AN LS LOCALISATION METHOD FOR MASSIVE MIMO TRANSMISSION SYSTEMS

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ABSTRACT

We present a novel localization method based on directional beams, as available in novel massive MIMO transmission techniques instead of radius information, and derive a least squares (LS) estimation method. The new method is a direct LS method that can be solved by a linear set of equations rather than an iterative method required for radius information. In a further step, we also show how to transform radius information into virtual beams to apply the proposed method. Finally, we evaluate the accuracy of the new methods by simulations.

Index Terms— Localisation, Least Squares, Massive MIMO

1. INTRODUCTION

For many years the localisation of a transmitter by means of information obtained by several receivers remains an interesting research problem with many applications in satellite localisation, indoor and outdoor detection of objects for various reasons. Classically, a device either receives from several fix stations with a-priori known positions $(x_m, y_m); m = 1, 2, ..., M$ (or transmits to them) as depicted in Figure 1. As the receiver strength decreases monotonically with distance, such observation can be mapped into an estimate of distance. In free space propagation the receiver strength information is simply given by the relation $E(d) \sim 1/d$ as a function of distance d, but depending on the transmission environment more involved relations can be incorporated [1]. To simplify matters, we will assume that distance estimates r_m are given to us. The setup is then fully described by a set of M triples $\{(x_m, y_m, r_m)\}_{m=1,2,...,M}$. The problem can be formulated as finding an optimal position estimate $(\hat{x}, \hat{y})$ with the minimal distance to the given values $r_1, r_2, ..., r_M$ in a Least Squares (LS) sense:

$$(x_{LS}, y_{LS}) = \arg\min_{(x,y)} \sum_{m=1}^{M} \left(\sqrt{(x-x_m)^2 + (y-y_m)^2} - r_m \right)^2. \quad (1)$$

However, due to the nonlinear nature of the problem, such expression is not very suitable for finding the desired point in a LS sense, in particular on low cost fixed-point devices.

1.1. Relation to Prior Work

There exists a multitude on survey papers as well as books, we just mention a few recent ones: [2–6]. Recently IEEE Proceedings had two issues devoted to localisation techniques [7, 8], containing several overview papers for various directions [9–11]. Essentially, there

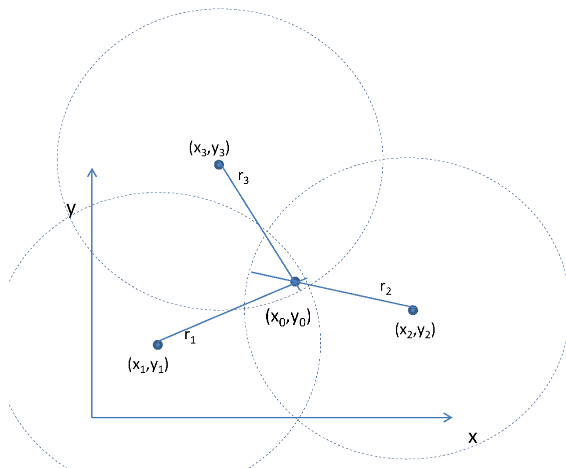

Fig. 1. Localisation scenario with $M = 3$ base stations to localise object at (x_0, y_0) by means of radius information.

are so-called range based methods and range-free methods. Range based methods are Time-of-Arrival (ToA) [11, 12] also often referred to as Time-of-Flight (ToF) or trilateration, Time-Difference-of-Arrival (TDoA) also called multi-lateration [13], Angle-of-Arrival (AoA) and the related triangulation methods [14], Received Signal Strength Indicator (RSSI) [1, 10] also in combination with fingerprints [15]. Often localisation methods are used in the context of target tracking. Our proposed method relates best to triangulation methods as only angular information is being used. However, due to our LS approach including an arbitrary number of reference points (bases stations), we are able to formulate the solution without trigonometric functions and make it more appropriate for modern signal processing applications.

