MASSIVE MIMO CHANNEL ESTIMATION WITH 1-BIT SPATIAL SIGMA-DELTA ADCS

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ABSTRACT

We consider channel estimation for an uplink massive multiple input multiple output (MIMO) system where the base station (BS) uses a first-order spatial Sigma-Delta ($\Sigma\Delta$) analog-to-digital converter (ADC) array. The $\Sigma\Delta$ array consists of closely spaced sensors which oversample the received signal and provide a coarsely quantized (1-bit) output. We develop a linear minimum mean squared error (LMMSE) estimator based on the Bussgang decomposition that reformulates the nonlinear quantizer model using an equivalent linear model plus quantization noise. The performance of the proposed $\Sigma\Delta$ LMMSE estimator is compared via simulation to channel estimation using standard 1-bit quantization and also infinite resolution ADCs.

Index Terms— Channel estimation, massive MIMO, $\Sigma\Delta$ ADCs, one-bit quantization.

1. INTRODUCTION

Massive MIMO systems provide high spatial resolution and throughput, but the cost and power consumption of the associated RF hardware, particularly the analog-to-digital converters (ADCs) can be prohibitive, especially at higher bandwidths and sampling rates. To save power and chip area, low resolution quantizers have been suggested. There has been extensive research on 1-bit ADCs for channel estimation in massive MIMO systems [1–4]. While it has been shown that 1-bit quantization causes only a minimal degradation, at medium to high SNR the loss is substantial.

To improve the performance of low-resolution sampling, one can of course increase the resolution of the quantizer. Simulations have shown that using ADCs with 3-5 bits of resolution in massive MIMO provides performance that is very close to that achievable with infinite precision, and still provides higher energy efficiency (e.g. see [5]). As an alternative to increasing the ADC resolution, one can increase the sampling rate at which the 1-bit quantizers operate. This approach has been studied in [6–8] and found to also be effective in reducing the medium-to-high-SNR performance loss for 1-bit quantization.

A well-known technique that combines 1-bit quantization and oversampling is the $\Sigma\Delta$ ADC, which to date has primarily found application in ultrasound imaging. The $\Sigma\Delta$ converter scheme consists of an oversampled modulator which is responsible for digitization of the analog signal and a negative feedback loop. This architecture provides noise shaping that alters the power spectral density of the quantization noise such that it is no longer uniform, as in regular quantization, but is shifted to higher frequencies. The quantization noise can then be processed by a digital low-pass filter and decimation stage so that it has a reduced effect on the signal. The use of $\Sigma\Delta$ ADCs in parallel architectures for MIMO systems has been studied in [9, 10].

A similar effect can be achieved by *oversampling in space* instead of time, i.e., using an antenna array whose elements are separated by less than one half wavelength. In spatial $\Sigma\Delta$ ADCs, the integration is performed by feeding the quantization error from one stage to the adjacent antenna input instead of feedback via a time delay. The spatial frequency, or the number of cycles per radian, is proportional to the sine of the angle of arrival measured from the array broadside. A low spatial frequency implies that signals are impinging from near broadside, and the $\Sigma\Delta$ modulator can be expected to exhibit lower quantization error than for signals from the endfire direction. The noise shaping characteristics of first and second-order spatial and cascaded (space-time) $\Sigma\Delta$ architectures have been the focus of prior work in this area [11–15].

In this paper, we consider optimal channel estimation for massive MIMO systems with first-order spatial $\Sigma\Delta$ ADCs. The model for this system is derived and the Bussgang decomposition is applied in order to find an equivalent linear signal-plus-quantization-noise representation that is the basis for the LMMSE channel estimation [1]. Our approach explicitly takes into account the spatial correlation between the quantized outputs of the $\Sigma\Delta$ ADC array. The feedback structure of the $\Sigma\Delta$ array complicates the calculation of the required covariance matrices needed for the Bussgang approach, rendering a closed-form solution impossible. However, the special structure of the data flow allows us to find a recursive solution for the covariance matrices, and hence the LMMSE estimator. The case with orthogonal pilots is considered since the block-diagonal nature of the correlation matrices can be leveraged. Our simulation results indicate

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that, at low-to-medium SNRs, the LMMSE channel estimator for the $\Sigma\Delta$ array yields channel estimates that are very close to those provided with infinite resolution, and significantly better than standard 1-bit quantization, with a very small increase in hardware complexity.

