

COOPERATIVE DETECTION VIA DIRECT LOCALIZATION IN MOBILE MULTI-AGENT NETWORKS

Kai Gu and Yuan Shen

Beijing National Research Center for Information Science and Technology
Department of Electronic Engineering, Tsinghua University, Beijing 100084, China

ABSTRACT

Cooperative detection is the foundation for high-level missions in mobile multi-agent networks. Although extensive research has focused on the design of detection schemes for fixed networks, the position uncertainty induced by the mobility of agents has not been fully investigated. In this paper, we develop a generalized likelihood ratio test (GLRT) via direct localization of target and mobile agents, where the joint processing gain from direct localization is utilized to mitigate the influence of position uncertainty. Then we derive a modified GLRT detector for theoretical performance analysis and reveal the relationship between position uncertainty and detection performance. Numerical results demonstrate the superior performance of the proposed GLRT detector to the conventional method using two-step localization.

Index Terms— Multi-agent networks, position uncertainty, generalized likelihood ratio test (GLRT), direct localization, cooperative detection

1. INTRODUCTION

Target detection based on multi-agent networks is playing a fundamental role in a wide range of applications including autonomous driving, environmental monitoring and geometric mapping [1–3]. Within the scope of fixed multi-agent networks, various schemes have been utilized to solve the target detection problem with multiple agents. In [4], a geometry perspective was proposed to detect an unknown signal based on observations from multiple sensors. The detection system consisting a surveillance array and a reference array was discussed for the detection of correlated subspace signals [5].

Compared with the fixed counterparts, mobile multi-agent networks have the advantages of broader coverages on the regions of interest and greater flexibility to changeable environments [6]. However, mobile multi-agent networks suffers from the inaccurate knowledge of agents' positions, which makes traditional detection schemes that require the exact position information of agents inapplicable to the mobile case. Thus, we propose a generalized likelihood ratio test (GLRT)

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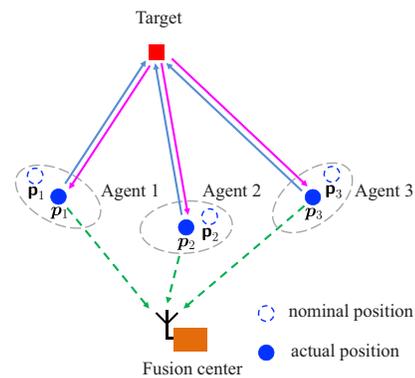


Fig. 1. Cooperative detection of a single target in a mobile multi-agent network: all the agents, which only have access to the nominal positions, transmit the received waveforms to the FC to develop a composite hypothesis test.

for target detection in mobile networks by taking position uncertainty into consideration.

Direct position determination is an effective algorithm for estimating target positions [7]. Unlike the conventional two-step localization method [8], direct localization exploits the spatial constraint that received waveforms are emitted or reflected by the same target. Thus, the joint processing gain provided by direct localization can be utilized to mitigate the adverse impact of position uncertainty on position estimation.

Inspired by the remarkable performance of direct localization, we develop a new GLRT for mobile multi-agent networks in this paper. Although our study is related to the cooperative detection schemes for fixed multi-agent networks [4, 5], it takes position uncertainty into consideration for the first time and implements GLRT via direct localization instead of conventional two-step methods. In this scheme, each agent samples the reflected waveforms and transmits the discrete signals to the fusion center (FC) for position estimation in a collaborative manner. The estimated positions are utilized to derive a GLRT to determine the existence of the target. Besides, we propose a modified GLRT to illustrate the detection performance degradation caused by position uncertainty.

The notations of lowercase (uppercase) letters in bold denote vectors (matrices) variables; $\mathbb{E}_{\mathbf{z}}\{\cdot\}$ denotes the expectation with respect to the random variable \mathbf{z} ; superscripts $(\cdot)^T$ and $(\cdot)^\dagger$ denote the transpose and conjugate transpose of their arguments, respectively; $\text{diag}\{\cdot\}$ denotes the diagonalization of a vector; $\text{Re}\{\cdot\}$ extracts the real part of the argument while $\text{Im}\{\cdot\}$ extracts the imaginary part; $\text{tr}\{\cdot\}$ denotes the trace of the argument; $\|\mathbf{x}\|_{\mathbf{A}}^2$ denotes the quadratic form of the argument with respect to the inverse of matrix \mathbf{A} , i.e., $\mathbf{x}^T \mathbf{A}^{-1} \mathbf{x}$.

