AN EFFICIENCY-IMPROVED TDOA-BASED DIRECT POSITION DETERMINATION METHOD FOR MULTIPLE SOURCES

Kegang Hao and Qun Wan

Department of Information and Communication Engineering
University of Electronic Science and Technology of China

ABSTRACT

It is well known that the direct position determination (D-PD) method outperforms the most common two-step localization method when the signal-to-noise ratio (SNR) is low. The advantage comes from the fact that the DPD method avoids estimating the intermediate parameters for localization. However, the DPD method has heavy computation load because of depending on exhaustive searching, especially in the multiple sources localization scenario where the problem is a high-dimensional optimization problem. In this paper, we constructed a cost function using the orthogonal relationship between received signals and noise. We reveal that the nature of the cost function is to verify the column-correlation of the matrix. Finally, we got the Determinant-based cost function which is more efficient and requires less computation resources in searching phase.

Index Terms— Direct position determination, efficient multiple sources localization, TDOA

1. INTRODUCTION

Multiple sources localization has been study for several decades and developed as the demand of Location-Based Services (LBS) in communication systems grew explosively. The emerging Technology such as Internet of Cars and Unmanned Aerial Vehicle require much more location information which promotes the development of multiple sources localization Technology ulteriorly. The classical localization methods are two-step processing [1, 2]. Firstly, intermediate parameters that rely on the locations of the sources are estimated from the received signals. These parameters are usually the angle of arrival (AOA), TDOA, Doppler frequency shift (DFS) or received signal strength (RSS). And then the previously estimated parameters are used to estimate the location of sources, using geometric or statistic considerations. The two-step method is suboptimal because the intermediate parameters are estimated by each of the observation stations independently and the global constraint that all intermediate parameters relate to the same source is not effective. Therefore the two-step method has to associate estimated parameters with their relevant sources in multiple sources localization. Fortunately, the DPD methods, proposed in [3] for single source and proposed in [4] for multiple sources, can solve the two previous problems of two-step method perfectly since it direct parameterizes the received signal data by the location of sources. There is no intermediate parameters to estimate and the association of the parameters and location of sources are realized implicitly. It is easy to know that the DPD method outperforms the two-step method, especially at low SNR. On the other hand, the localization capacity of DPD method is expanded by using all observations together to increase the degree of freedom of data. A DPD method uses the Maximum Likelihood (ML) criterion on the location-parameterized data model and ends with a cost function which requires the exhaustive searching. Therefore the DPD method can be considered achieving better performance by costing more computational resources. It is a high dimensional searching in multiple sources localization scenario, which costs too much computational resources. It is necessary to design the efficient algorithm.

In [4], the authors solved the high dimensional searching for multiple sources localization problem by two dimensional searching based on the subspace decomposition. In [5], the authors separated signal corresponding to each source one another and then estimated the position of each source using an iterative processing respectively. In [6], the authors decoupled the sources by Minimum Variance Distortionless Response (MVDR) approach and then acquire a high resolution localization algorithm. In [7], the authors solved the high-dimensional optimization by a sequence of low-dimensional optimizations using the alternating projection technique. These decoupling methods improved the efficiency of DPD for multiple sources but all end with calculating a low order Eigen-decomposition at every grid point. If the order is low, the computation load can be accepted even though the number of grid points could be large. However, the order is equal to the number of the observation stations. In TDOA-based multiple sources localization scenario, the number of the observation stations has to be larger than the number of

Corresponding author: wanqun@uestc.edu.cn.

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sources [8] which make it possible that the order of matrix for Eigen-decomposition is large and lots of computation resources are required. In order to reduce the computation load of the TDOA-based multiple sources localization method, we constructed a cost function using the orthogonal relationship between received signals and noise based on the subspace decomposition and reveal that the nature of the cost function is to verify the column-correlation of some matrix. Finally, we got a Determinant-based cost function which leads to less computation load than Eigen-decomposition-based one.