2. SYSTEM MODEL

We consider an uplink massive MIMO system with K single-antenna user terminals, and a BS equipped with $M > K$ antennas and a first-order spatial $\Sigma\Delta$ array. During the training period, all K users transmit their pilot sequences of length N simultaneously. The received signal, $\mathbf{X} \in \mathbb{C}^{M\times N}$, at the BS is

$$\mathbf{X} = \sqrt{\rho}\,\mathbf{H}\,\mathbf{\Phi}_t + \mathbf{N}, \tag{1}$$

where ρ is the SNR ratio common to all the users, $\mathbf{H} \in \mathbb{C}^{M\times K}$ is the channel matrix, $\mathbf{\Phi}_t \in \mathbb{C}^{K\times N}$ is the pilot matrix and $\mathbf{N}$ is the additive noise whose elements satisfy $[\mathbf{N}]_{ij} \sim \mathcal{CN}(0,1)$. We also define the vectorized channel as $\mathbf{h} = \mathrm{vec}(\mathbf{H})$. Following the Kronecker model, the channel covariance is assumed to satisfy $\mathbf{C_h} = \mathbf{C_S} \otimes \mathbf{C_H}$ where $\otimes$ represents the Kronecker product and $\mathbf{C_S} = \mathbb{E}[\mathbf{H}^H\mathbf{H}]$ and $\mathbf{C_H} = \mathbb{E}[\mathbf{H}\mathbf{H}^H]$ are the transmit and receive antenna array spatial covariance matrices respectively. Since we assume single-antenna uncorrelated users, $\mathbf{C_S} = \mathbf{I}_K$ and, thus, $\mathbf{C_h} = \mathbf{I}_K \otimes \mathbf{C_H}$. Vectorizing (1), we get

$$\begin{aligned}\mathbf{x} = \mathrm{vec}\left(\mathbf{X}\right) =& \sqrt{\rho}\left(\mathbf{\Phi}_t^T \otimes \mathbf{I}\right)\mathrm{vec}\left(\mathbf{H}\right) + \mathrm{vec}\left(\mathbf{N}\right)\\ =& \mathbf{\Phi}\mathbf{h} + \mathbf{n}.\end{aligned} \tag{2}$$

The first-order spatial $\Sigma\Delta$ converter array can be imple-

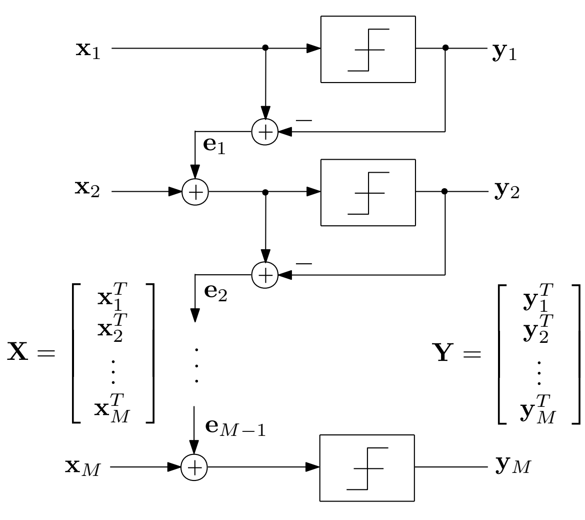

Fig. 1. A first-order $\Sigma\Delta$ array.