2. PROBLEM FORMULATION

2.1. System Model

Consider a 2-D network consisting of N_a mobile agents that perform a cooperative target detection task (see Fig. 1).¹ The position of each agent is assumed to be unchanged during a detection interval and is determined through triangulation with surrounding anchors. We denote the true position of the k -th agent as $\mathbf{p}_k = [x_k, y_k]^T$, $k = 1, 2, \dots, N_a$. \mathbf{z}_k is the corresponding nominal position, which is corrupted by an additive zero-mean Gaussian vector \mathbf{v}_k with covariance \mathbf{R}_k , i.e.,

$$f_p(\mathbf{z}_k | \mathbf{p}_k) = \frac{1}{2\pi\sqrt{\det\{\mathbf{R}_k\}}} \exp\left(-\frac{\|\mathbf{z}_k - \mathbf{p}_k\|_{\mathbf{R}_k}^2}{2}\right). \quad (1)$$

In addition, the target position is denoted as $\mathbf{p}_t = [x_t, y_t]^T$.

Since the detection performance in an environment without interference serves as an upper bound for interference-suppressing detectors, we consider a network where there exists no clutter interference. Each agent is equipped with a M -element uniform linear array (ULA). As a general case where each agent receives signals from all the agents, can be similarly analyzed, we investigate on a representative situation that agents only receive their own signals. The base-band signal received at the m -th element of the k -th agent is

$$\begin{aligned} \mathbf{r}_k^{(m)} &= \frac{1}{\sqrt{M}} \alpha_k e^{j\phi_k^{(m)}} \mathbf{D}_k \mathbf{s}_k + \mathbf{n}_k^{(m)} \\ \phi_k^{(m)} &= 2\pi \frac{d}{\lambda} (m-1) \sin \vartheta_k \end{aligned} \quad (2)$$

where α_k is the complex scaling factor that accounts for the combining effect of channel attenuation and target reflection; d is the element spacing; λ is the wavelength of the carrier; ϑ_k is the incident angle-of-arrival (AOA). The waveform-known signal \mathbf{s}_k transmitted by the k -th agent contains N_s snapshots with a total energy E . The additive noise $\mathbf{n}_k^{(m)}$ is modeled as a complex circular Gaussian vector distributed as $\mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_{N_s})$, where the noise variance σ_n^2 is assumed known and can be obtained from auxiliary data that only contains observation noise [9].

¹We first focus on the 2-D case and the proposed detection scheme can be extended to the 3-D case.

The unitary delay operator $\mathbf{D}_k = \mathbf{W}^\dagger \mathbf{T}_k \mathbf{W}$ is introduced where $\mathbf{T}_k = \text{diag}\{e^{-j\omega_1 \tau_k}, e^{-j\omega_2 \tau_k}, \dots, e^{-j\omega_{N_s} \tau_k}\}$, \mathbf{W} is the Discrete Fourier Transform (DFT) matrix, $w_i = 2\pi(i-1)f_s/N_s$ and f_s is the sampling frequency [10].

A concatenating form of the observation vector can be expressed as

$$\mathbf{r}_k = \alpha_k (\mathbf{a}_k \otimes \mathbf{D}_k) \mathbf{s}_k + \mathbf{n}_k \quad (3)$$

where

$$\begin{aligned} \mathbf{r}_k &= [\mathbf{r}_k^{(1)T}, \mathbf{r}_k^{(2)T}, \dots, \mathbf{r}_k^{(M)T}]^T \\ \mathbf{a}_k &= [e^{j\phi_k^{(1)}}, e^{j\phi_k^{(2)}}, \dots, e^{j\phi_k^{(M)}}]^T / \sqrt{M} \\ \mathbf{n}_k &= [\mathbf{n}_k^{(1)T}, \mathbf{n}_k^{(2)T}, \dots, \mathbf{n}_k^{(M)T}]^T. \end{aligned} \quad (4)$$

Remark 1. As the location information resides in both propagation delays and AOAs, the received waveforms can be expressed as functions of \mathbf{p}_t and $\{\mathbf{p}_k\}_{k=1}^{N_a}$ by variable substitution, which enables our proposed GLRT via direct localization of target and agents.

