A DATA-SELECTIVE LS SOLUTION TO TDOA-BASED SOURCE LOCALIZATION

J.A. Apolinário Jr., H. Yazdanpanah, A.S. Nascimento Fº

Military Institute of Engineering (IME)
Programs of Electrical/Defense Engineering
Rio de Janeiro, Brazil

M.L.R. de Campos

Federal University of Rio de Janeiro (UFRJ)
COPPE-Electrical Engineering Program
Rio de Janeiro, Brazil

ABSTRACT

In this paper, the localization of an emitter based on Time Difference of Arrival (TDoA) has been investigated. The classical least-squares (LS) algorithm, with a limited number of TDoA measurements, has been utilized for obtaining a closed-form solution to the source localization problem. Recently, an extension of the classical LS algorithm has been employed in an attempt to improve the precision of the localization technique by using a larger set of TDoA estimates. However, considering all TDoA values can eventually degrade the accuracy of the localization method due to the presence of heavily noisy measurements. In this work, by employing a data-selective approach, we have proposed a closed-form LS solution that disregards bad measurements. To this end, we have used two distinct objective functions, one to obtain a solution and a second one to test that particular solution among all possible ones within a subset of measurements. Simulation results indicate the superior performance of the proposed algorithm in the source localization problem.

Index Terms— Time difference of arrival, source localization, data selection, least squares.

1. INTRODUCTION

Source localization problem has attained a remarkable interest in the signal processing literature for the past few decades. It has applications in many fields such as telecommunications [1], radar [2], sonar [3], wireless sensor networks [4], mobile communications [5], military [6], etc. Most localization strategies are based on Received Signal Strength (RSS) [7], RF fingerprinting [8], Direction of Arrival (DoA) [9], Time of Arrival (ToA) [10], and Time Difference of Arrival (TDoA) [11–13] of the emitted signal.

The RSS approach utilizes the received signal energy for localization, and its precision can be degraded by fading of wireless signals [7]. The DoA technique measures the direction of arrival of the received signals and, for this purpose, it needs either directional antennas or antenna arrays. In general, this method requires costly antenna array, complex hardware, and its accuracy reduces in the case of signal reflection [14].

The ToA approach uses the signal travel time from the emitters to the receivers, whereas the TDoA strategy measures the difference of transmission time of a single signal between the receiver nodes; thus it requires at least three receiving sensors. The ToA technique requires a highly accurate synchronization of clocks in the emitter and receiver nodes; however, in the TDoA approach, only the receiver nodes must be synchronized [15, 16]. Therefore, the TDoA approach is more robust to reflections and easier to implement, having lower computational burden and requiring simpler antennae [17, 18].

Localization of an emitter based on TDoAs has had renewed interest with new technologies and applications, such as satellites [19], GPS (and other geo-positioning systems) [20], and mobile telephony [21]. The work in [22], an extension of the classical least squares (LS) solution presented in [23–25], has clearly shown that using a larger set of TDoA measurements tends to increase the accuracy of the localization method. The work presented herein has focused on the selection of TDoA measurements, taking into advantage the fact that it is better to discard bad measurements than using all available ones. The concept of data-selective approach is employed to obtain a closed-form LS solution to the TDoA-based source localization problem. It differs from a similar approach used in [26] which needs to perform a grid search instead of having a closed-form expression; moreover, this work was motivated by the use of data selection in the context of Direction of Arrival (DoA) estimation with signals from a microphone array [27]. The proposed method, in order to select a subset of TDoA measurements, uses a second cost function, other than the one used to obtain the position estimate corresponding to that subset. A recent work [28] uses a similar measurement to evaluate their results and, although using a different technique, enhances the need for outlier removal of TDoA measurements in order to have better localization accuracy.

The rest of the paper is organized as follows. Section 2 presents the fundamentals of TDoA-based source localization, and Section 3 describes the proposed approach. Section 4 gives the experimental results while Sections 5 summarizes the conclusions.

2. THE CLASSICAL LS APPROACH

In a 2D scenario, we consider M sensors with known positions given by $\mathbf{p}_m$, $1 \leq m \leq M$, and $N = \frac{M(M-1)}{2}$ TDoAs, from τ_{21} to $\tau_{M(M-1)}$. Assuming each TDoA given in number of samples, we define the range-difference Δd_{ij} between the distance from the unknown source and sensors i and j such that, letting v be the speed of propagation and f_s the sampling frequency, we have $\Delta d_{ij} = d_i - d_j = \frac{v\tau_{ij}}{f_s}$, $i > j$. The TDoA, τ_{ij}, is usually obtained from the peak of the cross-correlation of the signals acquired by the sensors.