mented as illustrated in Fig. 1, where $\mathbf{x}_m$ denotes the mth row of $\mathbf{X}$. The array consists of M 1-bit ADCs, where the input to the mth ADC is the quantization error from the previous stage subtracted from the signal received at the mth antenna. More specifically, the output of the mth $\Sigma\Delta$ ADC at the nth time instant, $y_m[n], n = 0, \ldots, N-1$, is

$$y_m[n] = \mathcal{Q}\left(x_m[n] + e_{m-1}[n]\right),$$

where $\mathcal{Q}(x) = 1/\sqrt{2}\left(\mathrm{sign}(x) + j\mathrm{sign}(x)\right)$ is the 1-bit quantization operation and $e_{m-1}[n]$ is the difference between the input and the output of the $(m-1)$th quantizer. The vectorized output of the $\Sigma\Delta$ array can be represented as

$$\mathbf{y} = \mathcal{Q}(\underbrace{\mathbf{U}\mathbf{x} - \mathbf{\Gamma}\mathbf{y}}_{\mathbf{z}}), \tag{3}$$

where $\mathbf{U} = \mathbf{I}_{MN} + \mathbf{\Gamma}$ and $\mathbf{\Gamma} = \mathbf{I}_N \otimes \mathbf{\Gamma}_d$ with

$$\mathbf{\Gamma}_d = \begin{bmatrix} 0 & 0 & 0 & 0 & \ldots & 0\\ 1 & 0 & 0 & 0 & \ldots & 0\\ 1 & 1 & 0 & 0 & \ldots & 0\\ & & & & \vdots & \\ 1 & 1 & 1 & 1 & \ldots & 0 \end{bmatrix}.$$

3. LMMSE CHANNEL ESTIMATION

We use the Bussgang decomposition to derive an LMMSE estimator of the channel from using measurements from the $\Sigma\Delta$ array. According to the Bussgang model, the output of the $\Sigma\Delta$ array can be represented as

$$\mathbf{y} = \mathbf{A}\mathbf{z} + \mathbf{q}, \tag{4}$$

where $\mathbf{q}$ is the quantization noise and $\mathbf{A}$ is a matrix chosen to make $\mathbf{q}$ uncorrelated with $\mathbf{z}$, i.e., $\mathbf{A} = \mathbf{C}_{\mathbf{zy}}^H\mathbf{C}_{\mathbf{z}}^{-1}$, where $\mathbf{C_{zy}}$ is the cross-correlation matrix between $\mathbf{z}$ and $\mathbf{y}$ and $\mathbf{C_z}$ is the auto-correlation matrix of $\mathbf{z}$. Let $\mathbf{\Sigma_z} = \mathrm{diag}(\mathbf{C_z})$ be the matrix formed by the diagonal elements of $\mathbf{C_z}$ and let $\mathbf{C_y}$ be the auto-correlation matrix of $\mathbf{y}$. For 1-bit quantization, the following relations can be obtained:

$$\begin{aligned}&\mathbf{A} = \sqrt{\frac{2}{\pi}}\mathbf{\Sigma}_{\mathbf{z}}^{-0.5},\\ &\mathbf{C_y} = \frac{2}{\pi}\left[\arcsin\left(\frac{\pi}{2}\mathbf{A}\mathrm{Re}(\mathbf{C_z})\mathbf{A}^H\right) + \right.\\ &\qquad\qquad\left. j\arcsin\left(\frac{\pi}{2}\mathbf{A}\mathrm{Im}(\mathbf{C_z})\mathbf{A}^H\right)\right],\\ &\mathbf{C_z} = \mathbf{U}\mathbf{C_x}\mathbf{U}^H + \mathbf{\Gamma}\mathbf{C_y}\mathbf{\Gamma}^H - \mathbf{U}\mathbf{C_{xy}}\mathbf{\Gamma}^H - \mathbf{\Gamma}\mathbf{C}_{\mathbf{xy}}^H\mathbf{U}^H.\end{aligned} \tag{5}$$