2. PROBLEM FORMULATION

Consider P transmitters whose positions are denoted by the vectors of coordinates $\boldsymbol{l}_p, p = 1, \ldots, P$ and N observation stations whose positions are denoted by the vectors of coordinates $\boldsymbol{p}_n, n = 1, \ldots, N$. Every observation station is equipped with single sensor. The complex envelope of the waveform observed by the n-th station is given as

$$x_n(t) = \sum_{p=1}^{P} \eta_{np} s_p(t - \tau_n(\boldsymbol{l}_p)) + w_n(t) \qquad 0 < t \leq T \tag{1}$$

where η_{np} is an unknown complex scalar representing the signal attenuation due to path loss between the n-th station and the p-th transmitter. $s_p(t - \tau_n(\boldsymbol{l}_p))$ is the p-th emission signal delayed by $\tau_n(\boldsymbol{l}_p) = \|\boldsymbol{p}_n - \boldsymbol{l}_p\|/c$, c is the light velocity. $w_n(t)$ represents zero-mean, white, circular complex Gaussian noise. The observed signal time interval $[0, T]$ can be partitioned into K sections, each of length T/K. It is assumed that $T/K \gg \max_p \tau_n(\boldsymbol{l}_p)$, which can be obtained by using long enough observation interval for the region of interest. Therefore the n-th observed signal in k-th section denoted by the $M \times 1$ vector of Fourier coefficients is given by

$$\bar{\boldsymbol{x}}_n(k) = \boldsymbol{A}_n(\boldsymbol{L})\bar{\boldsymbol{s}}(k) + \bar{\boldsymbol{w}}_n(k) \tag{2}$$

where

$$\begin{aligned}
\boldsymbol{A}_n(\boldsymbol{L}) &\triangleq [\eta_{n1}\boldsymbol{Q}_{n1} \quad \eta_{n2}\boldsymbol{Q}_{n2} \quad \cdots \quad \eta_{nP}\boldsymbol{Q}_{nP}] \in \mathbb{C}^{M\times MP} \\
\boldsymbol{Q}_{np} &\triangleq diag(e^{j2\pi f_1 \tau_n(\boldsymbol{l}_p)}, \ldots, e^{j2\pi f_M \tau_n(\boldsymbol{l}_p)}) \in \mathbb{C}^{M\times M} \\
\bar{\boldsymbol{s}}(k) &\triangleq [\bar{\boldsymbol{s}}_1^T(k) \quad \bar{\boldsymbol{s}}_2^T(k) \quad \bar{\boldsymbol{s}}_P^T(k)]^T \in \mathbb{C}^{MP}
\end{aligned} \tag{3}$$

therein $\bar{\boldsymbol{s}}_p(k) \in \mathbb{C}^M$ and $\bar{\boldsymbol{w}}_n(k) \in \mathbb{C}^M$represents M Fourier coefficients of the p-th emission signal and the n-th received noise respectively corresponding to frequencies $f_1, \ldots, f_M$. $\boldsymbol{L}$ is the set of all transmitters' position vectors.

Now, concatenate Fourier-coefficient vectors from all observation stations

$$\begin{aligned}
\bar{\boldsymbol{x}}(k) &\triangleq [\bar{\boldsymbol{x}}_1^T(k) \quad \bar{\boldsymbol{x}}_2^T(k) \quad \bar{\boldsymbol{x}}_N^T(k)]^T \in \mathbb{C}^{MN} \\
\boldsymbol{A}(\boldsymbol{L}) &\triangleq [\boldsymbol{A}_1^T(\boldsymbol{L}) \quad \boldsymbol{A}_2^T(\boldsymbol{L}) \quad \boldsymbol{A}_N^T(\boldsymbol{L})]^T \in \mathbb{C}^{MN\times MP} \\
\bar{\boldsymbol{w}}(k) &\triangleq [\bar{\boldsymbol{w}}_1^T(k) \quad \bar{\boldsymbol{w}}_2^T(k) \quad \bar{\boldsymbol{w}}_N^T(k)]^T \in \mathbb{C}^{MN}
\end{aligned} \tag{4}$$

