ALGEBRAIC SOLUTION FOR TDOA LOCALIZATION IN MODIFIED POLAR REPRESENTATION

Yimao Sun[1,2], K. C. Ho[2], Qun Wan[1]

[1]School of Information and Communication Engineering,
University of Electronic Science and Technology of China, Chengdu, China
[2]EECS Department, University of Missouri, MO 65211, USA

ABSTRACT

Time difference of arrival (TDOA) point positioning in the Cartesian coordinates is practical for a near-field source, and it will suffer from the thresholding effect when the source is in the far-field where only direction of arrival (DOA) can be obtained. Localization in the modified polar representation (MPR) is able to alleviate this problem, where point positioning and DOA estimation are unified into a single framework. The state-of-the-art literature only has an iterative realization of the maximum likelihood estimator (MLE) for this problem. This paper develops an algebraic closed-form positioning solution for MPR. The proposed algorithm avoids the initialization issue and is much more computationally efficient than the MLE with comparable accuracy. Simulation results validate the advocated performance.

Index Terms— Closed-form solution, DOA, localization, modified polar representation, TDOA.

1. INTRODUCTION

A basic task of many applications is the localization of a signal source [1–14]. Over the years, many localization approaches and algorithms have been developed and investigated. A common procedure for localization is to first derive the geometry related parameters from the observed signals at the sensors such as time of arrival (TOA) [1], time difference of arrival (TDOA) [4,15], angle of arrival (AOA) [16,17], or received signal strength (RSS) [3], and obtain the source location next by solving a set of non-linear equations. We consider here the TDOA measurement and focus on the latter.

The localization problem is typically studied in the Cartesian or polar coordinates for point positioning [18–20], where the source is assumed in the near-field with a circular wavefront. If the source is substantially far away that the wavefront becomes linear, direction of arrival (DOA) estimation becomes the research focus [21,22]. These two fields of research are conducted separately for years as they seem to be separate problems. Point localization is not feasible for a far-field source where the observed wavefront has negligible curvature, and DOA estimation is inaccurate when the wavefront does not appear linear. While near-field DOA estimation is possible using the Fresnel approximation [23], the approximation error could be significant [24] that limits the DOA accuracy. Traditional approach relies on the prior knowledge if the source is near or distant, so that a suitable choice between point localization and DOA estimation can be made. Nevertheless, such information about the source range is often not known in practice.

The recent work [25] analyzed the thresholding effect of TDOA positioning in the Cartesian coordinate model using the Abel bound [26] when the source range is large, and investigated the significant bias of DOA estimate when the far-field assumption is not fulfilled. The modified polar representation (MPR) [25] of the source location in terms of the arrival angle and inverse-range was then introduced that unifies point positioning and DOA estimation in the same framework, where the source is near or distant is irrelevant. It eliminates the ambiguity and the prior knowledge needed, and the problem can be solved with a common methodology regardless the source is located in the near- or far-field. The Maximum Likelihood Estimator (MLE) based on the Gauss-Newton iteration having initialization supplied by a coarse semidefinite relaxation (SDR) solution has been developed [25], where the thresholding effect caused by large source range is eliminated and the DOA bias resulted from small range-to-baseline ratio is restrained. Nevertheless, the iterative MLE is costly and could fail to perform when the source happens to be close to the sensors as the initial coarse solution is too far away from the actual.

This paper advances the previous research [25] and develops an algebraic closed-form solution to the localization problem in MPR. We propose a novel formulation of the problem from the TDOA measurements by representing the arrival angle as the sine and cosine functions of the angle [27]. The estimation problem becomes the weighted least-squares minimization with a quadratic constraint. We first solve the problem by ignoring the constraint and then improving the accuracy by nonlinear transformation and exploiting the constraint. Simulations verify that the proposed method attends the CRLB performance under Gaussian noise without suffering from the convergence and high complexity issues as in the iterative MLE. It also outperforms the closed-form point positioning methods from the literature especially when the source range becomes large.

We illustrate in the next section the localization problem. Section 3 reformulates the measurement equation and derives the proposed solution. Section 4 summarizes the CRLB and performance analysis. Section 5 presents the simulation and section 6 is the conclusion. $(*)^o$ denotes the true value of the variable $(*)$.

