TOA SOURCE NODE SELF-POSITIONING WITH UNKNOWN CLOCK SKEW IN WIRELESS SENSOR NETWORKS

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ABSTRACT

This paper investigates time-of-arrival (TOA) source node self-positioning with unknown clock skews in wireless sensor networks. For the source-to-anchor direction, source node clock skew does not affect the localization performance. When synchronized anchor nodes simultaneously transmit signals to a source node, the source node clock skew will degrade the localization performance. A semidefinite programming (SDP) algorithm that jointly estimates the source position and clock skew is proposed for the latter case. The proposed algorithm is better than the two kinds of existing schemes, namely, asynchronous TOA localization and TDOA localization. We also tune the algorithm to the case of anchor nodes position uncertainties. Simulation results validate the performance of the proposed algorithm.

Index Terms— Source localization, time-of-arrival (TOA), clock skew

1. INTRODUCTION

Source localization is important for wireless sensor networks (WSNs), [1, 2] global navigation satellite system (GNSS) [3], and intelligent transportation systems (ITS) [4]. Time-of-arrival (TOA) and time-difference-of-arrival (TDOA) based schemes typically achieve a higher accuracy than received signal strength (RSS) or angle of arrival (AOA) based schemes [5, 6, 7, 8, 9, 10, 11, 12, 13], but the former require stringent clock synchronization. Specifically, TOA requires all anchor nodes as well as the source node be time-synchronized; TDOA requires all anchor nodes be synchronized.

There are asynchronous TOA localization methods [14, 15, 16, 17], which all assume that the anchor nodes are synchronized and the source node has only a clock offset with the anchor nodes but clock skew is assumed known. In practice, however, the clock skew of the source node depends on the manufacturing process; a range for it is typical available [18] but its precise value is difficult to obtain. For the source-to-anchor direction, source node clock skew does not affect the localization performance. When the anchor nodes simultaneously transmit signals to the source node, source node clock skew will degrade the localization performance.

Gholami et al. [18] present a TDOA self-positioning model with unknown clock skews, which assumes that the anchor nodes are perfectly synchronized and transmit their signals at a common time instant. The source node then measures the TOAs of the received signals and forms a set of the TDOA measurements to localize itself. The TDOA measurements could eliminate the clock offset, they are still affected by the clock skew. A semidefinite programming (SDP) based algorithm is provided to estimate the source position while the clock skew is treated as a nuisance parameter. However, the construction of TDOA measurements leads to a high level of nonlinearity than the original estimating problem. Besides, it leads to correlated noise in TDOA, and strengthens the measurement noise by 3dB [14].

Our previous work [2] has developed a joint synchronization and localization method for WSNs using two-way exchanged time-stamps, which requires both anchor nodes and source node to transmit signals, i.e., it requires a duplex scheme. The asynchronous TOA localization methods in [14, 15, 16, 17] require either the source node to transmit signals to the anchor nodes, or the anchor nodes to transmit signals to the source node, i.e., they are simplex systems, which are less complex. The methods in [14, 15, 16, 17] just consider clock offset, though the clock skew is ignored.

In this paper, we consider the problem of source node self-positioning in simplex system with unknown clock skew. Specially, the source node exploits the signals transmitted simultaneously from synchronized anchor nodes to measure the TOAs and then jointly estimates its position and the clock skew. The contributions of this paper are as follows.

- Asynchronous TOA localization [14, 15, 16, 17] is extended to a more practical case: the clock skew of

*This work was supported in part by the National Natural Science Foundation of China under Grants 61471153 and 61471153, and China Scholarship Council.

source node is unknown.

- An SDP based algorithm is developed to solve this problem, which has better performance than the TDOA scheme described in [18].
- The SDP algorithm is tuned to the case of anchor nodes uncertainties, witch are considered in the TDOA scheme in [18].

Section II describes the measurement model and forms the maximum likelihood estimator (MLE) problem. The SDP based algorithm for joint position and clock skew estimation is developed in Section III. Section IV tune the SDP algorithm to the case of anchor node position uncertainties. Section V presents simulation results of the proposed algorithms and compare them with existing related algorithms.

2. PROBLEM STATEMENT

Consider a WSN with M time-synchronized anchor nodes and one independently operating source node. At a common time instant t_0, the anchor nodes transmit their signals to the source node. The source node measures the TOAs of the received signals. Let $\mathbf{s}_i \in \mathbb{R}^m$ denote the ith anchor node's position, and $\mathbf{u} \in \mathbb{R}^m$ the position of source node, where m is the location dimension, taking on the value of either 2 or 3.