We get

$$\bar{\boldsymbol{x}}(k) = \boldsymbol{A}(\boldsymbol{L})\bar{\boldsymbol{s}}(k) + \bar{\boldsymbol{w}}(k) \tag{5}$$

The column-space of $\boldsymbol{A}(\boldsymbol{L})$ is known as local manifold of the received signal. Without loss of generality, we can assume in $\boldsymbol{A}(\boldsymbol{L})$ that

$$\sum_{n=1}^{N} |\eta_{np}|^2 = 1 \tag{6}$$

The problem considered here may be stated briefly: Given a series of samples $\bar{\boldsymbol{x}}(1), \bar{\boldsymbol{x}}(2), \ldots, \bar{\boldsymbol{x}}(K)$ and the data model in (5), estimate all transmitters' position vectors $\boldsymbol{L}$ efficiently.

3. THE EFFICIENCY-IMPROVED DPD METHOD

3.1. Decoupling the transmitters

We start with approximating the covariance of the received signal using the covariance of examples $\hat{\boldsymbol{R}}$, which is a good approximation when the number of sections K is big enough

$$\hat{\boldsymbol{R}} \triangleq \frac{1}{K}\sum_{k=1}^{K} \bar{\boldsymbol{x}}(k)\bar{\boldsymbol{x}}^H(k) \tag{7}$$

And then apply the Eigen-decomposition technique

$$\hat{\boldsymbol{R}} = \boldsymbol{U}_S\boldsymbol{\Lambda}_S\boldsymbol{U}_S^H + \boldsymbol{U}_N\boldsymbol{\Lambda}_N\boldsymbol{U}_N^H \tag{8}$$

where the diagonal matrices $\boldsymbol{\Lambda}_S \in \mathbb{R}^{MP\times MP}$ and $\boldsymbol{\Lambda}_N \in \mathbb{R}^{M(N-P)\times M(N-P)}$ consist of the group of P bigger eigen-values and the rest of eigen-values respectively. The signal subspace consists of columns of $\boldsymbol{U}_S \in \mathbb{C}^{MN\times MP}$ and the noise subspace consists of columns of $\boldsymbol{U}_N \in \mathbb{C}^{MN\times M(N-P)}$. Note that the number of transmitters P can be estimated by model order determination based on frequency-domain MDL criterion, as described in [9].

The orthogonal relationship between the local manifold of the received signal and the noise subspace is given as

$$\boldsymbol{U}_N^H\boldsymbol{A}(\boldsymbol{L}) = \boldsymbol{0} \tag{9}$$

According to (3) and (4), the local manifold of the received signal $\boldsymbol{A}(\boldsymbol{L})$ can be divided into P blocks

$$\boldsymbol{A}(\boldsymbol{L}) \triangleq [\boldsymbol{V}(\boldsymbol{l_1}) \quad \boldsymbol{V}(\boldsymbol{l_2}) \quad \boldsymbol{V}(\boldsymbol{l}_P)] \tag{10}$$

then there are P orthogonal relationships corresponding to P transmitters respectively

$$\boldsymbol{U}_N^H\boldsymbol{V}(\boldsymbol{l}_p) = \boldsymbol{0}, p = 1, \ldots, P \tag{11}$$

where

$$\boldsymbol{V}(\boldsymbol{l}_p) \triangleq \begin{bmatrix} \eta_{1p}\boldsymbol{Q}_{1p} \\ \eta_{2p}\boldsymbol{Q}_{2p} \\ \vdots \\ \eta_{Np}\boldsymbol{Q}_{Np} \end{bmatrix} \tag{12}$$

It is worthy mentioning that the subspace-decomposition technique is also applied in [4] but we construct a more efficient cost function using the noise subspace instead of the signal subspace.