2. LOCALIZATION PROBLEM

TDOA localization has been well examined in many literatures [18, 25,28,29]. We follow the tradition and describe the positioning configuration in the Cartesian coordinates. For ease of illustration, the presentation is limited to the 2-D scenario. Extension to the 3-D case

Corresponding author: wanqun@uestc.edu.cn.

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is straightforward.

M sensors are deployed in space at known positions $\mathbf{s}_i \in \mathbb{R}^2$, $i = 1, 2, \cdots, M$. The source position is represented by $\mathbf{u}^o \in \mathbb{R}^2$, which is the unknown to be found. The sensors and the source are considered stationary during the observation period. The sensors are synchronized and they are able to provide the TDOAs of their received signals with respect to the one at $\mathbf{s}_1$, giving the measurements

$$r_{i1} = r_{i1}^o + n_{i1} \quad i = 2, 3, \cdots, M. \tag{1}$$

$$r_{i1}^o = r_i^o - r_1^o = \|\mathbf{u}^o - \mathbf{s}_i\| - \|\mathbf{u}^o - \mathbf{s}_1\| \tag{2}$$

is the true range difference when multiplying the TDOA with the propagation speed, r_i^o is the distance between the source and sensor i and $\| \bullet \|$ represents the L-2 norm. n_{i1} is the additive noise. The measurement vector is

$$\mathbf{r} = \mathbf{r}^o + \mathbf{n}\,, \tag{3}$$

where $\mathbf{r}^o = [r_{21}^o, r_{31}^o, \cdots, r_{M1}^o]^T$. The noise vector $\mathbf{n} = [n_{21}, n_{31}, \cdots, n_{M1}]^T$ is Gaussian distributed with zero-mean and known covariance matrix $\mathbf{Q}$. It is equal to $\mathbf{Q} = 0.5\sigma_n^2(\mathbf{I}_{M-1} + \mathbf{1}_{M-1}\mathbf{1}_{M-1}^T)$ for spectrally flat Gaussian random signal and noise with identical received signal-to-noise ratio (SNR) at the sensors [18], where $\mathbf{I}_{M-1}$ is an identity matrix of size $M-1$ and $\mathbf{1}_{M-1}$ is a $(M-1)\times 1$ vector of unity. The source localization problem is to determine $\mathbf{u}^o$ by $\mathbf{r}$.

For a given TDOA, (2) is a hyperbola governing the possible source positions, with $\mathbf{s}_i$ and $\mathbf{s}_1$ at the foci. The $M-1$ hyperbolas from all measurements intersect and yield the source position estimate. When the source is distant from the sensors, the hyperbolic branches become nearly parallel and may fail to intersect properly as perturbed by the measurement noise, causing the thresholding effect where the point location estimate can be very far from the actual. Nevertheless, we should still be able to determine the DOA of the source as it relates to the asymptotes of the branches.

MPR alleviates the thresholding problem by representing the source location as [25]

$$\widetilde{\mathbf{u}}^o = [\theta^o, g^o]^T\,, \tag{4}$$

where θ^o is the signal arrival direction and $g^o = 1/r_1^o$ is the inverse-range of the source, both with respect to $\mathbf{s}_1$. When the source is in the near-field, there is a one-to-one mapping between $\widetilde{\mathbf{u}}^o$ and $\mathbf{u}^o$ to obtain a unique point estimate. If the source is in the far-field, g^o will approach zero while θ^o will be the DOA. [25] has shown that using MPR will not result in the thresholding effect of an estimator regardless of the source range. Our aim is to obtain θ^o and g^o from the TDOA measurements $\mathbf{r}$ and the M sensor positions, using a computationally efficient solution rather than the iterative MLE [25].

3. SOLUTION

The proposed solution uses a pair of sine and cosine functions to represent the arrival angle and converts the problem to quadratic optimization with a quadratic constraint. We begin by using (2) in (1),

$$r_i^o = r_{i1} - n_{i1} + r_1^o\,. \tag{5}$$

Squaring both sides and rearranging give

$$r_i^{o2} - r_1^{o2} = (r_{i1} - n_i)^2 + 2(r_{i1} - n_i)r_1^o\,. \tag{6}$$

Realizing

$$r_j^{o2} = (\mathbf{u}^o - \mathbf{s}_j)^T(\mathbf{u}^o - \mathbf{s}_j) = \|\mathbf{u}^o\|^2 - 2\mathbf{s}_j^T\mathbf{u}^o + \|\mathbf{s}_j\|^2\,, \tag{7}$$