We note here that there is a complicated inter-relationship between $\mathbf{C_y}$ and $\mathbf{C_z}$ for the $\Sigma\Delta$ array that is the primary difficulty for calculating the matrix $\mathbf{A}$. We show below that, while it is not possible to obtain closed form expressions for $\mathbf{C_y}$ and $\mathbf{C_z}$, their elements can be computed recursively. To begin, note that

$$\begin{aligned}\mathbf{C_{xy}} =& \mathbb{E}[\mathbf{x}\mathbf{z}^H]\mathbf{A}^H + \mathbb{E}[\mathbf{x}\mathbf{q}^H]\\ =& \mathbf{C_x}\mathbf{U}^H\mathbf{A}^H - \mathbf{C_{xy}}\mathbf{\Gamma}^H\mathbf{A}^H + \mathbb{E}[\mathbf{x}\mathbf{q}^H]\\ =& (\mathbf{C_x}\mathbf{U}^H\mathbf{A}^H + \mathbb{E}[\mathbf{x}\mathbf{q}^H])(\mathbf{I} + \mathbf{\Gamma}^H\mathbf{A}^H)^{-1}.\end{aligned} \tag{6}$$

Using the fact that $\mathbf{z}$ and $\mathbf{q}$ are uncorrelated, $\mathbf{U}\mathbb{E}[\mathbf{x}\mathbf{q}^H] = \mathbf{\Gamma}\mathbb{E}[\mathbf{y}\mathbf{q}^H] = \mathbf{\Gamma}\mathbf{C_q}$. Since $\mathbf{C_q} = \mathbf{C_y} - \mathbf{A}\mathbf{C_z}\mathbf{A}^H$, (6) can be written as

$$\mathbf{C_{xy}} = (\mathbf{C_x}\mathbf{U}^H\mathbf{A}^H + \mathbf{U}^{-1}\mathbf{\Gamma}\mathbf{C_q})(\mathbf{I} + \mathbf{\Gamma}^H\mathbf{A}^H)^{-1}. \quad (7)$$

Substituting for $\mathbf{C_{xy}}$ in (5),

$$\begin{aligned}\mathbf{C_z} =& \underbrace{\mathbf{U}\mathbf{C_x}\mathbf{U}^H}_{\text{Term1}} + \underbrace{\mathbf{\Gamma}\mathbf{C_y}\mathbf{\Gamma}^H}_{\text{Term2}} \\ &- \underbrace{\mathbf{U}\mathbf{C_x}\mathbf{U}^H\mathbf{A}^H(\mathbf{I}+\mathbf{\Gamma}^H\mathbf{A}^H)^{-1}\mathbf{\Gamma}^H}_{\text{Term3}} \\ &- \underbrace{\mathbf{\Gamma}(\mathbf{I}+\mathbf{A}\mathbf{\Gamma})^{-1}\mathbf{A}\mathbf{U}\mathbf{C_x}\mathbf{U}^H}_{(\text{Term3})^{\text{H}}} \\ &- \underbrace{\mathbf{\Gamma}\mathbf{C_q}(\mathbf{I}+\mathbf{\Gamma}^H\mathbf{A}^H)^{-1}\mathbf{\Gamma}^H}_{\text{Term4}} - \underbrace{\mathbf{\Gamma}(\mathbf{I}+\mathbf{A}\mathbf{\Gamma})^{-1}\mathbf{C_q}\mathbf{\Gamma}^H}_{(\text{Term4})^{\text{H}}}.\end{aligned} \quad (8)$$

From (8), we see that the first term is known and terms 2, 3 and 4 are required to be computed. To simplify the subsequent derivation, we will assume that the pilots are orthogonal, and that the minimum possible number of pilots $N = K$ are used for channel estimation. This leads to $\mathbf{\Phi}^*\mathbf{\Phi}^T = \rho\mathbf{I}$, and will result in the matrices $\mathbf{U}\mathbf{C_x}\mathbf{U}^H$, $\mathbf{C_z}$ and $\mathbf{C_y}$ all being block diagonal. We illustrate the recursive algorithm for computing the necessary covariances using one of the blocks.