2.2. Cooperative Detection Problem

We first denote the generalized steering vector related to the positions of the k -th agent as $\mathbf{h}_k = (\mathbf{a}_k \otimes \mathbf{D}_k) \mathbf{s}_k$. Then the cooperative target detection problem can be converted into the following composite hypothesis testing

$$\begin{aligned} \mathcal{H}_1 : \mathbf{r}_k &= \mathbf{h}_k \alpha_k + \mathbf{n}_k, \mathbf{z}_k = \mathbf{p}_k + \mathbf{v}_k \\ \mathcal{H}_0 : \mathbf{r}_k &= \mathbf{n}_k, \mathbf{z}_k = \mathbf{p}_k + \mathbf{v}_k \end{aligned} \quad (5)$$

for $k = 1, 2, \dots, N_a$. In addition, the unknown parameters under two hypotheses are listed below as

$$\begin{aligned} \boldsymbol{\theta}_1 &= [\boldsymbol{\alpha}^T, \mathbf{p}_t^T, \mathbf{p}_1^T, \mathbf{p}_2^T, \dots, \mathbf{p}_{N_a}^T]^T \\ \boldsymbol{\theta}_0 &= [\mathbf{p}_1^T, \mathbf{p}_2^T, \dots, \mathbf{p}_{N_a}^T]^T \end{aligned} \quad (6)$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_{N_a}]^T$. We further denote $\mathbf{p}_a = [\mathbf{p}_t^T, \mathbf{p}_1^T, \mathbf{p}_2^T, \dots, \mathbf{p}_{N_a}^T]^T$ as the augmented position vector.

2.3. Generalized Likelihood Ratio Test

The log-likelihood function at the k -th agent under \mathcal{H}_1 after omitting the normalization constant can be written as

$$\ln f_1(\mathbf{r}_k, \mathbf{z}_k | \boldsymbol{\theta}_1) = -\frac{1}{\sigma_n^2} \|\mathbf{r}_k - \mathbf{h}_k \alpha_k\|_2^2 + \ln f_p(\mathbf{z}_k | \mathbf{p}_k) \quad (7)$$

while the log-likelihood function under \mathcal{H}_0 is given by

$$\ln f_0(\mathbf{r}_k, \mathbf{z}_k | \boldsymbol{\theta}_0) = -\frac{1}{\sigma_n^2} \|\mathbf{r}_k\|_2^2 + \ln f_p(\mathbf{z}_k | \mathbf{p}_k). \quad (8)$$

Due to the independence of noise in received waveforms, the GLRT at the FC can be expressed as

$$\Lambda \triangleq \max_{\boldsymbol{\theta}_1} \sum_{k=1}^{N_a} q_{1,k}(\boldsymbol{\theta}_1) - \max_{\boldsymbol{\theta}_0} \sum_{k=1}^{N_a} q_{0,k}(\boldsymbol{\theta}_0) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \xi \quad (9)$$

where

$$q_{1,k}(\boldsymbol{\theta}_1) \triangleq -\frac{2}{\sigma_n^2} \|\mathbf{r}_k - \mathbf{h}_k \alpha_k\|_2^2 - \|\mathbf{z}_k - \mathbf{p}_k\|_{\mathbf{R}_k}^2 \quad (10a)$$

$$q_{0,k}(\boldsymbol{\theta}_0) \triangleq -\frac{2}{\sigma_n^2} \|\mathbf{r}_k\|_2^2 - \|\mathbf{z}_k - \mathbf{p}_k\|_{\mathbf{R}_k}^2 \quad (10b)$$

and ξ is the given threshold. Note that the maximum likelihood estimation (MLE) of unknown parameters under each hypothesis is required to establish the above GLRT.