For the m-th sensor, we define d_m as the distance from the source, assumed it at the unknown position $\mathbf{p}$, to the m-th sensor. Therefore, we can write $\|\mathbf{p} - \mathbf{p}_1\|^2 = d_1^2$ and

$$\|\mathbf{p} - \mathbf{p}_m\|^2 = (d_1 + \Delta d_{m1})^2 . \quad (1)$$

Equation (1) leads to

$$(\mathbf{p}_m - \mathbf{p}_1)^{\mathrm{T}} \mathbf{p} + \Delta d_{m1} d_1 = b_{1m}, \quad (2)$$

This work was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brazil (CAPES) – Financing Code 88881.117868/2016-01, and also by the Conselho Nacional de Desenvolvimento Científico e Tecnológico – Brazil (CNPq).

where $b_{1m} = \frac{\|\mathbf{p}_m\|^2 - \|\mathbf{p}_1\|^2 - \Delta d_{m1}^2}{2}$, $2 \leq m \leq M$.

The previous equation should hold for the theoretical case of no measurement nor sensor calibration errors. A reasonable estimate of $\mathbf{p}$ can be obtained by minimizing a LS cost function given by

$$\xi_1 = \sum_{m=2}^{M} \left[\left((\mathbf{p}_m - \mathbf{p}_1)^{\mathrm{T}} \mathbf{p} + \Delta d_{m1} d_1 - b_{1m} \right]^2, \qquad (3)$$

which can be expressed as the squared norm of an error vector defined as follows.

$$\mathbf{e}_1 = \underbrace{\begin{bmatrix} (\mathbf{p}_2 - \mathbf{p}_1)^{\mathrm{T}} & \Delta d_{21} \\ (\mathbf{p}_3 - \mathbf{p}_1)^{\mathrm{T}} & \Delta d_{31} \\ \vdots & \vdots \\ (\mathbf{p}_M - \mathbf{p}_1)^{\mathrm{T}} & \Delta d_{M1} \end{bmatrix}}_{\mathbf{A}_1} \underbrace{\begin{bmatrix} \mathbf{p} \\ d_1 \end{bmatrix}}_{\mathbf{x}_1} - \underbrace{\begin{bmatrix} b_{12} \\ b_{13} \\ \vdots \\ b_{1M} \end{bmatrix}}_{\mathbf{b}_1} \qquad (4)$$

Note, from Equation (4) and assuming a 2D scenario, that $\mathbf{A}_1$ is an $(M-1) \times 3$ matrix such that, if we have at least four sensors ($M \geq 4$), an unconstrained least squares solution is obtained after equating to zero the gradient of $\xi_1 = \mathbf{e}_1^{\mathrm{T}} \mathbf{e}_1 = \|\mathbf{A}_1 \mathbf{x}_1 - \mathbf{b}_1\|^2$ with respect to vector $\mathbf{x}_1$. The estimated position of the source is then given by

$$\hat{\mathbf{p}} = [\mathbf{I} \;\; \mathbf{0}] \left(\mathbf{A}_1^{\mathrm{T}} \mathbf{A}_1 \right)^{-1} \mathbf{A}_1^{\mathrm{T}} \mathbf{b}_1, \qquad (5)$$

where $\mathbf{I}$ is a 2×2 identity matrix and $\mathbf{0}$ is a 2×1 null vector. This closed-form solution is equivalent to the one found in [25], also presented in [23, 24], without the constraint of having a reference sensor at the origin.

3. THE PROPOSED APPROACH

The solution in Equation (5), since matrix $\mathbf{A}_1$ is $(M-1) \times 3$ for a 2D estimation, works well for at least $M = 4$ sensors. One could think that only three sensors should be required for estimating the location of the source: nevertheless, $M = 3$ sensors may present the ambiguity of two possible solutions [29]. Therefore, for the purpose of this work, we assume that $M \geq 4$.

Another feature regarding the LS solution derived in the previous section is the fact that it uses only $M - 1$ from the total of $N = M(M-1)/2$ possibly available TDoAs measurements. In order to use a larger number of measurements, we start by presenting an extended version of the LS estimator which uses all available TDoAs. Although the localization of the sensors, $\mathbf{p}_i$, can be obtained with arbitrary precision, TDoAs, which are represented in the data matrix as Δd_{ij}, may be misestimated, especially in low SNR conditions. In the existence of outliers, working with a subset of TDoA measurements may provide a more accurate location estimation. This data-selective approach has been used successfully in Direction of Arrival (DoA) estimation [27] (LS closed solution) and source localization [26] (grid search), where presumably misestimated time delays and time differences are discarded.