using it for $j = i$ and $j = 1$ in (6) yields

$$\begin{aligned} -2(\mathbf{s}_i - \mathbf{s}_1)^T\mathbf{u}^o + &\|\mathbf{s}_i\|^2 - \|\mathbf{s}_1\|^2 \\ &= r_{i1}^2 + 2r_{i1}r_1^o - 2r_i^o n_{i1} - n_{i1}^2 \end{aligned} \tag{8}$$

where (1) and (2) have been used on the right side to combine the linear noise terms. Expressing $\mathbf{u}^o = (\mathbf{u}^o - \mathbf{s}_1) + \mathbf{s}_1$, dividing both sides by $2r_1^o$ and moving the first two terms from the right to the left, we obtain

$$\begin{aligned} -r_{i1} - (\mathbf{s}_i - \mathbf{s}_1)^T\bar{\mathbf{u}}^o - &\frac{1}{2}(r_{i1}^2 - \|\mathbf{s}_i - \mathbf{s}_1\|^2)g^o \\ &= -\frac{r_i^o}{r_1^o}n_{i1} - \frac{1}{2r_1^o}n_{i1}^2\,, \end{aligned} \tag{9}$$

where

$$\bar{\mathbf{u}}^o = (\mathbf{u}^o - \mathbf{s}_1)/r_1^o = [\cos\theta^o, \sin\theta^o]^T \tag{10}$$

is a unit vector pointing from $\mathbf{s}_1$ to $\mathbf{u}^o$ and $g^o = 1/r_1^o$. (9) is a nonlinear equation with respect to θ^o, nevertheless it is pseudo-linear in terms of the two variables $\bar{\mathbf{u}}^o$ and g^o.

Stacking (9) for $i = 2, 3, \cdots, M$ together yields the matrix equation

$$\mathbf{h}_1 - \mathbf{G}_1\boldsymbol{\psi}_1^o = \mathbf{B}_1\mathbf{n} - \mathbf{o}_1\,. \tag{11}$$

In (11),

$$\boldsymbol{\psi}_1^o = [\bar{\mathbf{u}}^{oT}, g^o]^T \tag{12a}$$

$$\mathbf{h}_1 = -\mathbf{r}\,, \tag{12b}$$

$$\mathbf{G}_1 = \begin{bmatrix} (\mathbf{s}_2 - \mathbf{s}_1)^T & 0.5(r_{21}^2 - \|\mathbf{s}_2 - \mathbf{s}_1\|^2) \\ \vdots & \vdots \\ (\mathbf{s}_M - \mathbf{s}_1)^T & 0.5(r_{M1}^2 - \|\mathbf{s}_M - \mathbf{s}_1\|^2) \end{bmatrix}, \tag{12c}$$

$$\mathbf{B}_1 = -\mathrm{diag}\left\{\left[\frac{r_2^o}{r_1^o}, \frac{r_3^o}{r_1^o}, \cdots, \frac{r_M^o}{r_1^o}\right]\right\}, \tag{12d}$$

and $\mathbf{o}_1 = (\mathbf{n} \odot \mathbf{n})/2r_1^o$ is the second order noise term where $\odot$ is the operation of element by element multiplication. The unknown is $\boldsymbol{\psi}_1^o$.

The weighted least-squares estimate [30] for $\boldsymbol{\psi}_1^o$ from (11) is

$$\boldsymbol{\psi}_1 = (\mathbf{G}_1^T\mathbf{W}_1\mathbf{G}_1)^{-1}\mathbf{G}_1^T\mathbf{W}_1\mathbf{h}_1\,. \tag{13}$$

$\mathbf{W}_1$ is the weighting matrix given by

$$\begin{aligned} \mathbf{W}_1 &= E\left[(\mathbf{B}_1\mathbf{n} - \mathbf{o}_1)(\mathbf{B}_1\mathbf{n} - \mathbf{o}_1)^T\right]^{-1} \\ &\simeq (\mathbf{B}_1\mathbf{Q}\mathbf{B}_1)^{-1}\,, \end{aligned} \tag{14}$$

where the noise terms higher than second order have been ignored. The covariance matrix of the solution, when neglecting the noise in $\mathbf{G}_1$, is [30]

$$\mathrm{cov}(\boldsymbol{\psi}_1) \simeq (\mathbf{G}_1^T\mathbf{W}_1\mathbf{G}_1)^{-1}\,. \tag{15}$$