Under \mathcal{H}_1 , the MLE of α_k is firstly obtained as

$$\hat{\alpha}_k = (\mathbf{h}_k^\dagger \mathbf{h}_k)^{-1} \mathbf{h}_k^\dagger \mathbf{r}_k = \frac{1}{E} \mathbf{h}_k^\dagger \mathbf{r}_k. \quad (11)$$

Second, we calculate the MLE of \mathbf{p}_a embedded in $\{\mathbf{h}_k\}_{k=1}^{N_a}$. In this phase, direct localization is employed to estimate the positions of target and mobile agents instead of the conventional two-step method. We denote the position estimation as $\hat{\mathbf{p}}_a$ and $\hat{\mathbf{h}}_k = \mathbf{h}_k(\hat{\mathbf{p}}_a)$. Thus the maximum of (10a) is given by

$$q_{1,k}(\hat{\boldsymbol{\theta}}_1) = -\frac{2}{\sigma_n^2} \left\| \mathbf{r}_k - \frac{1}{E} \hat{\mathbf{h}}_k^\dagger \mathbf{h}_k^\dagger \mathbf{r}_k \right\|_2^2 - \|\mathbf{z}_k - \hat{\mathbf{p}}_k\|_{\mathbf{R}_k}^2. \quad (12)$$

Similarly, the maximum of (10b) can be achieved by setting $\hat{\mathbf{p}}_k$ equal to \mathbf{z}_k under \mathcal{H}_0 , i.e.,

$$q_{0,k}(\hat{\boldsymbol{\theta}}_0) = -\frac{2}{\sigma_n^2} \|\mathbf{r}_k\|_2^2. \quad (13)$$

Combining (12) and (13) together, the test statistic of GLRT is given by

$$\Lambda = \sum_{k=1}^{N_a} \left(\frac{2}{\sigma_n^2 E} \left| \mathbf{r}_k^\dagger \hat{\mathbf{h}}_k \right|^2 - \|\mathbf{z}_k - \hat{\mathbf{p}}_k\|_{\mathbf{R}_k}^2 \right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \xi. \quad (14)$$

Remark 2. In order to construct the above GLRT, we implement direct localization to acquire the MLE of \mathbf{p}_a rather than conventional two-step methods. A simulated annealing method is first employed to find an initial point, followed by Newton's method to determine the high-precision solution.

The performance analysis of the GLRT in (14) is intractable because its distribution under \mathcal{H}_0 is not a closed form [11]. Besides, the threshold determination suffers from low accuracy and heavy computation cost. Thus, we propose a modified GLRT in the next section.

3. MODIFIED GLRT ANALYSIS

In this section we design a modified GLRT based on (14) to evaluate the detection performance analytically. Following the idea of cross validation [11], we divide the snapshots received by agents in half and use superscripts $(\cdot)^-$ and $(\cdot)^+$ to distinguish waveforms sampled at odd and even time indexes. The modified GLRT first obtains the direct localization of \mathbf{p}_a according to (14) with position uncertainty, denoted by $\hat{\mathbf{p}}_a^+$, from $\mathbf{r}^+ = \{\mathbf{r}_k^+\}_{k=1}^{N_a}$. Then we substitute $\hat{\mathbf{p}}_a^+$ into the test statistic with $\mathbf{r}^- = \{\mathbf{r}_k^-\}_{k=1}^{N_a}$ assuming all the position parameters are known and neglecting the terms regarding $\{\mathbf{z}_k\}_{k=1}^{N_a}$.

3.1. Threshold Determination under \mathcal{H}_0

After substituting $\hat{\mathbf{p}}_a^+$ into the original test statistic Λ and omitting the second part of (14), the modified test statistic Λ_M can be formulated as

$$\Lambda_M = \frac{2}{\sigma_n^2 E} \sum_{k=1}^{N_a} \left| (\mathbf{r}_k^-)^\dagger \hat{\mathbf{h}}_k^+ \right|^2 \quad (15)$$

where $\hat{\mathbf{h}}_k^+ = \mathbf{h}_k(\hat{\mathbf{p}}_a^+)$ and $(\hat{\mathbf{h}}_k^+)^\dagger \hat{\mathbf{h}}_k^+ = E$ always holds. Note that Λ_M depends on \mathbf{r}^- as well as $\{\mathbf{z}_k\}_{k=1}^{N_a}$ through $\hat{\mathbf{p}}_a^+$. Under \mathcal{H}_0 , the conditional distribution of Λ_M given $\hat{\mathbf{p}}_a^+$ is a chi-squared distribution with $2N_a$ degrees of freedom [9], i.e.,