1.2. Our Contribution

In this contribution we will follow a new path that is offered by future massive MIMO systems [16] as they will allow for very narrow beams, making an angle of arrival estimation much more precise and thus offer new parameters for designing position estimators. Different to the classic approach based on receiver strength, we will show that the new concept based on arrival angles allows for an explicit LS solution. Moreover, the method can also be based on receiver strength information as we will show in the second part of this paper and thus does not need a massive number of antennas to be applied.

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1.3. Paper Structure

This paper provides in Section 1 a short introduction into the localisation problem and related work. In Section 2 we propose a novel scheme in the context of massive MIMO transmission systems. In Section 3 we reformulate the original nonlinear LS problem into a form that can be solved by LS methods and thus allows to estimate position information from receiver strength information. We demonstrate the behavior of our methods on some experimental examples in Section 4 and eventually we conclude the contribution in Section 5.

2. LOCALIZATION SETUP IN MASSIVE MIMO SYSTEMS

Consider M base stations equipped with massive linear antenna arrays at positions $(x_m, y_m); m = 1, 2, ..., M$. Based on their estimation algorithm, the base stations derive an angle α_m, in which direction they estimate the desired object to be (due to the strongest receiver strength), the available information is thus the triple (x_m, y_m, α_m). The orientation angles α_m under which the base stations estimate the desired object to be are with respect to the coordinate system and not relating to the orientation of the linear array. They relate directly to the slope $a_m = \tan(\alpha_m)$ of the lines, in which direction the base station believes the desired object is located. Each base station with its estimate is thus fully described by the triple (x_m, y_m, a_m) which now replaces the original triple from the classic approach, each triple defines a line

$$(y - y_m) = a_m(x - x_m) \tag{2}$$

on which the object to be located presumably lies. We consider this mapping unique as the orientation of the array allows only a range of 180 degrees. Let the desired object be at true position (x_0, y_0) as depicted in Figure 2 with $M = 3$ base stations. We desire to minimize the Euclidean distance between a point (x, y) and the corresponding set of lines defined by $\{(x_m, y_m, a_m)\}_{m=1,2,...,M}$:

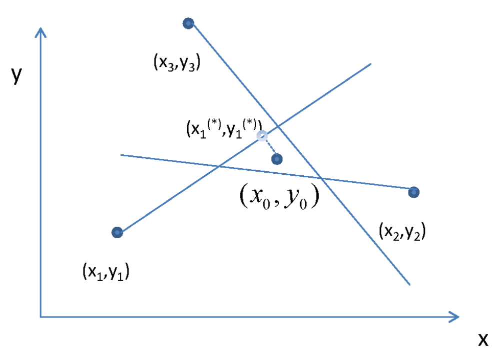

Fig. 2. Localization scenario with $M = 3$ base stations to localize object at (x_0, y_0) by means of direction information.

$$(\hat{x}, \hat{y}) = \arg\min_{(x,y)} \sum_{m=1}^{M} \text{dist}((x, y), \text{line}(x_m, y_m, a_m))^2, \tag{3}$$

with dist() denoting the Euclidean distance between a point and a line. Our desire is to develop a LS algorithm that delivers such an estimate $(\hat{x}, \hat{y})$ with the expectation that it minimizes the Euclidean distance between such estimate and the true position (x_0, y_0). The deviation between these two points is due to the estimation errors in the arrival angles α_m and equivalently in the slopes $a_m; m = 1, 2, ..., M$.

Let us denote the point that is closest to $(\hat{x}, \hat{y})$ along the line to the m-th base station located at (x_m, y_m) by $(x_m^{(*)}, y_m^{(*)})$. In Figure 2 this is exemplary shown for $(x_1^{(*)}, y_1^{(*)})$. Once this point is defined, the original problem (3) takes on the LS-form

$$(\hat{x}, \hat{y}) = \arg\min_{(x,y)} \sum_{m=1}^{M} (x - x_m^{(*)})^2 + (y - y_m^{(*)})^2. \tag{4}$$

Given an estimate $(\hat{x}, \hat{y})$ we can measure its accuracy by computing

$$D = \sqrt{\frac{1}{M} \sum_{m=1}^{M} (\hat{x} - x_0)^2 + (\hat{y} - y_0)^2}, \tag{5}$$

denoting the root mean square (RMS) value of such estimate. The following Lemma 2.1 provides us with the necessary information how to compute these required points $(x_m^{(*)}, y_m^{(*)})$.