3.1. The extended LS solution

As previously mentioned, the unconstrained LS solution presented in Equation (5) uses only $M - 1$ from a total of $N = \frac{M(M-1)}{2}$ TDoA measurements. Clearly, when we assume the possibility of similar measurement errors, using more measurements leads to a more accurate solution. To extend the number of measurements to be used in estimating the position of the source, we may define another cost function from an expression similar to Equation (1):

$$\|\mathbf{p} - \mathbf{p}_m\|^2 = \left(d_2 + \Delta d_{m2} \right)^2. \qquad (6)$$

From Equation (6), we define another cost function as the squared norm of an $(M-2) \times 1$ error vector $\mathbf{e}_2$:

$$\xi_2 = \sum_{m=3}^{M} \left((\mathbf{p}_m - \mathbf{p}_2)^{\mathrm{T}} \mathbf{p} + \Delta d_{m2} d_2 - b_{2m} \right)^2 = \mathbf{e}_2^{\mathrm{T}} \mathbf{e}_2, \qquad (7)$$

where $b_{2m} = \frac{\|\mathbf{p}_m\|^2 - \|\mathbf{p}_2\|^2 - \Delta d_{m2}^2}{2}$, $3 \leq m \leq M$, and

$$\mathbf{e}_2 = \underbrace{\begin{bmatrix} (\mathbf{p}_3 - \mathbf{p}_2)^{\mathrm{T}} & \Delta d_{32} \\ \vdots & \vdots \\ (\mathbf{p}_M - \mathbf{p}_2)^{\mathrm{T}} & \Delta d_{M2} \end{bmatrix}}_{\mathbf{A}_2} \underbrace{\begin{bmatrix} \mathbf{p} \\ d_2 \end{bmatrix}}_{\mathbf{x}_2} - \underbrace{\begin{bmatrix} b_{23} \\ \vdots \\ b_{2M} \end{bmatrix}}_{\mathbf{b}_2}. \qquad (8)$$

Similarly, we define matrices

$$\mathbf{A}_3 = \begin{bmatrix} (\mathbf{p}_4 - \mathbf{p}_3)^{\mathrm{T}} & \Delta d_{43} \\ \vdots & \vdots \\ (\mathbf{p}_M - \mathbf{p}_3)^{\mathrm{T}} & \Delta d_{M3} \end{bmatrix} \qquad (9)$$

to $\mathbf{A}_{M-1} = \begin{bmatrix} (\mathbf{p}_M - \mathbf{p}_{M-1})^{\mathrm{T}} & \Delta d_{M(M-1)} \end{bmatrix}$, and vectors $\mathbf{b}_3 = [b_{34} \; \cdots \; b_{3M}]^{\mathrm{T}}$, where $b_{3m} = \frac{\|\mathbf{p}_m\|^2 - \|\mathbf{p}_3\|^2 - \Delta d_{m3}^2}{2}$, $4 \leq m \leq M$, to

$$\mathbf{b}_{M-1} = \frac{\|\mathbf{p}_M\|^2 - \|\mathbf{p}_{M-1}\|^2 - \Delta d_{M(M-1)}^2}{2}, \qquad (10)$$

a scalar.

With these definitions, we form an extended cost function using all N TDoAs measurements:

$$\xi = \sum_{m=1}^{M} \xi_m, \qquad (11)$$

where $\xi_m = \mathbf{e}_m^{\mathrm{T}} \mathbf{e}_m$, and $\mathbf{e}_m = \mathbf{A}_m [\mathbf{p}^{\mathrm{T}} \; d_m]^{\mathrm{T}} - \mathbf{b}_m$.

From Equation (11) and the definitions of ξ_m, $\mathbf{e}_m$ and $\mathbf{A}_m$, we could express the extended cost function as the squared norm of an extended error vector, $\xi = \mathbf{e}^{\mathrm{T}} \mathbf{e}$, the extended error vector $\mathbf{e}$ being defined as

$$\underbrace{\begin{bmatrix} (\mathbf{p}_2 - \mathbf{p}_1)^{\mathrm{T}} & \Delta d_{21} & 0 & 0 \cdots 0 \\ \vdots & \vdots & \vdots & \vdots \quad \vdots \\ (\mathbf{p}_M - \mathbf{p}_1)^{\mathrm{T}} & \Delta d_{M1} & 0 & 0 \cdots 0 \\ (\mathbf{p}_3 - \mathbf{p}_2)^{\mathrm{T}} & 0 & \Delta d_{32} & 0 \cdots 0 \\ \vdots & \vdots & \vdots & \vdots \quad \vdots \\ (\mathbf{p}_M - \mathbf{p}_2)^{\mathrm{T}} & 0 & \Delta d_{M2} & 0 \cdots 0 \\ & \vdots & & \vdots \\ (\mathbf{p}_M - \mathbf{p}_{M-1})^{\mathrm{T}} & 0 & 0 & \cdots 0 \, \Delta d_{M(M-1)} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \mathbf{p} \\ d_1 \\ d_2 \\ \vdots \\ d_{M-1} \end{bmatrix}}_{\mathbf{x}} - \underbrace{\begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_{M-1} \end{bmatrix}}_{\mathbf{b}}, \qquad (12)$$

such that the extended LS solution is given by

$$[\hat{\mathbf{p}}^{\mathrm{T}} \; \hat{d}_1 \; \hat{d}_2 \; \cdots \; \hat{d}_{M-1}]^{\mathrm{T}} = \left(\mathbf{A}^{\mathrm{T}} \mathbf{A} \right)^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{b}. \qquad (13)$$

This extended closed-form LS solution using the full set of the available TDoA measurements has been introduced in [22].