3.2. Construct the efficient cost function

In subsection 3.1, the P transmitters are decoupled to satisfy P orthogonal relationships respectively. Furthermore, the P equations are the same except for the locations of transmitters and the attenuation. Therefore a basic cost function with respect to the location variable $\boldsymbol{l}$ and the attenuation variable $\boldsymbol{\eta_l}$ can be construct to verify those orthogonal relationships

$$f_0(\boldsymbol{l}, \boldsymbol{\eta_l}) \triangleq \|\boldsymbol{U}_N^H \boldsymbol{V}(\boldsymbol{l}, \boldsymbol{\eta_l})\|_F \tag{13}$$

where

$$\boldsymbol{\eta_l} \triangleq [\eta_{1\boldsymbol{l}} \quad \eta_{2\boldsymbol{l}} \quad \cdots \quad \eta_{N\boldsymbol{l}}]^T, \qquad \|\boldsymbol{\eta_l}\| = 1$$

and $\|\cdot\|_F$ denotes the Frobenius norm. the $\|\boldsymbol{\eta_l}\| = 1$ is from (6). Dividing $\boldsymbol{U}_N^H$ into N blocks,

$$\boldsymbol{U}_N^H \triangleq \begin{bmatrix} \boldsymbol{G}_1 & \boldsymbol{G}_2 & \cdots & \boldsymbol{G}_N \end{bmatrix} \tag{14}$$

and substituting (12) and (14) in the cost function (13), we get

$$\begin{aligned} f_0(\boldsymbol{l}, \boldsymbol{\eta_l}) &= \|\sum_{n=1}^{N} \eta_{n\boldsymbol{l}} \boldsymbol{G}_n \boldsymbol{Q}_n(\boldsymbol{l})\|_F \qquad (15) \\ &= \|\sum_{n=1}^{N} \eta_{n\boldsymbol{l}} \mathrm{Vec}(\boldsymbol{G}_n \boldsymbol{Q}_n(\boldsymbol{l}))\|_F \end{aligned}$$

where the notation $\mathrm{Vec}(\cdot)$ denotes the vectorization operation. The second equation is true because $\|\cdot\|_F$ is an entrywise norm.

Defining

$$\Psi(\boldsymbol{l}) \triangleq \begin{bmatrix} \mathrm{Vec}(\boldsymbol{G}_1\boldsymbol{Q}_1(\boldsymbol{l}))^T \\ \mathrm{Vec}(\boldsymbol{G}_2\boldsymbol{Q}_2(\boldsymbol{l}))^T \\ \vdots \\ \mathrm{Vec}(\boldsymbol{G}_N\boldsymbol{Q}_N(\boldsymbol{l}))^T \end{bmatrix}^T \in \mathbb{C}^{M^2(N-P)\times N}$$

the cost function in (15) can be rewritten as

$$f_0(\boldsymbol{l}, \boldsymbol{\eta_l}) = \|\Psi(\boldsymbol{l})\boldsymbol{\eta_l}\|_F \tag{16}$$

The purpose of the cost function is to find out the set of solution pairs $(\boldsymbol{l}_p, \boldsymbol{\eta}_{\boldsymbol{l}_p})$ of equation $f_0(\boldsymbol{l}, \boldsymbol{\eta_l}) = 0$ which means the orthogonal relationships in (11). Equivalently, we have the homogeneous linear equations

$$\Psi(\boldsymbol{l})\boldsymbol{\eta_l} = \mathbf{0} \tag{17}$$

Given $\|\boldsymbol{\eta_l}\| = 1$, equation (17) having nontrivial solutions means that the columns of $\Psi(\boldsymbol{l})$ are linear correlation, i.e., $rank(\Psi(\boldsymbol{l})) < N$. Therefore, the new objective function is just with respect to the location variable $\boldsymbol{l}$

$$f_1(\boldsymbol{l}) \triangleq \begin{cases} \mathrm{rank}(\Psi(\boldsymbol{l})) < N & \text{if} \quad \boldsymbol{l} \in \boldsymbol{L} \\ \mathrm{rank}(\Psi(\boldsymbol{l})) = N & \text{if} \quad \boldsymbol{l} \notin \boldsymbol{L} \end{cases} \tag{18}$$