Lemma 2.1 *Given point* $(\hat{x}, \hat{y})$, *point* $\left(x_m^{(*)}, y_m^{(*)}\right)$ *along a line* $(y - y_m) = a_m(x - x_m)$ *closest to* $(\hat{x}, \hat{y})$ *is given by*

$$x_m^{(*)} = \frac{1}{1 + a_m^2}(\hat{x} + a_m\hat{y}) - \frac{a_m}{1 + a_m^2}(y_m - a_m x_m), \tag{6}$$

$$y_m^{(*)} = \frac{a_m}{1 + a_m^2}(\hat{x} + a_m\hat{y}) + \frac{1}{1 + a_m^2}(y_m - a_m x_m). \tag{7}$$

Proof: The line is defined by point (x_m, y_m) and slope $a_m = \tan(\alpha_m)$ as given in (2). The closest point along such line to $(\hat{x}, \hat{y})$ is given by $\min_{x,y}(y - \hat{y})^2 + (x - \hat{x})^2$:

$$\min_{x,y}(y-\hat{y})^2+(x-\hat{x})^2 = \min_{x,y}(y_m+a_m(x-x_m)-\hat{y})^2+(x-\hat{x})^2$$

Differentiation with respect to x and setting to zero for $x = x_m^*$ leads to

$$\begin{aligned} 0 &= a_m(y_m - \hat{y} + a_m(x - x_m)) + x - \hat{x} && (8) \\ &= (1 + a_m^2)x + a_m(y_m - \hat{y} - a_m x_m) - \hat{x}, \\ x_m^{(*)} &= \frac{\hat{x} - a_m(y_m - \hat{y} - a_m x_m)}{1 + a_m^2}, && (9) \\ y_m^{(*)} &= y_m + a_m \left(\frac{\hat{x} - a_m(y_m - \hat{y} - a_m x_m)}{1 + a_m^2} - x_m \right) && (10) \end{aligned}$$

with the y coordinate obtained from plugging in x into (2). A simple reformulation leads to the result of Lemma 2.1.

Let us reformulate (6) and (7) by using the following short notation: $A_{0,m} = \frac{1}{1+a_m^2}, A_{1,m} = \frac{a_m}{1+a_m^2}, A_{2,m} = \frac{a_m^2}{1+a_m^2}, L_m = y_m - a_m x_m$:

$$x_m^{(*)} = A_{0,m}\hat{x} + A_{1,m}\hat{y} - A_{1,m}L_m, \tag{11}$$

$$y_m^{(*)} = A_{1,m}\hat{x} + A_{2,m}\hat{y} + A_{0,m}L_m. \tag{12}$$

Now, minimizing the Euclidean distance in (4) simplifies to

$$\begin{aligned} (\hat{x}, \hat{y}) &= \arg\min_{(x,y)} \sum_{m=1}^{M} (x - x_m^{(*)})^2 + (y - y_m^{(*)})^2, && (13) \\ &= \arg\min_{(x,y)} \sum_{m=1}^{M} (A_{2,m}x - A_{1,m}y + A_{1,m}L_m)^2 \\ &\quad +(-A_{1,m}x + A_{0,m}y - A_{0,m}L_m)^2, && (14) \end{aligned}$$

whose solution we present in the following lemma.