3.2. The data selection approach

The extended LS cost function ξ, in Equation (11), uses all N TDoA measurements. Whenever all measurements contain similar errors, using them all leads to a more accurate position estimation and this was, certainly, the main motivation for the extended version of this method. However, when estimating the TDoAs using the peak of the cross-correlation between the signals of two sensors, it is quite often the case, due to noise or any other interfering signal, that a secondary peak is chosen instead of the correct one. When it happens, the corresponding TDoA uncertainty may not be small, degrading the position estimate. In that case, a scheme to select a subset of measurements may lead to better results. This corresponds to removing rows of matrix $\mathbf{A}$ (and from vector $\mathbf{b}$) in Equation (12) and, consequently, in Equation (13). For example, if we suspect that τ_{31} is incorrect, we remove the second row of $\mathbf{A}$ and the second element of $\mathbf{b}$, those corresponding to Δd_{31}.

In order to find a position estimate from a given subset with n TDoAs, out of the overall $N = \frac{M(M-1)}{2}$ possibilities, we minimize a different LS cost function, $\mathbf{e}_n^{\mathrm{T}}\mathbf{e}_n$, where $\mathbf{e}_n$ is a subset of $\mathbf{e}$ with only n elements. This data-selective approach has been employed successfully in [27] for DoA estimation of low SNR audio signals. The choice of n depends on the SNR and there might be several ways to do it. In the experiments carried out in this work, we used $M = 5$ sensors, and calculated the position estimate for each possible combination of N TDoA measurements taken $n = 6$ at a time, i.e., $\frac{N!}{n!(N-n)!}$ estimates. In order to choose which combination (or subset $\mathcal{S}_n$ of n TDoA measurements) provides the best solution, we proposed another cost function ξ_n, as defined below.

Once the value of n has been chosen, for each subset $\mathcal{S}_n$ of n TDoAs, we minimize $\mathbf{e}_n^{\mathrm{T}}\mathbf{e}_n$ and obtain $\mathbf{x}_n = \left(\mathbf{A}_n^{\mathrm{T}}\mathbf{A}_n\right)^{-1}\mathbf{A}_n^{\mathrm{T}}\mathbf{b}_n$, where $\mathbf{A}_n$ and $\mathbf{b}_n$ are obtained from Equation (12) after removing the $N-n$ rows for $\mathbf{A}_n$ and $\mathbf{b}_n$ corresponding to each combination to be tested. As matrix $\mathbf{A}$ is sparse, after removing some rows, it may also be required to remove any eventual null column in order to guarantee that a solution is obtained for that candidate subset. The estimated source position is

$$\hat{\mathbf{p}}_n = [\mathbf{I}\ \ \mathbf{0}]\underbrace{\left(\mathbf{A}_n^{\mathrm{T}}\mathbf{A}_n\right)^{-1}\mathbf{A}_n^{\mathrm{T}}\mathbf{b}_n}_{\mathbf{x}_n}. \tag{14}$$

For all values of $\hat{\mathbf{p}}_n$ obtained from the $\frac{N!}{n!(N-n)!}$ possibilities, we must calculate the squared error and choose, as optimal, the estimate $\hat{\mathbf{p}}_n$ which provides the smallest squared error. However, our experience indicates that replacing $\mathbf{x}_n$ into $\mathbf{e}_n$ to choose the smallest value of $\mathbf{e}_n^{\mathrm{T}}\mathbf{e}_n$ does not yield the most accurate results. Better results are obtained if we calculate $\Delta\hat{d}_{ij} = \hat{d}_i - \hat{d}_j = \|\hat{\mathbf{p}}_n - \mathbf{p}_i\| - \|\hat{\mathbf{p}}_n - \mathbf{p}_j\|$ for each sensor pair $\{i,j\} \in \mathcal{S}_n$. The proposed cost function becomes

$$\xi_n = \frac{1}{n}\sum_{\{i,j\}\in\mathcal{S}_n}\left(\Delta\hat{d}_{ij} - \frac{v\tau_{ij}}{f_s}\right)^2, \tag{15}$$

where all $\tau_{ij} \in \mathcal{S}_n$ correspond to the TDoA measurements belonging to the combination under test.