$$f(\Lambda_M | \hat{\mathbf{p}}_a^+, \mathcal{H}_0) = f_{\chi_{2N_a}^2}(\Lambda_M) \quad (16)$$

where $f_{\chi_{2N_a}^2}(\cdot)$ is the corresponding probability density function (PDF), independent of $\hat{\mathbf{p}}_a^+$. Therefore, the unconditional distribution $f(\Lambda_M | \mathcal{H}_0)$ is the same as $f_{\chi_{2N_a}^2}(\Lambda_M)$ and the modified threshold ξ_M for a false alarm rate P_{fa} is determined by $\xi_M = Q_{\chi_{2N_a}^2}^{-1}(P_{fa})$ where $Q_{\chi_{2N_a}^2}(\cdot)$ is its right-tail probability function.

3.2. Distribution Approximation under \mathcal{H}_1

Under \mathcal{H}_1 , the modified test statistic in (15) follows a non-central chi-squared distribution conditioned on the direct localization $\hat{\mathbf{p}}_a^+$ with a noncentrality parameter λ_M

$$f(\Lambda_M | \hat{\mathbf{p}}_a^+, \mathcal{H}_1) = f_{\chi_{2N_a}^{\prime 2}(\lambda_M(\hat{\mathbf{p}}_a^+))}(\Lambda_M) \quad (17)$$

$$\lambda_M(\hat{\mathbf{p}}_a^+) = \frac{2}{\sigma_n^2 E} \sum_{k=1}^{N_a} \left| \alpha_k \mathbf{h}_k^\dagger \hat{\mathbf{h}}_k^+ \right|^2. \quad (18)$$

We further approximate the distribution of Λ_M under \mathcal{H}_1 following the concept in [12], given by

$$f(\Lambda_M | \mathcal{H}_1) = f_{\chi_{2N_a}^{\prime 2}(\bar{\lambda}_M)}(\Lambda_M) \quad (19)$$

in which $\bar{\lambda}_M = \mathbb{E}_{\hat{\mathbf{p}}_a^+} \{\lambda_M(\hat{\mathbf{p}}_a^+)\}$ is the expected noncentrality parameter over $\hat{\mathbf{p}}_a^+$. Next, we define effective deflection coefficient $\gamma_k = \mathbb{E}_{\hat{\mathbf{p}}_a^+} \{|\mathbf{h}_k^\dagger \hat{\mathbf{h}}_k^+|^2\}$ and have

$$\bar{\lambda}_M = \frac{2}{\sigma_n^2 E} \sum_{k=1}^{N_a} |\alpha_k|^2 \gamma_k. \quad (20)$$

Furthermore, the upper bound $\bar{\lambda}_M^*$ can be achieved when γ_k equals $\tilde{\gamma}_k$ as

$$\bar{\lambda}_M^* = \frac{2}{\sigma_n^2 E} \sum_{k=1}^{N_a} |\alpha_k|^2 \tilde{\gamma}_k \quad (21)$$

where $\tilde{\gamma}_k$ is the optimal effective deflection coefficient corresponding to the case in which $\hat{\mathbf{p}}_a^+$ reaches its Cramér-Rao lower bound (CRLB), illustrated in the following proposition.

Proposition 1. Given that position uncertainty is much smaller than the distance between the target and the agent, i.e., $\sqrt{\text{trace}\{\mathbf{R}_k\}} \ll c\tau_k$, the optimal effective deflection coefficient $\tilde{\gamma}_k$ is given by

$$\tilde{\gamma}_k = \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} |\bar{s}_{k,i}|^2 |\bar{s}_{k,j}|^2 \exp\left(-\frac{1}{2} \rho_{ij}^2 \mathbf{u}_k^T \mathbf{J}_e^{-1}(\mathbf{p}_a) \mathbf{u}_k\right)$$

where $\bar{s}_{k,i}$ is the signal in frequency domain; $\rho_{ij} = \omega_i - \omega_j$; $\mathbf{u}_k = \partial\tau_k / \partial\mathbf{p}_a^T$; $\mathbf{J}_e(\mathbf{p}_a)$ is the equivalent Fisher information matrix (EFIM) of \mathbf{p}_a .