The local time of the source node, τ, is related to the reference time (i.e., synchronized anchor nodes' time), t, as

$$\tau = \omega t + \theta \tag{1}$$

where ω is the clock skew of source node and θ is the clock offset of source node.

In the line-of-sight propagation conditions, the received TOA measurements in the source node can be expressed as [2, 18]

$$\tau_i = \omega t_0 + \theta + w(\frac{d_i}{c} + n_i),\ i = 1, 2, \cdots, M. \tag{2}$$

where τ_i is the TOA measurement from the ith anchor node, t_0 is the unknown transmission time at the anchor nodes, d_i is the distance between the source node and the ith anchor node, and n_i is the measurement noise, assumed to be a zero-mean Gaussian variable [14].

Let $x = t_0 + \frac{\theta}{\omega}$. Eq. (2) can be rewritten as

$$\tau_i = \omega x + \omega(\frac{d_i}{c} + n_i) \tag{3}$$

To make the analysis easier, the time measurements will be converted to range measurements as

$$r_i = \omega z + \omega(d_i + e_i),\ i = 1, 2, \cdots, M. \tag{4}$$

where $r_i = \tau_i c$, $z = xc$, and $e_i = n_i c$.

The MLE problem assuming independent Gaussian noise is expressed as

$$\min_{\mathbf{u},\omega,z} \sum_{i=1}^{M} \frac{(\frac{r_i}{\omega} - \|\mathbf{u} - \mathbf{s}_i\| - z)^2}{\sigma_i^2} \tag{5}$$

where $\mathbf{u}$, ω, and z are the unknown parameters, $\|\cdot\|$ represents the Euclidean norm, and σ_i^2 is the known variance of range measurement noise e_i. The above MLE problem is not easy to solve due to its nonlinearity and nonconvexity.

The Cramér-Rao lower bound (CRLB) provides a benchmark for any unbiased estimator [19]. Define the unknown parameter vector $\boldsymbol{\zeta} = [\mathbf{u}^T, \omega, z]^T$. Its Fisher information matrix (FIM) can be computed as [2]

$$\mathbf{I}(\boldsymbol{\zeta}) = \frac{1}{\omega^2}\mathbf{q}\mathbf{Q}^{-1}\mathbf{q}^T \tag{6}$$

where

$$\mathbf{q}(:, i) = \left[\omega\frac{(\mathbf{u} - \mathbf{s}_i)^T}{\|\mathbf{u} - \mathbf{s}_i\|}, z + \|\mathbf{u} - \mathbf{s}_i\|, \omega\right]^T \tag{7a}$$

$$\mathbf{Q} = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \cdots, \sigma_M^2). \tag{7b}$$

So, the CRLB of $\boldsymbol{\zeta}$ is $\mathbf{I}^{-1}(\boldsymbol{\zeta})$.

3. LOCALIZATION ALGORITHM

Here an SDP based method is proposed to jointly estimate the position and clock skew of the source node.

First, (5) can be expressed as

$$\min_{\mathbf{u},a,z,\mathbf{d}} (a\mathbf{r} - \mathbf{d} - z\mathbf{1}_M)^T\mathbf{Q}^{-1}(a\mathbf{r} - \mathbf{d} - z\mathbf{1}_M) \tag{8a}$$

$$\text{s.t. } d_i = \|\mathbf{u} - \mathbf{s}_i\|,\ i = 1, 2, \cdots, M. \tag{8b}$$

where $a = \frac{1}{\omega}$, $\mathbf{r} = [r_1, r_2, \cdots, r_M]^T$, $\mathbf{d} = [d_1, d_2, \cdots, d_M]^T$, $\mathbf{1}_M$ is an $m \times 1$ vector whose elements are all 1's. Let $\mathbf{A} = [\mathbf{r}, -\mathbf{1}_M]$, and $\mathbf{y} = [a, z]^T$. Then (8) can be recast as

$$\min_{\mathbf{u},\mathbf{y},\mathbf{d}} (\mathbf{A}\mathbf{y} - \mathbf{d})^T\mathbf{Q}^{-1}(\mathbf{A}\mathbf{y} - \mathbf{d}) \tag{9a}$$

$$\text{s.t. } d_i = \|\mathbf{u} - \mathbf{s}_i\| \tag{9b}$$

Let the gradient of the objective function in (9) with respect to $\mathbf{y}$ equal zero:

$$-2\mathbf{A}^T\mathbf{Q}^{-1}(\mathbf{A}\mathbf{y} - \mathbf{d}) = 0, \tag{10}$$

which results in

$$\mathbf{y} = (\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{d}. \tag{11}$$