First, we note that multiplication by $\mathbf{\Gamma}$ on the left and $\mathbf{\Gamma}^H$ on the right is equivalent to computing a running sum across the rows and columns and shifting the result diagonally across. For example, consider the first block of $\mathbf{\Gamma}\mathbf{C_y}\mathbf{\Gamma}^H$. The (m,n)th element of this block is

$$\left[\mathbf{\Gamma}\mathbf{C_y}\mathbf{\Gamma}^H\right]_{mn} = \sum_{i=1}^{m-1}\sum_{j=1}^{n-1}[\mathbf{C_y}]_{ij}. \quad (9)$$

Next, consider the product $(\mathbf{I} + \mathbf{\Gamma}^H\mathbf{A}^H)^{-1}\mathbf{\Gamma}^H$. Let the first $M \times M$ block of $\mathbf{A}$ be $\mathbf{A}_d$. We note that $\mathbf{\Gamma}_d^H\mathbf{A}_d^H$ is a nilpotent matrix that satisfies $(\mathbf{\Gamma}_d^H\mathbf{A}_d^H)^M = 0$. This gives us:

$$(\mathbf{I} + \mathbf{\Gamma}_d^H\mathbf{A}_d^H)^{-1} = \mathbf{I} - \sum_{k=1}^{M-1}(\mathbf{\Gamma}_d^H\mathbf{A}_d^H)^k. \quad (10)$$

Using mathematical induction, it can be shown that $(\mathbf{I} + \mathbf{\Gamma}_d^H\mathbf{A}_d^H)^{-1}\mathbf{\Gamma}_d^H$ has the following structure:

$$(\mathbf{I} + \mathbf{\Gamma}_d^H\mathbf{A}_d^H)^{-1}\mathbf{\Gamma}_d^H = \begin{bmatrix} 0 & 1 & (1-\mathbf{A}_{2,2}) & \dots & \prod_{k=2}^{M-1}(1-\mathbf{A}_{k,k}) \\ 0 & 0 & 1 & \dots & \prod_{k=3}^{M-1}(1-\mathbf{A}_{k,k}) \\ & & & \vdots & \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}. \quad (11)$$

We can use (11) to analyze the structure of Term3 and Term4 in (8). More specifically,

$$\begin{aligned}[\text{Term3}]_{mn} =& [\mathbf{U}\mathbf{C_x}\mathbf{U}^H]_{m,n-1}\mathbf{A}_{n-1,n-1} + \\ & [\mathbf{U}\mathbf{C_x}\mathbf{U}^H]_{m,n-2}\mathbf{A}_{n-2,n-2}(1-\mathbf{A}_{n-1,n-1}) + \\ & \cdots + [\mathbf{U}\mathbf{C_x}\mathbf{U}^H]_{m,1}\mathbf{A}_{1,1}\prod_{k=2}^{n-1}(1-\mathbf{A}_{k,k}) \\ [\text{Term4}]_{mn} =& \sum_{i=1}^{m-1}[\mathbf{C_q}]_{i,n-1} + \sum_{i=1}^{m-1}[\mathbf{C_q}]_{i,n-2}(1-\mathbf{A}_{n-1,n-1}) \\ & + \cdots + \sum_{i=1}^{m-1}[\mathbf{C_q}]_{i,1}\prod_{k=2}^{n-1}(1-\mathbf{A}_{k,k}).\end{aligned} \quad (12)$$

From (9)-(12), it can be seen that to compute $[\mathbf{C_z}]_{mn}$, it is only necessary to know the elements of the matrices $\mathbf{C_y}$, $\mathbf{C_q}$ and $\mathbf{A}$ up to and including the indices $m-1$ and $n-1$. It is also worth noting that the first column of Term3, and the first row and first column of Term4 consist of zeroes.

The shaping of the quantization noise to higher spatial frequencies can be exploited by means of a low-pass spatial filter, or beamformer, which implies that the majority of signal energy from the users of interest should arrive from a sector centered near the broadside of the array (as is typically the case in sectored cellular systems). The beamformed output is given by

$$\mathbf{r} = \mathbf{G}\mathbf{y}, \quad (13)$$

where $\mathbf{G} \in \mathbb{C}^{M' \times MN}$ with $M' < MN$. The specific choice of $\mathbf{G}$ will depend on the degree of oversampling in the array as well as the size of the sector from which the user signals are assumed to come from.