As an example, for $M = 5$ sensors, we have $N = 10$ TDoA measurements, choosing the best estimate using only $n = 6$ measurements requires a total of $\frac{10!}{6!4!} = 210$ combinations. A pseudocode of the proposed data-selective (DS) algorithm for estimating source localization is presented in Algorithm 3.1. An additional test restricts further the number of possible subsets: a subset $\mathcal{S}_n$ will be considered valid only if all its TDoA measurements are smaller than their maximum possible value, i.e., only if the absolute value of each of its Δd_{ij} is smaller than the distance between sensors i and j.

Algorithm 3.1: DS TDoA SOURCE LOCALIZATION (τ_{ij})

Let $\mathbf{p}_m$, $1 \le m \le M$, be the positions of M sensors
$N \leftarrow \frac{M(M-1)}{2}$
Let τ_{ij} be the N available TDoAs (in # samples)
Set v and f_s, **and choose** n (under current investigation)
$\Delta d_{ij} \leftarrow \frac{v\tau_{ij}}{f_s}$ for all N TDoAs
$|\Delta d_{ij}|_{\max} \leftarrow \|\mathbf{p}_i - \mathbf{p}_j\|$ for all N TDoAs
Form matrix $\mathbf{A}$ and vector $\mathbf{b}$ as in Eq. (12)
$\hat{\mathbf{p}}_n \leftarrow [\mathbf{I}\ \ \mathbf{0}]\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$
$\hat{d}_m \leftarrow \|\hat{\mathbf{p}}_n - \mathbf{p}_m\|$ for all $1 \le m \le M$ sensors
$\Delta\hat{d}_{ij} \leftarrow \hat{d}_i - \hat{d}_j$ for all N possible pairs $\{i,j\}$
$\xi_{min} \leftarrow \frac{1}{N}\sum_{\{i,j\}}\left(\Delta\hat{d}_{ij} - \Delta d_{ij}\right)^2$
for each subset of measurements $\mathcal{S}_n$
do {
  if all $\{|\Delta d_{ij}| \le |\Delta d_{ij}|_{\max}, \{i,j\} \in \mathcal{S}_n\}$
  then {
    Adjust $\mathbf{A}_n$ and $\mathbf{b}_n$ according to $\mathcal{S}_n$
    $\hat{\mathbf{p}}_n \leftarrow [\mathbf{I}\ \ \mathbf{0}]\left(\mathbf{A}_n^{\mathrm{T}}\mathbf{A}_n\right)^{-1}\mathbf{A}_n^{\mathrm{T}}\mathbf{b}_n$
    $\hat{d}_m \leftarrow \|\hat{\mathbf{p}}_n - \mathbf{p}_m\|$ for $1 \le m \le M$
    $\Delta\hat{d}_{ij} \leftarrow \hat{d}_i - \hat{d}_j$ for $\{i,j\} \in \mathcal{S}_n$
    $\xi_n \leftarrow \frac{1}{n}\sum_{\{i,j\}\in\mathcal{S}_n}\left(\Delta\hat{d}_{ij} - \Delta d_{ij}\right)^2$
    if $\xi_n < \xi_{min}$
      then { $\xi_{min} \leftarrow \xi_n$; $\mathbf{p}_o \leftarrow \hat{\mathbf{p}}_n$ }
return $(\mathbf{p}_o)$

4. EXPERIMENTAL RESULTS

In this section, we evaluate the performance of the proposed algorithm in two experiments. The first one consists of a simulation with two sets of sensors positioned in two different ways. The same positions were used in the second experiment where a set of microphones recorded audio signals in an indoor environment.

4.1. Simulation experiment

For this experiment, we used two distinct 2-D geometries of five sensors with perfectly known positions. In Geometries 1 and 2, the source (also with assumed known position) is located inside and outside the polygon restricted by the five receivers, respectively. We observed, from experience, that the errors to be added to the correct TDoAs are of two types. The first type, relatively small, corresponds to errors in the estimation process due to, e.g., environmental noise, and can be modeled with normal distribution. From recordings carried out for this work, the estimated variance (σ_n^2) of the TDoA error type one ranged from 28 to 48, which, defining $\sigma_\tau^2 = \frac{1}{N}\sum_{ij}\tau_{ij}^2$, τ_{ij} as the correct TDoA (obtained from the known positions and speed of sound), results in a TDoA-to-noise ratio, $\mathrm{TNR}_{\mathrm{dB}} = 10\log\frac{\sigma_\tau^2}{\sigma_n^2}$, from 24.2 to 30dB. Therefore, in our simulations, we varied $\mathrm{TNR}_{\mathrm{dB}}$ from 10 to 40dB.

