda \pi D^2 \gg 1$.

Proof. See Section 4. □

This theorem allows a simple and convenient characterization of the network performance. Unlike the known results for MMSE, this bound is valid for finite number of antennas, and also bounds the performance in the presence of thermal noise. A bound on the average user rate, $\bar{R}$ can also be easily obtained from Theorem 1, by averaging over the user-BS distance (which have an exponential distribution).

However, the bound of Theorem 1 requires a choice of the canceling radius, D. To make the bound even more useful, the next corollary gives an approximation of the optimal canceling radius, D_{apx}. Using D_{apx} in (11) gives a simpler bound on the maximal network performance.

Corollary 1. *Let $D_{\mathrm{opt}} = \arg\max_D R_{\mathrm{LB}}(r; D)$ be the optimal canceling radius, D. A good approximation for D_{opt} is the solution of:*

$$D^{\alpha} (\alpha - 2) \sigma^2 + \alpha\pi D^2 \lambda = (\alpha - 2) (L - 1). \quad (12)$$

Proof of Corollary 1. The proof is based on replacing K in (11) by its expectation $E[K] = \pi\lambda D^2$ resulting with:

$$R_{\mathrm{LB}}(r; D) \approx E\left[\log_2\left(1 + \frac{r^{-\alpha}\left(L - \pi\lambda D^2 - 1\right)}{\frac{2\pi\lambda}{\alpha-2} D^{2-\alpha} + \sigma^2}\right)\right] \quad (13)$$

To find the optimal D, we evaluate the derivative with respect to D and compare to zero, which results in (12). ☐

In the following we will term the solution of (12) as D_{apx} and define $R_{\mathrm{bnd}}(r) = R_{\mathrm{LB}}(r; D_{\mathrm{apx}})$. Note that $E[R|r_0 = r] \geq R_{\mathrm{LB}}(r; D)$ for any D and hence also $E[R|r_0 = r] \geq R_{\mathrm{bnd}}(r)$.

While D_{apx} is given only implicitly, as the solution of an equation, we can gain much insight from it by considering the interference limited regime and the noise limited regime. In the interference limited regime, $\sigma^2 \ll \frac{2\pi\lambda}{\alpha-2} D^{2-\alpha}$. Thus, we can neglect the first term on the left hand side of (12) and get:

$$D_{\mathrm{apx}} \approx \sqrt{\frac{(\alpha - 2)(L - 1)}{\alpha\pi\lambda}}. \quad (14)$$

On the other hand, in the noise limited regime, $\sigma^2 \gg \frac{2\pi\lambda}{\alpha-2} D^{2-\alpha}$, and we can neglect the second term on the left hand side of (12), and we get:

$$D_{\mathrm{apx}} \approx \left(\frac{L - 1}{\sigma^2}\right)^{\frac{1}{\alpha}}. \quad (15)$$

Note that (14) and (15) give important insight on the network as they tell us the radius of 'important' interferers. Here, an 'important' interferer is an interferer that has higher power than the noise, and hence should be canceled. Alternatively, we can translate the same parameter to the average number of 'important' interferers, given by $\pi\lambda D_{\mathrm{apx}}^2$. For example, in the interference limited case this average number of interferers is $(\alpha - 2)/\alpha \cdot (L - 1)$, which is about half of the available degrees of freedom for $\alpha = 4$ but only one fifth of them for $\alpha = 2.5$.

3.2. Asymptotic analysis

When L grows to infinity, we can derive an even simpler expression. From (12) (or more conveniently from (14) and (15)) we can see that when L grows to infinity, the optimal D also grows to infinity. Thus, from Theorem 1, the bound is tight. We can further simplify it by observing that

$$\lim_{D\to\infty} \frac{K}{\lambda\pi D^2} = 1. \quad (16)$$

Next we need to differentiae between two cases. if $\sigma^2 > 0$, then asymptotically, the mobile will always be in the noise limited regime. Substituting (15) and (16) into (11), The user rate in noise limited regime converges to: $\lim_{L\to\infty} \frac{E[R|r_0=r]}{R_{\mathrm{PZF},\sigma^2>0}(r)}$ where:

$$R_{\mathrm{PZF},\sigma^2>0}(r) = \log_2\left(1 + \frac{r^{-\alpha} L}{\sigma^2}\right). \quad (17)$$

which shows that even though we consider infinite number of interfers, the PZF asymptotically achives the no-interference bound.