Substituting (11) into (9) yields

$$\min_{\mathbf{u},\mathbf{d}} (\mathbf{H}\mathbf{d})^T\mathbf{Q}^{-1}(\mathbf{H}\mathbf{d}) \tag{12a}$$

$$\text{s.t. } d_i = \|\mathbf{u} - \mathbf{s}_i\| \tag{12b}$$

where $\mathbf{H} = \mathbf{A}(\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Q}^{-1} - \mathbf{I}_M$ and $\mathbf{I}_M$ is an identity matrix.

By letting $\mathbf{D} = \mathbf{d}\mathbf{d}^T$, (12a) can be rewritten as

$$(\mathbf{Hd})^T\mathbf{Q}^{-1}(\mathbf{Hd}) = \text{tr}(\mathbf{d}\mathbf{d}^T\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H}) = \text{tr}(\mathbf{D}\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H}). \tag{13}$$

where $\text{tr}(\cdot)$ denotes the trace of a matrix.

Further, by letting $y_s = \mathbf{u}^T\mathbf{u}$, from (12b), we have

$$\mathbf{D}_{i,i} = d_i^2 = \|\mathbf{u} - \mathbf{s}_i\|^2 = \mathbf{y}_s - 2\mathbf{u}^T\mathbf{s}_i + \mathbf{s}_i^T\mathbf{s}_i,\ i = 1, 2, \ldots, M \tag{14}$$

and

$$\begin{aligned}\mathbf{D}_{i,j} &= d_i d_j = \|\mathbf{u} - \mathbf{s}_i\|\,\|\mathbf{u} - \mathbf{s}_j\| \geq |(\mathbf{u} - \mathbf{s}_i)^T(\mathbf{u} - \mathbf{s}_j)| \\ &= |\mathbf{y}_s - \mathbf{u}^T(\mathbf{s}_i + \mathbf{s}_j) + \mathbf{s}_i^T\mathbf{s}_j|,\ 1 \leq i < j \leq M. \end{aligned} \tag{15}$$

It is easy to verify that the column vectors of $\mathbf{A}$ are the eigenvectors of $\mathbf{H}$ corresponding to the zero eigenvalue. As a result, $\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H}$ is not a positive definite matrix. An SDP-based localization algorithm is finally expressed as

$$\min_{\mathbf{u}, y_s, \mathbf{d}, \mathbf{D}} \text{tr}(\mathbf{D}\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H}) + \eta\text{tr}(\mathbf{D}) \tag{16a}$$

$$\text{s.t. } \mathbf{D}_{i,i} = \mathbf{y}_s - 2\mathbf{u}^T\mathbf{s}_i + \mathbf{s}_i^T\mathbf{s}_i, \tag{16b}$$

$$\|\mathbf{u} - \mathbf{s}_i\| \leq d_i, \tag{16c}$$

$$\mathbf{D}_{i,j} \geq |\mathbf{y}_s - \mathbf{u}^T(\mathbf{s}_i + \mathbf{s}_j) + \mathbf{s}_i^T\mathbf{s}_j|, \tag{16d}$$

$$\begin{bmatrix} 1 & \mathbf{d}^T \\ \mathbf{d} & \mathbf{D} \end{bmatrix} \succeq \mathbf{0} \tag{16e}$$

$$\begin{bmatrix} \mathbf{I}_m & \mathbf{u} \\ \mathbf{u}^T & \mathbf{y_s} \end{bmatrix} \succeq \mathbf{0} \tag{16f}$$

where η is a positive parameter, which can be selected using an approach similar to the one described in [17].

The next step would be to calculate $\mathbf{y}$, i.e., (11) using the value of $\hat{\mathbf{u}}$ obtained. The clock skew is estimated from $\mathbf{y}$ as

$$\hat{\omega} = \frac{1}{\mathbf{y}(1)}. \tag{17}$$

4. LOCALIZATION ALGORITHM WITH POSITION UNCERTAINTIES

When the locations of the anchor nodes are not precise, which is mostly the case in practice, the sensor positions with errors can be expressed as [17, 20]

$$\mathbf{s}_i = \mathbf{s}_i^0 + \boldsymbol{\beta}_i,\ i = 1, 2, \cdots, M. \tag{18}$$