The second type of errors is present if we fail (by far) to identify the correct TDoA. The TDoA is usually estimated from the cross-correlation, $r_{ij}(\tau)$, between two signals (pair $\{i,j\}$ out of the N possible pairs of sensors) by finding the lag (τ) corresponding to the peak of $r_{ij}(\tau)$. It may happen that the highest peak of the cross-correlation is not due to the time difference of arrival, but due to environmental noise, interference, multipath, defective sensor, to name

Table 1: Results from simulation and from the experiment with recorded-audio signals (localization errors in m)

TNR / # outliers	Geometry	Conv. LS	Ext. LS	DS-LS
10dB / 3	1	4.1885	4.0103	**1.3808**
	2	5.4328	3.5410	**0.3731**
20dB / 2	1	2.1831	2.2836	**0.1312**
	2	2.2328	1.7517	**0.1073**
30dB / 1	1	0.8383	0.9316	**0.0355**
	2	0.5861	0.5022	**0.0337**
40dB / 0	1	0.0140	**0.0095**	0.0110
	2	0.0108	**0.0065**	0.0108

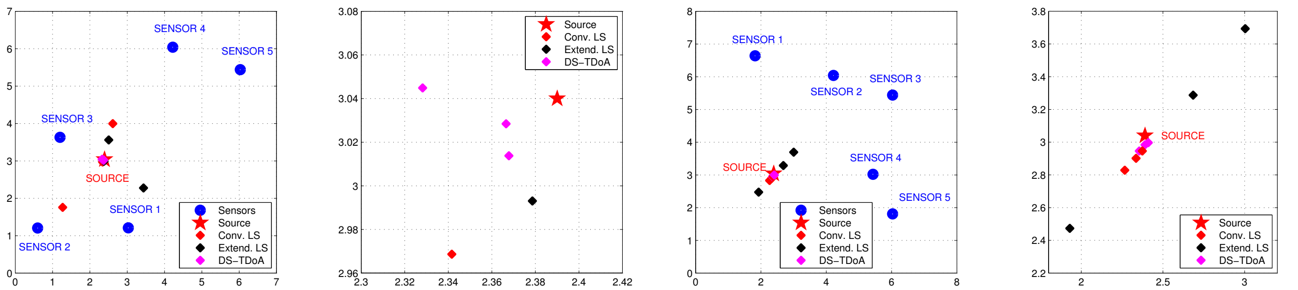
(a) Simulation (10,000 indep. runs averaged).

Signal	Conv. LS	Ext. LS	DS-LS
GCC-Classic			
Music	0.0862	0.0483	**0.0343**
White Noise	1.7013	1.2965	**0.0620**
Gunshot	0.9814	0.5321	**0.0262**
GCC-PhaT			
Music	2.0591	1.1654	**0.0741**
White Noise	0.8666	0.4700	**0.0258**
Gunshot	1.0075	0.6458	**0.0815**

(b) Audio signals: Geometry 1.

Signal	Conv. LS	Ext. LS	DS-LS
GCC-Classic			
Music	32.8392	13.3874	**0.5281**
White Noise	0.1509	1.1406	**0.0975**
Gunshot	1.1604	0.8929	**0.0422**
GCC-PhaT			
Music	0.1491	0.3849	**0.0560**
White Noise	0.2447	0.8966	**0.1003**
Gunshot	0.0950	0.7301	**0.0471**

(c) Audio signals: Geometry 2.



(a) Results Geometry 1. (b) Results Geometry 1 zoomed. (c) Results Geometry 2. (d) Results Geometry 2 zoomed.

Fig. 1: Audio source localization with two sets of 5 microphones (three signals, TDoAs estimated with GCC-PhaT).

a few. In this case, we may find very large errors, modeled as follows. In our simulations, we modeled the strong additive error with a uniform distribution between -2500 and 2500 samples (assuming the TDoA given in number of samples), to be added only to a certain number of TDoAs. We varied this number from 0 (case without errors of type 2) to 3 (most critical case simulated herein). In cases for which errors of type 2 are to be inserted, we randomly select 1–3 TDoAs, out of N, to be contaminated with strong uniformly distributed additive error. Estimation of source position and calculation of localization error are performed as usual.

Algorithms Conventional LS, Extended LS, and DS-LS are employed to estimate the source position, and Table 1a presents the respective localization errors. The localization error is defined as $\|\mathbf{p}_s - \mathbf{p}_o\|$, where $\mathbf{p}_s$ is the vector with the correct position of the source. Results were averaged from an ensemble of 10,000 independent runs, for geometries 1 and 2, and for four different scenarios of TNRs (in dB) and number of TDoA outliers (errors of type 2). For most scenarios, the proposed algorithm with data selection presented the most accurate results.