Lemma 2.2 *The minimum sum squared distance is found for point $(\hat{x}^{(B)}, \hat{y}^{(B)})$ that satisfies the following set of linear equations:*

$$\begin{bmatrix} \sum_{m=1}^{M} A_{2,m} & -\sum_{m=1}^{M} A_{1,m} \\ -\sum_{m=1}^{M} A_{1,m} & \sum_{m=1}^{M} A_{0,m} \end{bmatrix} \begin{bmatrix} \hat{x}^{(B)} \\ \hat{y}^{(B)} \end{bmatrix} = \begin{bmatrix} -\sum_{m=1}^{M} A_{1,m} L_m \\ \sum_{m=1}^{M} A_{0,m} L_m \end{bmatrix}. \quad (15)$$

The so obtained estimation $(\hat{x}^{(B)}, \hat{y}^{(B)})$ is independent on the knowledge of (x_0, y_0) and only obtained by using known information $\{(x_m, y_m, a_m)\}_{m=1,2,...,M}$.

3. AN ALTERNATIVE LS APPROACH WITH VIRTUAL BASE STATIONS BASED ON RECEIVE STRENGTH

As mentioned in the introduction, the direct approach to solve the localisation problem when receiver strength information is given, results in a nonlinear LS problem. Illustrated in Figure 3 we propose thus a different method. Given the location of M base stations, we consider all $M^{(*)} = \binom{M}{2} = \frac{M(M-1)}{2}$ pairs of them. For each pair (base station location (x_i, y_i) and (x_k, y_k) with distance $d_{i,k}$) the receiver strength defines the radius of circles. We compute the line that is perpendicular to the connecting line between base station i and k at position $(x_{i,k}^{(*)}, y_{i,k}^{(*)})$. This point is chosen to divide the connection between base station i and k according to the radius information (r_i, r_k) provided. Note that we need to distinguish two cases. The first case is given if the sum of the two radii is smaller than the distance between the corresponding circles, i.e., $d_{2,3} > r_2 + r_3$ as shown in Figure 3 between the two base stations 2 and 3. The second case is its opposite $d_{1,2} \le r_1 + r_2$ as shown between the base stations 1 and 2.

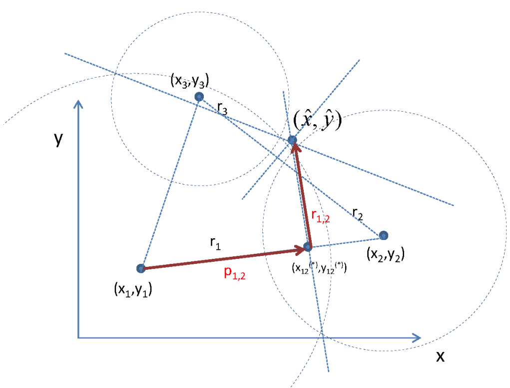

Fig. 3. Localization scenario with $M = 3$ base stations to localize object at $(\hat{x}, \hat{y})$ by means of radius information, translated into direction information.

Given all distances $d_{i,k} = \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2}$ for $i = 1, 2, ..., M, i \neq k$, the distance $p_{i,k}$ along the connecting line from starting point (x_i, y_i) to $(x_{i,k}^{(*)}, y_{i,k}^{(*)})$ is given by

$$p_{i,k} = \begin{cases} p_{i,k}^{(+)} = \frac{d_{i,k} - r_k + r_i}{2} & ;\text{if } r_k + r_i < d_{i,k} \\ p_{i,k}^{(-)} = \frac{|d_{i,k}^2 - r_k^2 + r_i^2|}{2d_{i,k}} & ;\text{if } d_{i,k} \le r_i + r_k \end{cases}. \quad (16)$$

The line from base station i to base station k can be described by

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \gamma \begin{bmatrix} x_k - x_i \\ y_k - y_i \end{bmatrix}. \quad (17)$$

Setting $\gamma = p_{i,k}/d_{i,k}$, we find the desired point $(x_{i,k}^{(*)}, y_{i,k}^{(*)})$. This leads us to our next statement.