The elements of $\mathbf{B}_1$ require the distances between $\mathbf{u}^o$ and $\mathbf{s}_i$. To avoid the use of the Cartesian coordinates $\mathbf{u}^o$ where the source range from $\mathbf{s}_1$, $1/g^o$, is meaningless for a far-field source, we shall express them in terms of $\bar{\mathbf{u}}^o$ defined in (10) and g^o. Substituting $\mathbf{u}^o = (\mathbf{u}^o - \mathbf{s}_1) + \mathbf{s}_1$ and using $1/r_1^o = g^o$ result in

$$\frac{r_i^o}{r_1^o} = \left\|\frac{\mathbf{u}^o - \mathbf{s}_1}{r_1^o} + \frac{\mathbf{s}_1 - \mathbf{s}_i}{r_1^o}\right\| = \|\bar{\mathbf{u}}^o + g^o(\mathbf{s}_1 - \mathbf{s}_i)\|\,. \tag{16}$$

r_i^o/r_1^o is now well defined, and it approaches unity and $\mathbf{B}_1$ becomes identity when the source is in the far-field.

The weighting matrix $\mathbf{W}_1$ is not known since $\mathbf{B}_1$ requires $\bar{\mathbf{u}}^o$ and g^o that are the unknowns to be found. Nevertheless, we can approximate $\mathbf{W}_1$ by setting $\mathbf{B}_1$ to identity to generate a preliminary solution for $\boldsymbol{\psi}_1$. A better $\mathbf{W}_1$ is constructed from the preliminary solution to obtain the final $\boldsymbol{\psi}_1$ estimate.

The solution development so far converts the source position in MPR $\widetilde{\mathbf{u}}^o$ to the unknown vector $\boldsymbol{\psi}_1^o$ defined in (12a) whose solution is (13). The first two elements of $\boldsymbol{\psi}_1^o$ are not independent according to (10) and the natural constraint from trigonometry that

$$\boldsymbol{\psi}_1^o(1)^2 + \boldsymbol{\psi}_1^o(2)^2 = 1 \tag{17}$$

has been ignored. We next utilize this constraint.

The estimate $\boldsymbol{\psi}_1$ from (13) can be expressed as

$$\boldsymbol{\psi}_1 = \boldsymbol{\psi}_1^o + \Delta\boldsymbol{\psi}_1 \,, \tag{18}$$

where $\Delta\boldsymbol{\psi}_1$ is the estimation error. Squaring both sides of the first two elements results in

$$\begin{aligned}&\boldsymbol{\psi}_1(1:2)\odot\boldsymbol{\psi}_1(1:2) - \boldsymbol{\psi}_1^o(1:2)\odot\boldsymbol{\psi}_1^o(1:2)\\ &= 2\boldsymbol{\psi}_1^o(1:2)\odot\Delta\boldsymbol{\psi}_1(1:2) + \Delta\boldsymbol{\psi}_1(1:2)\odot\Delta\boldsymbol{\psi}_1(1:2)\,.\end{aligned} \tag{19}$$

In (19), $\boldsymbol{\psi}_1^o(2)$ can be expressed in terms of $\boldsymbol{\psi}_1^o(1)$ according to (17).

Let

$$\boldsymbol{\psi}_2^o = [\boldsymbol{\psi}_1^o(1)^2, g^o]^T \tag{20}$$

be the vector of independent unknowns. Using (19) and the last row of (18), we can construct the linear matrix equation

$$\mathbf{h}_2 - \mathbf{G}_2\boldsymbol{\psi}_2^o = \mathbf{B}_2\Delta\boldsymbol{\psi}_1 + \mathbf{o}_2 \,, \tag{21}$$

$$\mathbf{h}_2 = \begin{bmatrix}\psi_1(1)^2\\ \psi_1(2)^2-1\\ \psi_1(3)\end{bmatrix}, \mathbf{G}_2 = \begin{bmatrix}1 & 0\\ -1 & 0\\ 0 & 1\end{bmatrix}, \tag{22a}$$

$$\mathbf{B}_2 = \mathrm{diag}\left\{[\,2\boldsymbol{\psi}_1^o(1:2)^T\,,1\,]^T\right\}, \tag{22b}$$

and $\mathbf{o}_2 = [\Delta\boldsymbol{\psi}_1(1:2)^T\odot\Delta\boldsymbol{\psi}_1(1:2)^T,\, 0]^T$ is the second order error of (21).