4.2. Experiment with recorded audio signals

A 2-D experiment of an audio source localization was carried out in a laboratory with approximated dimensions 7m×7m×5.8m and estimated reverberation time around 1.1s. Two sets of 5 microphones were employed, one with the source (loudspeaker) inside the perimeter of the group of sensors, and another with the source placed outside the perimeter. For each case, three signals were played: *music*, *white noise*, and a sequence of three *gunshots*. The durations of the first two signals are close to 18 seconds, and all signals were sampled at a rate equal to 44,100 samples per second.

We have used the peak of the GCC (classical and Phase Transform) [30] to estimate each of the $N = 10$ TDoAs. Figure 1 shows how each set of microphones was disposed in the lab and the results of the localization procedures (TDoAs estimated with GCC-PhaT). Results for both GCC-Classic and GCC-PhaT are shown, in terms of localization error (in meters), in Tables 1b (geometry 1) and 1c (geometry 2).

Microphone geometry 1 is more convenient to localize the audio source and therefore presents the best results in broad terms. The second geometry is intrinsically less accurate and presented worse results, as expected [29, 31].

In some cases, a single TDoA outlier was already present in the Conventional LS algorithm, and the result of the Extended LS algorithm improved it. In other cases, the Conventional LS approach already has $M-1$ (4 in our experiments) good TDoA estimates, and the Extended LS algorithm eventually included one or more outliers such that results were worsened.

The GCC-Classic algorithm did not yield viable results for TDoAs for the music signal in geometry 2. All combinations of $n = 6$ TDoA measurements resulted in corresponding $|\Delta d_{ij}|$ larger than the maximum allowed values $|\Delta d_{ij}|_{\max}$. In this particular case, we used $n = 3$ TDoA measurements which gave the best result. However, this was not a problem when GCC-PhaT was used.

From both experiments, in different scenarios, with simulated as well as recorded data, and with varying TNRs, we observed that the proposed DS-LS source-localization algorithm yielded better results than both the conventional and the extended LS algorithms, only exception occurring for very small TDoA measurement errors.

5. CONCLUSION

This work proposed a new TDoA-based source localization algorithm. The new algorithm uses the concept of data selection for discarding TDoA measurements in order to minimize the chance of taking into account outliers in the estimation. In several experiments carried out so far, of which two were described here, the new algorithm showed better results for both simulated as well as real-world data. The advantages of data selection were more pronounced when recorded signals were very noisy or strongly reverberated, for the method has improved capability to mitigate the deleterious effects of wrong TDoA measurements.

6. REFERENCES

[1] Y. Huang and J. Benesty, *Audio Signal Processing for Next-Generation Multimedia Comn. Systems*, Springer US, 2004.

[2] E. Bosse, R. M. Turner, and D. Brookes, "Improved radar tracking using a multipath model: maximum likelihood compared with eigenvector analysis," *IEE Proc. - Radar, Sonar and Navigation*, vol. 141, no. 4, pp. 213–222, Aug. 1994.

[3] F. V. F. de Lima and C. M. Furukawa, "Development and testing of an acoustic positioning system - description and signal processing," in *2002 IEEE Ultrasonics Symposium*, Munich, Germany, Oct. 2002, vol. 1, pp. 849–852.

[4] I. Stojmenovic, *Handbook of Sensor Networks: Algorithms and Architectures*, Wiley Series on Parallel and Distributed Computing. Wiley, 2005.

[5] A. Jagoe, *Mobile Location Services: The Definitive Guide*, Prentice Hall, 2003.

[6] A. M. C. R. Borzino, J. A. Apolinário Jr., and M. L. R. de Campos, "Estimating direction of arrival of long range gunshot signals," in *2014 International Telecommunications Symposium (ITS)*, São Paulo, Brazil, Aug. 2014, pp. 1–5.

[7] A. J. Weiss, "On the accuracy of a cellular location system based on RSS measurements," *IEEE Transactions on Vehicular Technology*, vol. 52, no. 6, pp. 1508–1518, Nov. 2003.

[8] R. S. Campos, L. Lovisolo, and M. L. R. de Campos, "Wi-Fi multi-floor indoor positioning considering architectural aspects and controlled computational complexity," *Expert Systems with Applications*, vol. 41, no. 14, pp. 6211–6223, Oct. 2014.

[9] A. M. C. R. Borzino, J. A. Apolinário Jr., and M. L. R. de Campos, "Robust DOA estimation of heavily noisy gunshot signals," in *2015 IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Brisbane, QLD, Australia, Apr. 2015, pp. 449–453.

[10] J. Shen, A. F. Molisch, and J. Salmi, "Accurate passive location estimation using TOA measurements," *IEEE Transactions on Wireless Communications*, vol. 11, no. 6, pp. 2182–2192, June 2012.