The case that was studied more is the noise free case, $\sigma^2 = 0$. In this case, substituting (16) and (14), into (11) gives $\lim_{L\to\infty} \frac{E[R|r_0=r]}{R_{\mathrm{PZF},\sigma^2=0}(r)}$ where:

$$R_{\mathrm{PZF},\sigma^2=0}(r) = \log_2\left(1 + \left(\frac{(\alpha - 2)(L - 1)}{\alpha\pi\lambda r^2}\right)^{\alpha/2}\right). \quad (18)$$

As mentioned above, the scaling of the SNR with L was reported already in [17]. The result above is more detailed as it gives the exact asymptote of the SINR.

This result also allows a comparison to the performance of the MMSE receiver for this case, which is given in [18]:

$$R_{\mathrm{MMSE},\sigma^2=0}(r) = \log_2\left(1 + \left(\frac{\alpha L}{2\pi^2\lambda r^2} \sin\frac{2\pi}{\alpha}\right)^{\alpha/2}\right). \quad (19)$$

Both (18) and (19) show that the SINR scales as $L^{\alpha/2}$. Thus, we can compare their SINR (the term inside the logarithm) and see that the ratio depends only on the the path loss exponent:

$$\frac{SINR_{\mathrm{PZF},\sigma^2=0}(r)}{SINR_{\mathrm{MMSE},\sigma^2=0}(r)} = \left(\frac{1 - \frac{2}{\alpha}}{\frac{\alpha}{2\pi} \sin\frac{2\pi}{\alpha}}\right)^{\alpha/2}. \quad (20)$$

This asymptotic SINR loss is easy to evaluate and ranges between 0.82 for $\alpha = 2.5$ and 0.6 for $\alpha = 4$.

4. PROOF OF THEOREM 1

The bound is derived using the chain rule for expectations and successive application of the Jensen inequality, using the concavity of $\log_2(1+1/x)$. Conditioning on $\Psi = \{r, \mathbf{w}, \mathbf{h}_0, K\}$:

$$\begin{aligned} &E[R|r_0 = r] \quad (21) \\ &= E\left[E\left[\log_2\left(1 + \frac{|\mathbf{w}^H\mathbf{h}_0|^2}{\sum\limits_{j>K} |\mathbf{w}^H\mathbf{h}_j|^2 + \|\mathbf{w}\|^2\sigma^2}\right)\middle|\Psi\right]\right] \\ &\geq E\left[\log_2\left(1 + \frac{|\mathbf{w}^H\mathbf{h}_0|^2}{E\left[\sum\limits_{j:r_j>D} |\mathbf{w}^H\mathbf{h}_j|^2 \middle| \mathbf{w}\right] + \|\mathbf{w}\|^2\sigma^2}\right)\right] \end{aligned}$$

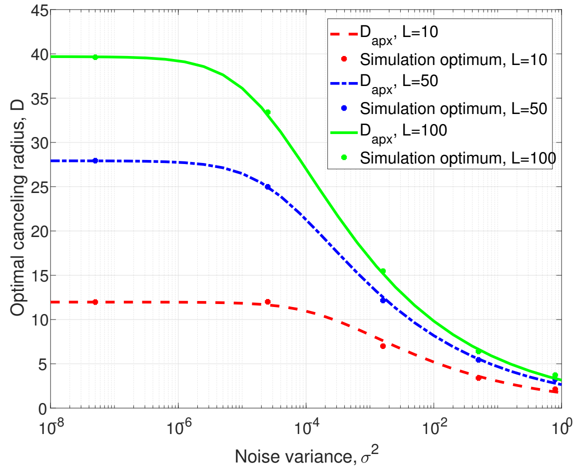

Fig. 1. Optimal and approximate cancellation radius as a function of the noise variance for different numbers of BS antennas. The path loss exponent is $\alpha = 4$.