The WLS solution for $\boldsymbol{\psi}_2^o$ is

$$\boldsymbol{\psi}_2 = (\mathbf{G}_2^T\mathbf{W}_2\mathbf{G}_2)^{-1}\mathbf{G}_2^T\mathbf{W}_2\mathbf{h}_2 \,. \tag{23}$$

The weighting matrix $\mathbf{W}_2$ is

$$\begin{aligned}\mathbf{W}_2 &= E\left[(\mathbf{B}_2\Delta\boldsymbol{\psi}_1)(\mathbf{B}_2\Delta\boldsymbol{\psi}_1)^T\right]^{-1}\\ &\simeq (\mathbf{B}_2\,\mathrm{cov}(\boldsymbol{\psi}_1)\,\mathbf{B}_2)^{-1},\end{aligned} \tag{24}$$

where only the linear noise term is used and the noise in $\mathbf{B}_2$ is assumed negligible. $\mathrm{cov}(\boldsymbol{\psi}_1)$ is given by (15). The corresponding covariance matrix of $\boldsymbol{\psi}_2$ is

$$\mathrm{cov}(\boldsymbol{\psi}_2) = (\mathbf{G}_2^T\mathbf{W}_2\mathbf{G}_2)^{-1}\,. \tag{25}$$

The final source location estimate in MPR comes from a direct mapping of $\boldsymbol{\psi}_2$, which is

$$\widetilde{\mathbf{u}} = \begin{bmatrix}\tan^{-1}\left(\frac{\mathrm{sgn}(\psi_1(2))\sqrt{1-\psi_2(1)}}{\mathrm{sgn}(\psi_1(1))\sqrt{\psi_2(1)}}\right)\\ \psi_2(2)\end{bmatrix}. \tag{26}$$

$\tan^{-1}(a/b)$ is the arc-tangent operation with the quadrant of the pair (a,b) taken into consideration.

Let us obtain the covariance matrix of $\widetilde{\mathbf{u}}$. Expressing the estimate $\boldsymbol{\psi}_2$ as $[\cos\theta^{o2},\, g^o]^T$ according to (20) and taking differentials at the true values yield, according to the definition of $\widetilde{\mathbf{u}}^o$ in (4),

$$\Delta\boldsymbol{\psi}_2 = \mathbf{D}_2\Delta\widetilde{\mathbf{u}} \tag{27}$$

where

$$\mathbf{D}_2 = \begin{bmatrix}-\sin 2\theta^o & 0\\ 0 & 1\end{bmatrix}. \tag{28}$$

Thus, after using (25),

$$\mathrm{cov}(\widetilde{\mathbf{u}}) = \mathbf{D}_2^{-1}\,\mathrm{cov}(\boldsymbol{\psi}_2)\mathbf{D}_2^{-T} = (\mathbf{D}_2^T\mathbf{G}_2^T\mathbf{W}_2\mathbf{G}_2\mathbf{D}_2)^{-1} \tag{29}$$

where $\mathbf{W}_2$ is given by (24).

4. CRLB AND ANALYSIS

Under the first order analysis where bias is negligible compared to variance, the CRLB [30] provides a performance bound for the localization problem. From the Gaussian noise model considered, the CRLB for the source position in MPR is

$$\mathrm{CRLB}(\widetilde{\mathbf{u}}^o) = \left(\frac{\partial\mathbf{r}^{oT}}{\partial\widetilde{\mathbf{u}}^o}\mathbf{Q}^{-1}\frac{\partial\mathbf{r}^o}{\partial\widetilde{\mathbf{u}}^{oT}}\right)^{-1}. \tag{30}$$

Recall that $r_1^o = \|\mathbf{u}^o - \mathbf{s}_1\| = 1/g^o$, the range difference (2) can be expressed in terms of the elements of $\widetilde{\mathbf{u}}^o$, θ^o and g^o, as

$$r_{i1}^o = (\|\bar{\mathbf{u}}^o - g^o(\mathbf{s}_i - \mathbf{s}_1)\| - 1)/g^o \tag{31}$$

and $\bar{\mathbf{u}}$ is related to the source arrival angle by (10). From (4), the partial derivative is

$$\frac{\partial\mathbf{r}^o}{\partial\widetilde{\mathbf{u}}^{oT}} = \mathbf{P}^T\mathbf{M} + \mathbf{L}\,, \tag{32}$$