[11] H. He, J. Chen, J. Benesty, Y. Zhou, and T. Yang, "Robust multichannel TDOA estimation for speaker localization using the impulsive characteristics of speech spectrum," in *2017 IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, New Orleans, LA, USA, Mar. 2017, pp. 6130–6134.

[12] M. S. Brandstein, J. E. Adcock, and H. F. Silverman, "A closed-form location estimator for use with room environment microphone arrays," *IEEE Transactions on Speech and Audio Processing*, vol. 5, no. 1, pp. 45–50, Jan. 1997.

[13] S. R. Drake and K. Doğançay, "Geolocation by time difference of arrival using hyperbolic asymptotes," in *2004 IEEE International Conference on Acoustics, Speech, and Signal Processing*, May 2004, vol. 2, pp. II–361–II–364.

[14] S. S. Haykin and J. H. Justice, *Array signal processing*, Prentice-Hall signal processing series. Prentice-Hall, 1985.

[15] H.-J. Kolb, A. Kleschenkow, S. Warzügel, and D. Kolb, "Method and apparatus for TDOA-locating with reduced data rates," European Patent 2642313A1 Sept. 2013.

[16] Y. Jiang, Q. Hu, and D. Yang, "Analysis of positioning error for two-dimensional location system," *Mathematical Problems in Engineering*, vol. 2013, pp. 1–8, Oct. 2013.

[17] B. Jin, X. Xu, and T. Zhang, "Robust time-difference-of-arrival (TDOA) localization using weighted least squares with cone tangent plane constraint," *Sensors*, vol. 2018, no. 3, pp. 1–16, Mar. 2018.

[18] A. Canclini, P. Bestagini, F. Antonacci, M. Compagnoni, A. Sarti, and S. Tubaro, "A robust and low-complexity source localization algorithm for asynchronous distributed microphone networks," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 23, no. 10, pp. 1563–1575, Oct. 2015.

[19] Y. Xue, X. Li, L. Xu, and Y. Ren, "Research on position differential method of dual-satellites TDOA and FDOA in passive location system," in *2012 IEEE International Frequency Control Symposium Proceedings*, Baltimore, MD, USA, May 2012, pp. 1–5.

[20] Z. Li, D. C. Dimitrova, T. Braun, and D. Rosário, "Highly accurate evaluation of GPS synchronization for TDOA localization," in *2013 IFIP Wireless Days (WD)*, Valencia, Spain, Nov. 2013, pp. 1–3.

[21] J. F. Bull, "Wireless geolocation," *IEEE Vehicular Technology Magazine*, vol. 4, no. 4, pp. 45–53, Dec. 2009.

[22] M. Khalaf-Allah, "An extended closed-form least-squares solution for three-dimensional hyperbolic geolocation," in *2014 IEEE Symposium on Industrial Electronics Applications (ISIEA)*, Sept. 2014, pp. 7–11.

[23] J. Smith and J. Abel, "Closed-form least-squares source location estimation from range-difference measurements," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 35, no. 12, pp. 1661–1669, Dec. 1987.

[24] J. Smith and J. Abel, "The spherical interpolation method of source localization," *IEEE Journal of Oceanic Engineering*, vol. 12, no. 1, pp. 246–252, Jan. 1987.

[25] P. Stoica and J. Li, "Lecture notes - source localization from range-difference measurements," *IEEE Signal Processing Magazine*, vol. 23, no. 6, pp. 63–66, Nov. 2006.

[26] F. B. da Silva and W. A. Martins, "Robust TDOA-based sound source localization," in *XXXIII Simpósio Brasileiro de Telecomunicações (SBrT)*, Sept. 2015.

[27] A. M. C. R. Borzino, J. A. Apolinário Jr., and M. L. R. de Campos, "Consistent DOA estimation of heavily noisy gunshot signals using a microphone array," *IET Radar, Sonar Navigation*, vol. 10, no. 9, pp. 1519–1527, 2016.

[28] M. Compagnoni, A. Pini, A. Canclini, P. Bestagini, F. Antonacci, S. Tubaro, and A. Sarti, "A geometrical –- statistical approach to outlier removal for TDOA measurements," *IEEE Transactions on Signal Processing*, vol. 65, no. 15, pp. 3960–3975, Aug. 2017.

[29] M. Compagnoni, R. Notari, F. Antonacci, and A. Sarti, "A comprehensive analysis of the geometry of TDOA maps in localization problems," *Inverse Problems*, vol. 30, no. 3, 2014.

[30] C. Knapp and G. Carter, "The generalized correlation method for estimation of time delay," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 24, no. 4, pp. 320–327, Aug. 1976.

[31] W. Meng, W. Xiao, and L. Xie, "Optimal sensor pairing for TDOA based source localization in sensor networks," in *2011 8th International Conference on Information, Communications Signal Processing*, Dec. 2011, pp. 1–5.