Lemma 3.3 *The perpendicular line between two base stations at (x_i, y_i) and (x_k, y_k) is defined by*

$$-\frac{x_i - x_k}{y_i - y_k}\left(x - x_{i,k}^{(*)}\right) = \left(y - y_{i,k}^{(*)}\right) \quad (18)$$

with the point

$$\begin{bmatrix} x_{i,k}^{(*)} \\ y_{i,k}^{(*)} \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \frac{p_{i,k}}{d_{i,k}} \begin{bmatrix} x_k - x_i \\ y_k - y_i \end{bmatrix}. \quad (19)$$

We thus have a new set of $M^{(*)}$ lines defined by $\{(x_{i,k}^{(*)}, y_{i,k}^{(*)}, a_{i,k}^{(*)}\}$ with $a_{i,k}^{(*)} = -\frac{x_i - x_k}{y_i - y_k}$ which can be used as before for the location estimate.

4. EXPERIMENTAL RESULTS

Figure 4 displays our experimental setup. We start with $M = 3$ base stations that are positioned closely together on the right of the figure. We then increase their number up to 12 by adding more neighbors along a circle until they are finally equidistantly distributed along a circle. By this we cover two extreme scenarios, all base stations are clumped together and all base stations are nicely spaced apart. The desired objects/users to be localized are positioned parallel to the x-axis, starting from the very left (far from the three initial base stations) to the very right (close to the three initial base stations). We expect that for few base stations the estimation algorithms become better with closer position to the base stations, and that with increasing number of base stations the estimates should improve.

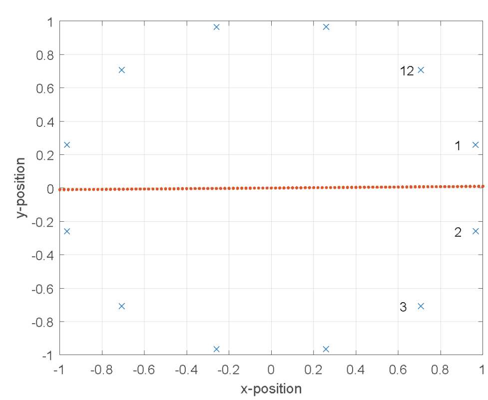

Fig. 4. Experimental setup starting with three base stations (circles) on the right and gradually increasing their number. The receiver location is along the x axis (crosses).

To generate random scenarios we added zero-mean Gaussian noise to the true user position (only in the x coordinate) with standard deviation $\sigma_n = \{0.001, 0.01, 0.1\}$. Figure 5 shows the result of D_{BF}^2 from Eq. (5) of 1 000 MC runs with $M = 3, 4, ..., 12$ base stations. Here the subscript BF refers to the beam-forming algorithm. As expected the accuracy increases with the number of base stations but surprisingly for a few base stations there is a "sweet spot" for which the precision is higher. Only for equally spaced base stations the precision is independent of the position. This becomes even clearer when inspecting Figure 6 in which the precision is displayed for $M = 3$ and $M = 12$ base stations when varying the noise variance.

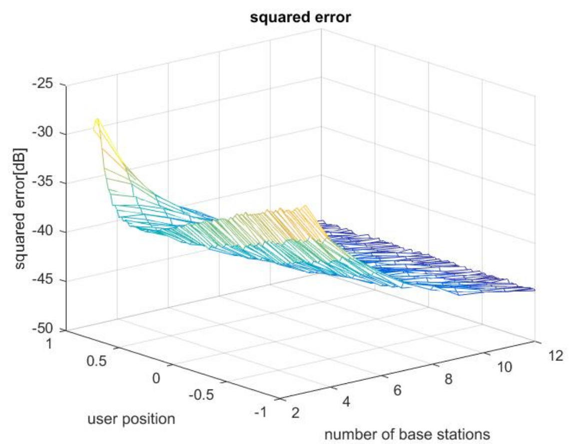

Fig. 5. MSE D^2_{BF} for beam-forming in massive MIMO with $\sigma_n = 0.01$, 3 to 12 base stations.