The LMMSE estimate of the channel from the beamformed array output $\mathbf{r}$ is thus given by

$$\begin{aligned}\hat{\mathbf{h}} =& \mathbf{C_{hr}}\mathbf{C_r}^{-1}\mathbf{r} \\ =& \mathbf{C_{hy}}\mathbf{G}^H\left(\mathbf{G}\mathbf{C_y}\mathbf{G}^H\right)^{-1}\mathbf{G}\mathbf{y}.\end{aligned} \quad (14)$$

Here,

$$\begin{aligned}\mathbf{C_{hy}} =& \mathbb{E}[\mathbf{h}\mathbf{z}^H]\mathbf{A}^H + \mathbb{E}[\mathbf{h}\mathbf{q}^H] \\ =& \mathbb{E}[\mathbf{h}\mathbf{x}^H\mathbf{U}^H - \mathbf{h}\mathbf{y}^H\mathbf{\Gamma}^H]\mathbf{A}^H + \mathbb{E}[\mathbf{h}\mathbf{q}^H] \\ =& \mathbf{C_h}\mathbf{\Phi}^H\mathbf{U}^H\mathbf{A}^H - \mathbf{C_{hy}}\mathbf{\Gamma}^H\mathbf{A}^H + \mathbb{E}[\mathbf{h}\mathbf{q}^H] \\ =& \left(\mathbf{C_h}\mathbf{\Phi}^H\mathbf{U}^H\mathbf{A}^H + \mathbb{E}[\mathbf{h}\mathbf{q}^H]\right)\left(\mathbf{I} + \mathbf{\Gamma}^H\mathbf{A}^H\right)^{-1},\end{aligned} \quad (15)$$

where $\mathbb{E}[\mathbf{h}\mathbf{q}^H]$ can be obtained from $\mathbf{C_{xq}}$ by noting that $\mathbf{\Phi}^\dagger\mathbf{x} = \mathbf{h} + \mathbf{\Phi}^\dagger\mathbf{n}$. We ignore the noise term due to the asymptotic orthogonality of $\mathbf{n}$ and $\mathbf{\Phi}$, to get $\mathbb{E}[\mathbf{h}\mathbf{q}^H] = \mathbf{\Phi}^\dagger\mathbf{C_{xq}}$.

Space constraints do not allow a more detailed description of the steps required to compute the necessary covariance matrices, but the algorithm for computing the LMMSE channel estimate for the $\Sigma\Delta$ array is summarized in Algorithm 1.

Algorithm 1: Channel estimation for 1-bit $\Sigma\Delta$ array

1. Initialize $\text{diag}(\mathbf{C_y}) = 1$, $[\mathbf{C_z}]_{1,1}$, $\mathbf{A}_{1,1}$, and $[\mathbf{C_q}]_{1,1}$:
$[\mathbf{C_z}]_{1,1} = [\mathbf{U}\mathbf{C_x}\mathbf{U}^H]_{1,1}, \quad \mathbf{A}_{1,1} = \sqrt{\frac{2}{\pi}}\,(\mathbf{C_z})^{-0.5}$
$[\mathbf{C_q}]_{1,1} = [\mathbf{C_y}]_{1,1} - \mathbf{A}_{1,1}^2[\mathbf{C_z}]_{1,1}$.

2. For $m = 2$ to M, repeat:

(i) Compute the mth column of Term3 using $\mathbf{A}$ & (12).

(ii) Compute $[\text{Term4}]_{ij}$ for $i, j = 1 : m$, using $[\mathbf{C_q}]_{kl}$, $k, l = 1 : m-1$ and (12).

(iii) Compute $[\mathbf{C_z}]_{m,m}$ using (8), and $\mathbf{A}_{m,m}$, $[\mathbf{C_y}]_{m,m}$, $[\mathbf{C_q}]_{m,m}$ from (5).

(iv) Compute $[\mathbf{C_z}]_{nm}$, $[\mathbf{C_y}]_{nm}$ and $[\mathbf{C_q}]_{nm}$, $n = 1 : m-1$.