Next, two linear algebraic lemmas, Theorem 5.2.1 and equation 5.2.27 in [10], are used to get the final efficient cost function

$$F(\boldsymbol{l}) \triangleq \det(\Psi^H(\boldsymbol{l})\Psi(\boldsymbol{l})) \begin{cases} = 0 & \text{if} \quad \boldsymbol{l} \in \boldsymbol{L} \\ > 0 & \text{if} \quad \boldsymbol{l} \notin \boldsymbol{L} \end{cases} \tag{19}$$

Considering the estimation error of the noise subspace, we can achieve a pseudo spectrum over the region of interest

$$P(\boldsymbol{l}) = \frac{1}{\det(\Psi^H(\boldsymbol{l})\Psi(\boldsymbol{l}))} \tag{20}$$

The P higher peaks of $P(\boldsymbol{l})$ correspond to the locations set of multiple transmitters $\boldsymbol{L}$.

Compared with the eigenvalue-based pseudo spectrum proposed in [4],

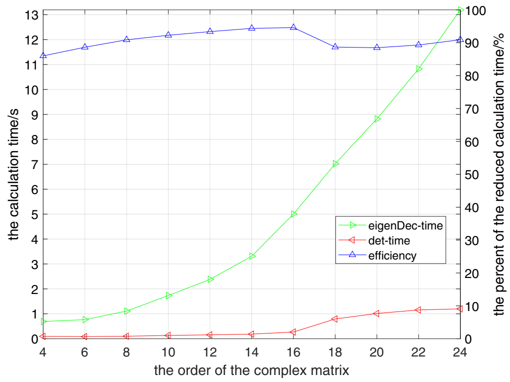
$$Q(\boldsymbol{l}) = \lambda_{max}(\boldsymbol{D}) \tag{21}$$

where matrix $\boldsymbol{D}$ has the same size as $\Psi^H(\boldsymbol{l})\Psi(\boldsymbol{l})$. it is determinant that is computed in our pseudo spectrum instead of Eigen-decomposition at every grid point. Note that the complexity of Eigen-decomposition is about $O(N^3)$ while that of determinant algorithm in [11] is about $O(N^2)$. Furthermore, the calculation will be performed thousands of times because of thousands of location points in region of interest, which make the calculation in searching phase costs the main part of the total calculation time. Therefore, our improved method will save much more calculation time and get more efficient.

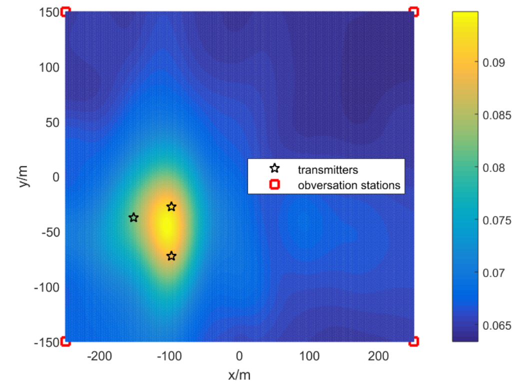
4. NUMERICAL SIMULATION RESULTS

In order to present the improved efficiency of our method (20), Fig. 1a shows that the calculation time of determinant operation (red line) and Eigen-decomposition (green line) increase with the order of the complex matrix and the improvement of efficiency at every order value. The order is varied from 4 to 24. At each order value, the two operations were performed twenty thousand of times respectively, which is about the amount of searching a region of $200 \times 100m^2$ by $1m^2$. It is clearly seen that the improved efficiency (blue line) is more than 85% when the order of the complex matrix varies between 4 and 24.