Proof. The proof is omitted due to limited space. The results can be obtained following the Taylor expansion of τ_k and expressing the deviation as a zero-mean Gaussian random variable with the EFIM as its covariance [12]. \square

4. NUMERICAL RESULTS

In this section, we demonstrate our proposed cooperative GLRT detection scheme in a scenario where four agents cooperatively detect an unknown target. The four agents locate at the corners of a 1.2 km \times 1.2 km square area while the target is randomly generated within a 60 m \times 60 m square area. The number of snapshots N_s and elements of a array M are set to be 32 and 10, respectively. The array element spacing is set to be half of the wavelength. The standard deviations of position uncertainties for four agents are set to be [3 m, 3 m, 5 m, 5 m]. The signal-to-noise ratio (SNR) is defined as the ratio of the signal energy to the noise variance. A conventional detection scheme that employs a two-step estimation method is compared with the proposed scheme.

4.1. Evaluation of Target Localization Accuracy

With respect to the two-step method, we estimate the parameters of delays and AOAs and then locate the target based on the estimated intermediate parameters [8]. The CRLB of the target position is also calculated for comparison. Fig. 2 shows the CRLB and the root mean squared errors (RMSEs) of the target position estimation. It can be observed that the proposed method has a superior localization performance to the two-step method, which can be interpreted as the joint processing gain from direct localization.

4.2. Cooperative Detection Performance

We compare the performance of the modified GLRT based on direct localization with its competitor based on the two-step estimation [8]. The false alarm rate is set as $P_{fa} = 10^{-3}$. Fig. 3 depicts the probability of detection (P_D) as a function of SNR. Both curves are plotted based on 10^5 Monte Carlo experiments. In addition, the analytical curve of the optimal detector determined by λ_M^* is also depicted as an upper bound

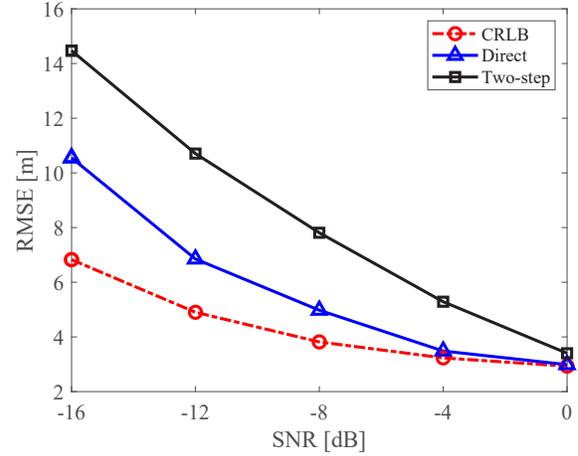


Fig. 2. CRLB and RMSEs of target localization

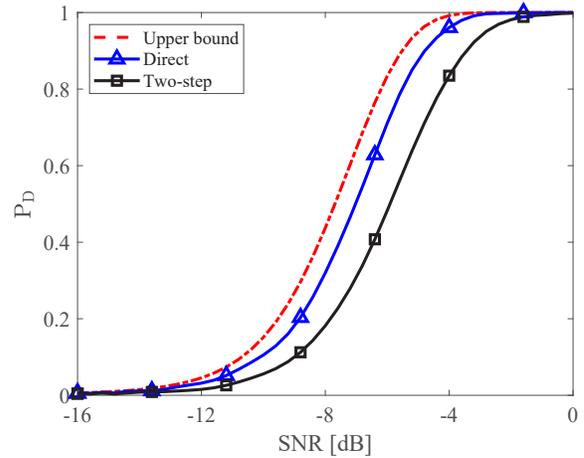


Fig. 3. Detection performance comparison

of the detection performance. As expected, the proposed cooperative detection scheme using direct localization outperforms the conventional scheme and mitigates the influence of position uncertainty by narrowing the SNR gap between the curves of upper bound and two-step for more than 50% when P_D equals 0.5.

5. CONCLUSION

In this paper, we proposed a cooperative detection scheme for mobile multi-agent networks. Direct localization was utilized to enhance the detection performance by jointly estimating target and agent positions. The influence of position uncertainty on the detection performance was also explained through the modified GLRT. Numerical results illustrate that the proposed scheme outperforms the conventional method when considering position uncertainty.

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