3. Estimate the channel, $\hat{\mathbf{h}}$, from (14).

4. SIMULATION RESULTS

We compare the NMSE of the $\Sigma\Delta$ MMSE channel estimator against that of the regular 1-bit Bussgang LLMSE (BLMMSE) [1] and the LMMSE estimator using unquantized measurements $\mathbf{X}$. The receive array is a uniform linear array (ULA) where the antennas are spaced at $\lambda/8$, i.e. the spatial oversampling factor is 4. The channel for user k is assumed to be a line-of-sight LOS channel defined by the direction of arrival (DOA) θ_k and the distance d between the antennas:

$$\beta_k \left[1, e^{-j2\pi d \sin\theta_k/\lambda}, \ldots, e^{-j2\pi d(M-1)\sin\theta_k/\lambda}\right]^T / \sqrt{M}, \tag{16}$$

where β_k is the large-scale fading coefficient associated with user k. Thus, the channel is spatially correlated and $\mathbf{C_H}$ is non-diagonal. The NMSE of the channel estimate is evaluated over 100 independent realizations of the source DOAs, which are assumed to be uniformly distributed in $[-30°, 30°]$. The spatial filter $\mathbf{G}$ is chosen to eliminate interference and spatial quantization noise in the interval between $-45°$ and $45°$. The filter tap length, L, is related to M' as $M' = N(M - L + 1)$. The weights are generated by Woodward's approach for pattern sampling [16]. Furthermore, $\mathbf{G}$ is applied to all the three algorithms used in the comparison so as to exploit the fact that the signals originated from the sector close to the array broadside.

Fig. 2 shows the NMSE of the channel estimate as a function of SNR with $M = 32$ antennas and $K = 5$ users. Orthogonal pilots are used with $N = 5$ and $L = 6$. It is observed that at low-to-medium SNRs, the performance of the $\Sigma\Delta$ channel estimate is almost identical to that of the unquantized MMSE channel estimate. However, the gap between the two widens as the SNR increases. Similarly, in Fig. 3, we plot the NMSE of the channel estimate for $M = 64$, $N = K = 8$, and $L = 16$. In both Fig. 2 and Fig. 3, it is seen that LMMSE channel estimation with the $\Sigma\Delta$ array offers an advantage over the conventional 1-bit quantizer array.

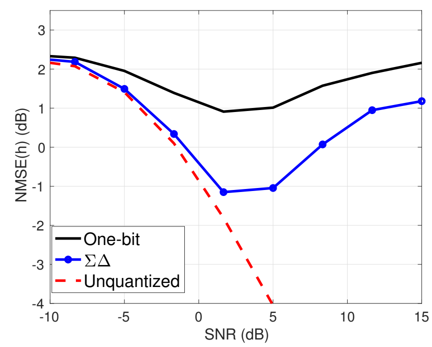

Fig. 2. NMSE of channel estimate, $M = 32, N = K = 5$.

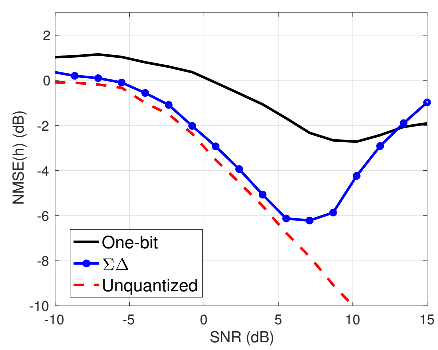

Fig. 3. NMSE of channel estimate, $M = 64, N = K = 8$.

5. CONCLUSION

In this paper, we considered channel estimation in massive MIMO using spatial $\Sigma\Delta$ modulation. We used the Bussgang decomposition to derive a new LMMSE channel estimator that takes into account the effect of correlation between quantizer outputs. We derived simple expressions for the Bussgang linearized model and provided a detailed method to compute the required correlation matrices. For oversampled arrays, the quantization noise is shifted to spatial frequencies higher than the Nyquist rate and is removed by low-pass beamforming. To study the performance of this LMMSE estimator, we considered a simple LOS channel model. The simulation results suggest that spatial $\Sigma\Delta$ modulation can significantly reduce the quantization error associated with the use of low-resolution ADCs in massive MIMO systems. In particular, the spatial $\Sigma\Delta$ converter provides an advantage in situations where the users are confined to sectors relatively close to the array broadside and can be constructed with low resolution hardware that requires minimal additional circuitry.

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