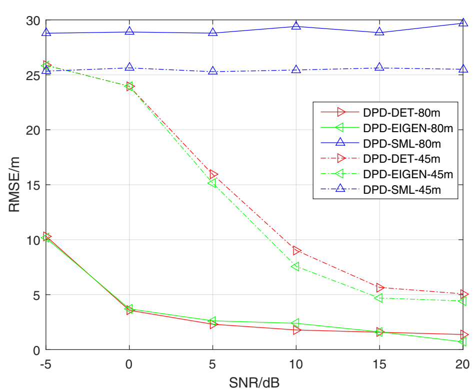
Consider four single sensors located at coordinates (-250,-150),(-250,150),(250,150) and (250,-150)m, and two transmitters transmitting unknown Gaussian signals. The channel attenuation is a circular, complex Gaussian, random variable, with mean of 1 and standard deviation of 0.1 and independent



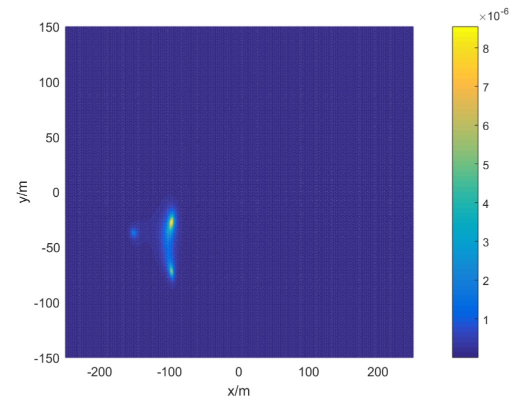
(a) calculation time vs. the order of the matrix



(a) the DPD-SML spectrum



(b) RMSE vs. SNR



(b) the DPD-DET spectrum

Fig. 1: the performance of the proposed spectrum

Fig. 2: the resolution of the proposed spectrum

among the pairs of sensor and transmitter. The location estimation is based on $K = 1280$ sections of $M = 32$ Fourier coefficients associated with frequencies up to 2MHz. the S-NR is varied from -10dB to 20dB. we performed 200 Monte Carlo simulations at each SNR to compare the performance of our pseudo spectrum (20), denoted by DPD-DET and the one in [4], denoted by DPD-EIGEN. The single emitter ML estimator in [3, 6], denoted by DPD-SML, is also compared with the proposed method in this paper. Fig. 1b shows two group of the Root Mean Square of Error (RMSE). The full line group denotes the distance of two transmitters is 80m and the imaginary line group denotes 45m.

It can be seen that the accuracy of DPD-DET and DPD-EIGEN is approximate at every SNR value in both groups and they both outperform the SML estimator, which indicates that the proposed method and DPD-EIGEN decoupled the transmitters while the SML estimator attempted to fit both signals.

The derivation of our efficient objective function is based on the subspace decomposition which is the fundamental of the classical Multiple Signal Classification (MUSIC) algorithm. It is well-known that MUSIC has the super resolution. In order to examine the resolution of DPD-DET, consider three neighboring transmitters located at (-97.5,-72.5),(-97.5,-27.5) and (-152.5,-37.5)m respectively and increasing four more observation stations. Fig. 2a and Fig. 2b demonstrate the spectrums of SML estimator and the DPD-DET spectrum (20) at SNR of 20dB. It is clearly seen that there are three sharp peaks in the DPD-DET spectrum (20) while only one wide main lobe in the spectrum of SML estimator.

5. CONCLUSION

In this paper, we constructed a determinant-based cost function using the noise subspace. The improved estimator reserves the accuracy and super resolution of the MUSIC-like method and the reduced calculation load was more than 85% in our simulation, which make the improved method reliable and more real-time. Furthermore, the idea of the determinant-based objective function can be also used in AOA-based localization.

6. REFERENCES

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