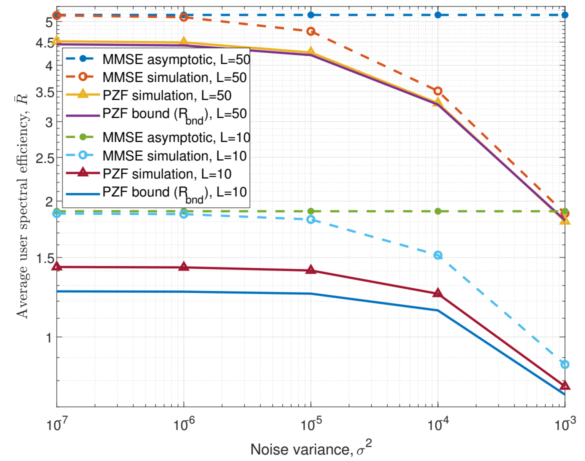

Fig. 2. Average user rates and bounds for PZF and MMSE receivers ($\alpha = 4$).

where the first line used the fact that the PZF receiver zeros the interference from the nearest K users, and the inequality used the Jensen inequality and the fact that for $j > K$ the channel vector $\mathbf{h}_j$ is statistically independent of $\mathbf{w}$ and $\mathbf{h}_0$.

To evaluate the residual interference expectation in the denominator we use the i.i.d. Gaussian distribution of $\mathbf{g}_i$ in (2), and note that given $\mathbf{w}$, the product $\mathbf{w}^H\mathbf{h}_j$ for any $j > K$ has a Gaussian distribution with zero mean and variance $\|\mathbf{w}\|^2 r_j^{-\alpha}$. Thus, we have:

$$E\Big[\sum_{j:r_j>D} |\mathbf{w}^H\mathbf{h}_j|^2\Big|\mathbf{w}\Big] = \left\|\mathbf{w}^H\right\|^2 \cdot E\left[\sum_{j:r_j>D} r_j^{-\alpha}\right] = \left\|\mathbf{w}^H\right\|^2 \frac{2\pi\lambda}{\alpha-2} D^{2-\alpha} \tag{22}$$

where the second line uses the HPPP properties (see for example [19]–[21]). Note that [20] also showed that the term in the expectation actually converges to a constant when $E[K]$ is large enough. Thus, the Jensen inequality in (23) is tight when $\lambda D^2 \gg 1$.

For the last stage of the proof, we note that given K, $2|\mathbf{w}^H\mathbf{h}_0|^2/\|\mathbf{w}\|^2 r_0^{-\alpha}$ has a Chi square distribution with $2(L-K)$ degrees of freedom. Thus, $E[\|\mathbf{w}\|^2/|\mathbf{w}^H\mathbf{h}_0|^2|K] = r_0^\alpha/(L-K-1)$. The proof is completed by substituting (22) in (23), conditioning on K and using the Jensen inequality again for $\log_2(1+1/x)$:

$$E[R|r_0 = r] \geq E\left[E\left[\log_2\left(1 + \frac{|\mathbf{w}^H\mathbf{h}_0|^2/\left\|\mathbf{w}\right\|^2}{\frac{2\pi\lambda}{\alpha-2}D^{2-\alpha} + \sigma^2}\right)\middle|K\right]\right]$$

which leads to (11) and completes the proof.

5. NUMERICAL EXAMPLES

In this section we demonstrate the usefulness of our novel bound. First, we show that the approximation of (14) is quite good even when the number of antennas is as low as 10. Fig. 1 depicts the approximate canceling radius of (14) (marked as D_{apx}), and the actual optimal cancelation radius, obtained from brute force optimization of the simulation results. As can be seen, the approximation is good both in the presence and in the absence of noise.

Fig. 2 depict the network performance and bounds vs. noise variance for $L = 10$ and $L = 50$ BS antennas. The figure shows that the MMSE asymptotic expression gives a good approximation even for 10 antennas (although this wasn't proved analytically). On the other hand, the novel PZF bound is proved to be a lower bound for all scenarios. This bound is up to 12% from the actual performance at $L = 10$ and only up to 1.5% for $L = 50$. Note that when the noise becomes significant, the novel lower bound still gives a good characterization of the performance, which cannot be obtained from the asymptotic MMSE expression.

6. CONCLUSIONS

We presented a novel lower bound on the performance of the PZF receiver in the uplink of a cellular network. The novel bound is easy to evaluate and can characterize the network performance at any network scenario, including finite number of antennas and in the presence of thermal noise. We also derived an asymptotic expression for the SINR loss of PZF compared to MMSE receiver. This loss ranges between 0.82 at $\alpha = 2.5$ and 0.6 at $\alpha = 4$.

7. REFERENCES

[1] Jakob Hoydis, Stephan ten Brink, and Mérouane Debbah, "Massive MIMO in the UL/DL of cellular networks: How many antennas do we need?," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 160–171, Feb. 2013.