where $\mathbf{s}_i^0$ is the actual but unknown sensor position and $\boldsymbol{\beta}_i$ represents the position error, which is assumed to be a zero-mean Gaussian vector with a known covariance $\delta_i^2\mathbf{I}_2$ [20]. Note that n_i and $\boldsymbol{\beta}_i$ are assumed to be mutually independent. The MLE problem is expressed as

$$\min_{\mathbf{u}, \omega, z, \mathbf{s}_i^0} \sum_{i=1}^{M} \frac{(\frac{r_i}{w} - \|\mathbf{u} - \mathbf{s}_i^0\| - z)^2}{\sigma_i^2} + \sum_{i=1}^{M} \frac{\|\mathbf{s}_i - \mathbf{s}_i^0\|^2}{\delta_i^2} \tag{19}$$

where z and $\mathbf{s}_i^0$ are the nuisance parameters.

Eq. (19) can be expressed as

$$\min_{\mathbf{X}, \mathbf{y}, \mathbf{d}} (\mathbf{Ay} - \mathbf{d})^T\mathbf{Q}^{-1}(\mathbf{Ay} - \mathbf{d}) + \left\|(\mathbf{B} - \mathbf{X}(:, 2:M+1))\mathbf{W}^{\frac{1}{2}}\right\|_F^2 \tag{20a}$$

$$\text{s.t. } d_i = \|\mathbf{X}(:, 1) - \mathbf{X}(:, i+1)\| \tag{20b}$$

where $\mathbf{X} = [\mathbf{u}, \mathbf{s}_1^0, \mathbf{s}_2^0, \ldots, \mathbf{s}_M^0]$, $\mathbf{B} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_M]$, $\mathbf{W} = \text{diag}\left([\delta_1^{-2}, \delta_2^{-2}, \cdots, \delta_M^{-2}]\right)$, and $\|\cdot\|_F$ represents the Frobenius norm.

Similar to the derivation of (12), (20) can be recast as

$$\min_{\mathbf{X}, \mathbf{d}} (\mathbf{Hd})^T\mathbf{Q}^{-1}(\mathbf{Hd}) + \left\|(\mathbf{B} - \mathbf{X}(:, 2:M+1))\mathbf{W}^{\frac{1}{2}}\right\|_F^2 \tag{21a}$$

$$\text{s.t. } d_i = \|\mathbf{X}(:, 1) - \mathbf{X}(:, i+1)\| \tag{21b}$$

Let $\mathbf{Y} = \mathbf{X}^T\mathbf{X}$. The SDP-based localization algorithm with anchor node position errors are expressed as

$$\min_{\mathbf{d}, \mathbf{D}, \mathbf{X}, \mathbf{Y}} \text{tr}(\mathbf{D}\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H}) - 2tr(\mathbf{W}\mathbf{A}^T\mathbf{X}(:, 2:M+1)) + \text{tr}(\mathbf{W}\mathbf{Y}(2:M+1, 2:M+1)) + \eta tr(\mathbf{D}) \tag{22a}$$

$$\text{s.t. } \mathbf{D}_{i,i} = \mathbf{Y}(1,1) - 2\mathbf{Y}(1, i+1) + \mathbf{Y}(i+1, i+1), \tag{22b}$$

$$\|\mathbf{X}(:, 1) - \mathbf{X}(:, i+1)\| \leq d_i, \tag{22c}$$

$$\mathbf{D}_{i,j} \geq |\mathbf{Y}(1,1) - \mathbf{Y}(1, i+1) - \mathbf{Y}(1, j+1) + \mathbf{Y}(i+1, j+1)|,\ 1 \leq i < j \leq M. \tag{22d}$$

$$\begin{bmatrix} 1 & \mathbf{d}^T \\ \mathbf{d} & \mathbf{D} \end{bmatrix} \succeq \mathbf{0} \tag{22e}$$

$$\begin{bmatrix} \mathbf{I}_m & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0}. \tag{22f}$$

Once the position estimate $\hat{\mathbf{u}}$ is obtained, the clock skew ω can be calculated by using (17).

5. SIMULATION RESULTS

In the simulation, 'Proposed' denotes the proposed SDP algorithms. For the ideal case of perfect anchor positions, we will compare (16) with the algorithm in [17], which assumes $\omega = 1$, and with the algorithm in [18], which uses the TDOA measurements; for the more realistic case that anchor positions have errors, we compare (22) with the algorithm in [17]. All the SDP-based algorithms are computed by CVX toolbox

[21], using SeDuMi as a solver, and the precision is set to best. Results are obtained from 500 Monte Carlo experiments.