$$\mathbf{P} = [\mathbf{p}_2, \mathbf{p}_3, \cdots, \mathbf{p}_M]\,, \quad \mathbf{p}_i = \frac{\bar{\mathbf{u}}^o - g^o(\mathbf{s}_i - \mathbf{s}_1)}{\|\bar{\mathbf{u}}^o - g^o(\mathbf{s}_i - \mathbf{s}_1)\|}\,, \tag{33}$$

$$\mathbf{M} = \frac{1}{g^o}\begin{bmatrix}-\sin\theta^o & -\cos\theta^o/g^o\\ \cos\theta^o & -\sin\theta^o/g^o\end{bmatrix}, \tag{34a}$$

$$\mathbf{L} = [\mathbf{O}_{(M-1)\times 2}, \mathbf{1}_{M-1}/g^{o2}]\,. \tag{34b}$$

After substituting (14)-(15), (22a)-(22b), (24) and going through some algebraic manipulations, it can be shown analytically that (29) is equal to (30) when the measurement noise n_{i1} is relatively small compared to the measurement r_{i1}. The proposed estimator is able to attain the CRLB performance in the small error region.

5. SIMULATIONS

Simulations use a configuration with $M = 8$ sensors located randomly at $\mathbf{s}_1 = [-0.54, -12.22]^T$ m, $\mathbf{s}_2 = [9.89, -14.01]^T$ m, $\mathbf{s}_3 = [-7.22, -9.21]^T$ m, $\mathbf{s}_4 = [1.08, -14.33]^T$ m, $\mathbf{s}_5 = [-3.32, -13.86]^T$ m, $\mathbf{s}_6 = [-13.61, 9.98]^T$ m, $\mathbf{s}_7 = [-7.31, 3.63]^T$ m, $\mathbf{s}_8 = [16.31, -2.36]^T$ m. The true source angle with respect to $\mathbf{s}_1$ is arbitrarily set to 74.36 deg. The number of ensemble runs is 1000. We evaluate the localization performance in terms of mean-square error (MSE) and bias of the source angle and inverse-range estimates. The proposed method is compared with the closed-form solutions from the literature: Chan-Ho method [18] and SCWLS [31], the iterative solution MLE-MPR [25] and the CRLB.

We first examine the performance for a near-field source with the noise power increases from 10^{-7} m^2 to 10^2 m^2. The range is fixed at 100 m, where the unknown source position in the Cartesian coordinates is $\mathbf{u}^o = [26.41, 84.08]^T$ m. Fig. 1 illustrates the MSE performance. The proposed method yields θ and g estimates reaching the CRLB accuracy if the noise level σ_n^2 is not higher than 1 m^2. MLE-MPR is able to achieve the CRLB performance when $\sigma_n^2 \leqslant 0.1\,\mathrm{m}^2$ and it diverges afterwards, which is caused by the inadequate initial guess from SDR-MPR to start the iteration. The Cartesian coordinate methods Chan-Ho and SCWLS perform well at small noise level and are inferior to the proposed method when $\sigma_n^2 > 0.1\,\mathrm{m}^2$.

Fig. 2 shows the bias behavior. The proposed solution has bias level a little higher than MLE-MPR in the angle estimate, but the

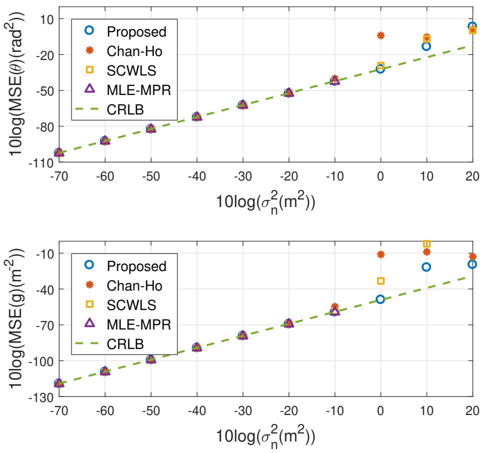

Fig. 1: MSE performance of angle and inverse-range estimates vs. noise power.