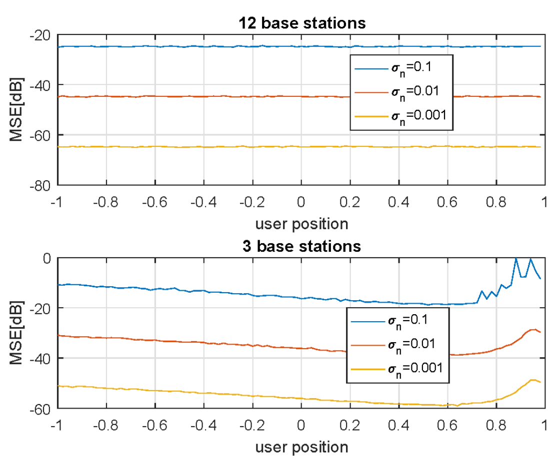

Fig. 6. MSE D^2_{BF} for beam-forming in massive MIMO with $\sigma_n = \{0.001, 0.01, 0.1\}$, 3 and 12 base stations.

We repeat now the experiment using RSSI information converted into directional information instead of directional information as explained in the previous section. Figure 7 displays the corresponding MSE metric D^2_{VB} for this experiment, the subscript VB denoting the virtual beam-forming approach. It is very illustrative to discuss the results. In general the precision improves with the number of base stations but the precision depends very much on the position even for many equidistantly distributed base stations.

We finally compare the two methods under various base station numbers and positions. Figure 8 shows a comparison of the two methods when all $M = 12$ base stations are employed and only three. As expected with growing error estimation variance, the corresponding error in location estimation also grows. While the obtained curves for beam-forming are superior, the RSSI curves can achieve the same quality depending on the user position. Note that $M^{(*)} \gg M$ but the radius based method never exceeds the quality of the beam based method.

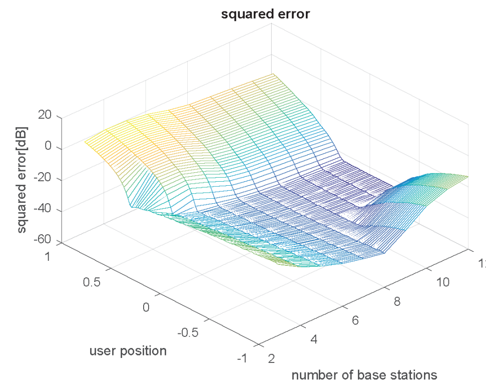

Fig. 7. MSE D^2_{VB} using virtual beams derived from radius information for $\sigma_n = 0.01$.

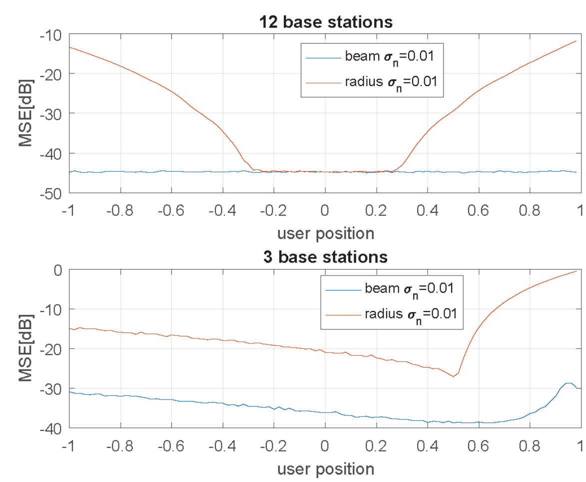

Fig. 8. Comparison of beam based and radius based method (converted into virtual beams).

5. CONCLUSIONS

We proposed a novel LS based method for localisation under the assumption that precise direction information from electromagnetic beams is available, which is expected in modern massive MIMO transmission systems. Furthermore, we introduce a conversion method from radius to beam information that allows to applying the new method also to classic setups in which only receiver strength information is available. The advantage of our method is that the beam-based formulation allows a closed form LS solution, suitable for low cost fixed-point processors, while the radius based form does not.

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