There are six anchor nodes, and their true positions are $[0,0]^T m$, $[400,0]^T m$, $[800,0]^T m$, $[800,800]^T m$, $[400,800]^T m$, $[0,800]^T m$. t_0, ω, and θ are uniformly distributed within $[10,40]ns$, $[0.995,1.005]$, and $[1,10]ns$, respectively. Both TOA measurement errors and anchor node position errors are assumed to be independent and identically distributed, i.e., $\sigma_i^2 = \sigma^2, \delta_i^2 = \delta^2$, and η is set to 10^{-7}, 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3} for the proposed algorithms and the algorithms in [17].

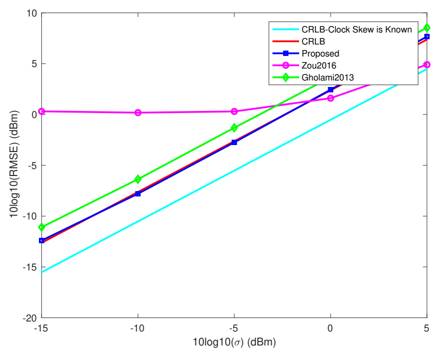

Fig. 1. RMSE of position vs. σ, $\mathbf{u} = [200,100]^T m$.

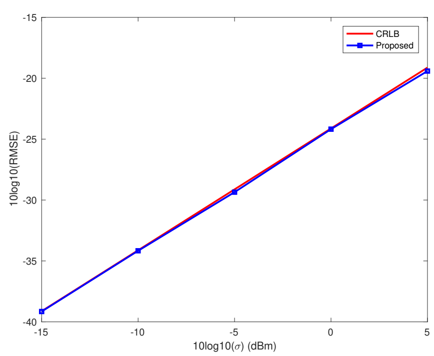

Fig. 2. RMSE of clock skew vs. σ, $\mathbf{u} = [200,100]^T m$.

The results for the case of accurate anchor node positions are shown in Figs. 1−2. Figs. 3-4 show the results for the case of inaccurate anchor node positions.

Fig. 1 reveals that when the clock skew is unknown, the position CRLB is higher than the CRLB when the clock skew is known, because adding an unknown parameter will degrade the position estimation performance. In Fig. 1, the proposed algorithm can attain the CRLB, and the method in [18] is $1.3dBm$ above the CRLB. Besides, it can be seen that the method in [17] is above the CRLB when σ is smaller than $0dBm$. This is because the method in [17] is a biased estimator.

Fig. 2 shows the performance of clock skew estimation. It

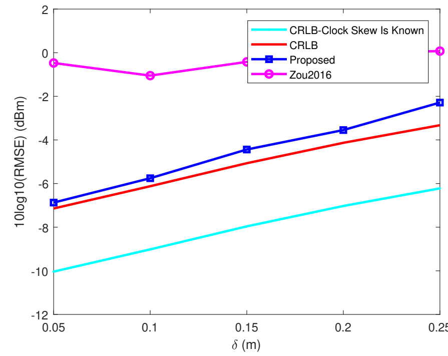

Fig. 3. RMSE of position vs. δ, $\mathbf{u} = [200,100]^T m$, $\sigma = 0.1m$.

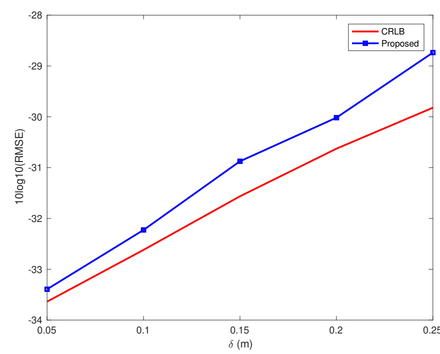

Fig. 4. RMSE of clock skew vs. δ, $\mathbf{u} = [200,100]^T m$, $\sigma = 0.1m$.

is observed from the figure that the proposed algorithm could attain the CRLB of clock skew.

Fig. 3 shows that the position estimate with the proposed algorithm (22) is more accurate than with the algorithm in [17]. Fig. 4 shows that the clock skew estimation of the proposed algorithm is near the CRLB (deviation is within 1.2 dB).

6. CONCLUSIONS

This paper addresses the problem of TOA-based source node self-positioning with unknown clock skew. We have developed an SDP based algorithm to jointly estimate the source node's position and clock skew. We have also considered the presence of anchor node position errors. Simulation results have shown that the proposed algorithm outperform existing methods when the anchor node positions are accurate or when they have errors.

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