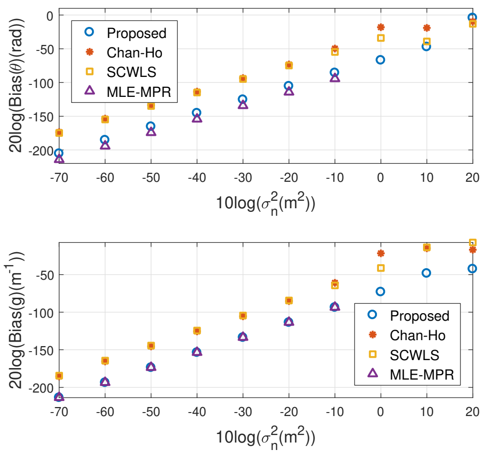

Fig. 2: Bias of angle and inverse-range estimates vs. noise power.

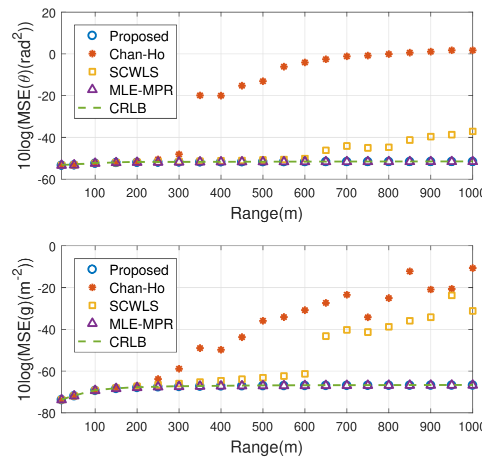

Fig. 3: MSE performance of angle and inverse-range estimates vs. source range.

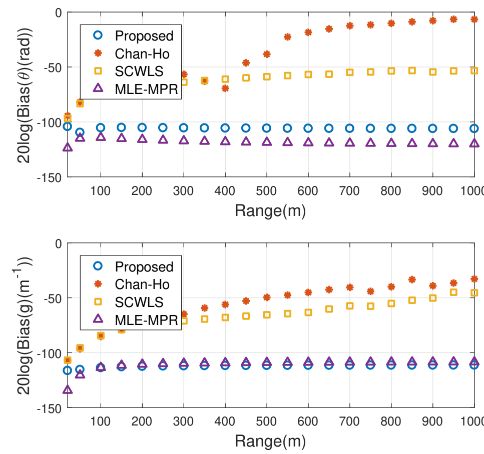

Fig. 4: Bias of angle and inverse-range estimates vs. source range.

bias is better than those of Chan-Ho and SCWLS by nearly 30 dB for both estimates.

The benefit of MPR is that it is able to provide robust angle and inverse-range estimates even when the source is far away from the sensors. Fig. 3 and Fig. 4 illustrate the performance as the source range increases from 20 m to 1000 m, where the noise power is kept at $10^{-3}\mathrm{m}^2$. The proposed method reaches the CRLB accuracy in Fig. 3 and has comparable performance with MLE-MPR. Chan-Ho and SCWLS suffer from the thresholding effect when the source range is beyond 250 m and 600 m respectively. In Fig. 4, the bias of proposed solution is larger than that of MLE-MPR in angle estimate. It is, however, much less than those from Chan-Ho and SCWLS as the source range increases.

The proposed method has complexity $O(M^2N+MN^2+N^3)$, where M is the number of sensors and $N = 2$ is the localization dimension. TABLE 1 tabulates the relative computation times of the algorithms for the simulation in Fig. 1 at noise power $\sigma_n^2 = 0.01\ \mathrm{m}^2$, where all algorithms are able to provide good estimates. The computation times are recorded from Matlab 2017b implementation executed on a typical PC with i7-4790 processor. The processing time of proposed solution is significantly less than that of MLE-MPR and is comparable with the Chan-Ho solution.

Table 1: Relative computation times for the proposed solution, MLE-MPR, Chan-Ho and SCWLS.

Algorithm	Proposed	MLE-MPR	Chan-Ho	SCWLS
Rel. Proc. Time	1	1605	0.9	8.2

6. CONCLUSION

This paper proposes an algebraic closed-form solution for TDOA localization in MPR that unifies the positioning of a source regardless it is near or distant from the sensors. The solution is obtained by representing the arrival angle with a pair of sine and cosine functions, reformulating the measurement equation and applying two linear weighted least-squares optimizations. It performs better than MLE-MPR [25] for a near-field source at higher noise level and is more computationally efficient. It also outperforms the closed-form solutions from the literature. Both analysis and simulation illustrate the performance of the proposed solution in reaching the CRLB accuracy over the small error region.

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