[2] Erik G Larsson, Ove Edfors, Fredrik Tufvesson, and Thomas L Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[3] Lu Lu, Geoffrey Ye Li, A Lee Swindlehurst, Alexei Ashikhmin, and Rui Zhang, "An overview of massive MIMO: Benefits and challenges," *IEEE journal of selected topics in signal processing*, vol. 8, no. 5, pp. 742–758, 2014.

[4] Andreas F Molisch, Vishnu V Ratnam, Shengqian Han, Zheda Li, Sinh Le Hong Nguyen, Linsheng Li, and Katsuyuki Haneda, "Hybrid beamforming for massive MIMO: A survey," *IEEE Communications Magazine*, vol. 55, no. 9, pp. 134–141, 2017.

[5] M. Haenggi, J.G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1029–1046, 2009.

[6] Moe Z Win, Pedro C Pinto, and Lawrence A Shepp, "A mathematical theory of network interference and its applications," *Proceedings of the IEEE*, vol. 97, no. 2, pp. 205–230, 2009.

[7] Yaniv George, Itsik Bergel, and Ephraim Zehavi, "Upper bound on the ergodic rate density of ALOHA wireless ad-hoc networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 6, pp. 3004–3014, 2015.

[8] Amogh Rajanna, Itsik Bergel, and Mostafa Kaveh, "Performance analysis of rateless codes in an ALOHA wireless ad hoc network," *IEEE Trans. Wireless Commun.*, vol. 14, no. 11, pp. 6216–6229, 2015.

[9] Jeffrey G. Andrews, François Baccelli, and Radha Krishna Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3122–3134, Apr. 2011.

[10] Thomas David Novlan, Harpreet S. Dhillon, and Jeffrey G. Andrews, "Analytical modeling of uplink cellular networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2669–2679, Jun. 2013.

[11] Kaibin Huang and Jeffrey G Andrews, "An analytical framework for multicell cooperation via stochastic geometry and large deviations," *IEEE Trans. Inf. Theory*, vol. 59, no. 4, pp. 2501–2516, Apr. 2013.

[12] Anjin Guo and Martin Haenggi, "Spatial stochastic models and metrics for the structure of base stations in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 11, pp. 5800–5812, 2013.

[13] Bartlomiej Blaszczyszyn, Mohamed Kadhem Karray, and Holger Paul Keeler, "Using Poisson processes to model lattice cellular networks," in *Proceedings of IEEE INFOCOM*, 2013, pp. 773–781.

[14] Bartłomiej Błaszczyszyn, Mohamed Kadhem Karray, and H Paul Keeler, "Wireless networks appear Poissonian due to strong shadowing," *IEEE Trans. Wireless Commun.*, vol. 14, no. 8, pp. 4379–4390, Aug. 2015.

[15] Yi Zhong, Tony QS Quek, and Xiaohu Ge, "Heterogeneous cellular networks with spatio-temporal traffic: Delay analysis and scheduling," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 6, pp. 1373–1386, 2017.

[16] Siddhartan Govindasamy and Itsik Bergel, "Uplink performance of multi-antenna cellular networks with co-operative base stations and user-centric clustering," *IEEE Transactions on Wireless Communications*, vol. 17, no. 4, pp. 2703–2717, 2018.

[17] N. Jindal, J.G. Andrews, and S. Weber, "Multi-antenna communication in Ad Hoc networks: Achieving MIMO gains with SIMO transmission," *IEEE Trans. Commun.*, vol. 59, no. 2, pp. 529–540, Feb. 2011.

[18] Siddhartan Govindasamy, D.W. Bliss, and David H. Staelin, "Asymptotic spectral efficiency of the uplink in spatially distributed wireless networks with multi-antenna base stations," *IEEE Trans. Commun.*, vol. 61, no. 7, pp. 100–112, Jun. 2013.

[19] François Baccelli and Bartlomiej Blaszczyszyn, *Stochastic geometry and wireless networks: Volume I theory*, vol. 1, Now Publishers Inc, 2010.

[20] Yaniv George, Itsik Bergel, and Ephraim Zehavi, "The ergodic rate density of ALOHA wireless ad-hoc networks," *IEEE Trans. Wireless Commun.*, vol. 12, no. 12, pp. 6340–6351, Dec. 2013.

[21] Yiftach Richter and Itsik Bergel, "Analysis of the simulated aggregate interference in random ad-hoc networks," in *Signal Processing Advances in Wireless Communications (SPAWC), 2014 IEEE 15th International Workshop on*, 2014, pp. 374–378.