ROBUST BEAMSPACE DESIGN FOR DIRECT LOCALIZATION

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ABSTRACT

Direct localization systems with large-scale antenna-arrays can greatly improve the localization accuracy by jointly processing all the observed signals. However, it incurs high communication overhead due to high-dimensional array signal transmission. In this paper, we propose a robust beamspace design technique in the presence of parameter uncertainty that can achieve high-accuracy positioning only with limited communication overhead. The beamspace design problem is formulated as a robust optimization in order to guarantee the worst-case performance in terms of the squared position error bound (SPEB). Since the problem is non-convex, we relax it to a convex programming and further prove that the solution of the relaxed problem converges to the optimal solution of the original problem. Simulation results validate the effectiveness and robustness of the proposed beamspace.

***Index Terms*—** Large-scale antenna arrays, beamspace design, direct localization, array signal processing

1. INTRODUCTION

High-accuracy localization is essential for a wide range of applications, and the global positioning system (GPS) is the most popular localization system. However, there exists GPS-challenged environment where the localization accuracy degrades sharply [1, 2]. Network localization is a promising alternative which typically composes two steps: estimating position-related parameters at each base station, and translating parameters to the position at the fusion center [1,2]. Note that this two-step localization system may suffer from information loss during metrics estimation, e.g., neglecting the geometric relationships among the metrics from the same user.

The direct position determination (DPD) system with large-scale antenna arrays has a great potential to improve the network localization accuracy. This is because more antennas bring higher spatial resolution and DPD avoids potential information loss by eliminating the intermediate parameter estimation step. However, with the growing antenna number, the DPD system will incur long-latency for requiring all the high-dimensional array signals to be transmitted to the fusion center [3]. Here, to decrease communication overhead, a suitable array signal dimension reduction method is demanded.

In array signal processing field, the beamspace technology has been widely used to reduce array signal dimension. The discrete Fourier transform (DFT) sequence is the most popular one, while its spatial resolution degrades by sidelobes of the other beams [4]. The discrete prolate spheroidal sequence (DPSS) in [5–8] minimizes the average Frobenius-norm error between the original and decompressed signal covariance matrices. However, these approaches are not necessarily optimal in terms of the mean squared error (MSE). The MSE is commonly regarded as the performance metric and its lower bound, the Cramér-Rao bound (CRB), has been widely used to evaluate the performance of localization system [9–11]. In terms of the CRB, the work in [9] proposes a precision lossless beamspace scheme with perfect agent position knowledge, while only a heuristic scheme is provided when the knowledge is imprecise. To the best of authors' knowledge, existing works have not proposed an effective beamspace design strategy for high-accuracy localization.

The main contributions of this paper are as follows. First, we formulate a robust optimization problem to guarantee the worst-case localization performance in the presence of position uncertainty. Next, we propose a convex relaxation of the robust problem, and prove that the solution of the relaxed problem converges to the optimal solution of the original problem. Third, we propose a robust beamspace design technique that achieves high accuracy with orders of magnitude lower signal dimension.

Notation: We use upper and lower case boldface to denote matrices and vectors, respectively; $[\boldsymbol{x}]_k$ denotes the kth element of $\boldsymbol{x}$; $[\boldsymbol{X}]_{i,j}$ is an element at the ith row and jth column of matrix $\boldsymbol{X}$; $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote the transpose and the conjugate transpose, respectively; $\boldsymbol{I}_n$ is an $n \times n$ identity matrix.

2. PRELIMINARY

2.1. System Model

Consider a 2-D wireless localization network with N_{a} single-antenna agents and N_{b} base stations where each is equipped with an array of M antennas. The base stations have pre-

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cise self-position knowledge while the agent positions are estimated in the fusion center by jointly processing all the narrow-band signals observed at each base station. Let $\boldsymbol{p}_i = [p_{ix}, p_{iy}]^{\mathrm{T}}$ ($i \in \mathcal{N}_{\mathrm{a}} = \{1, 2, \ldots, N_{\mathrm{a}}\}$) and $\boldsymbol{q}_j = [q_{jx}, q_{jy}]^{\mathrm{T}}$ ($j \in \mathcal{N}_{\mathrm{b}} = \{1, 2, \ldots, N_{\mathrm{b}}\}$) denote the position of the ith agent and jth base station, respectively. The far-field condition is assumed as in [3–5], that the incident angles to all array-antennas at one base station are identical.

The waveform-unknown received *element space signal* at the jth base station is written as[1]

$$\mathsf{r}_j(k) = \boldsymbol{A}(\boldsymbol{\theta}_j)\mathsf{s}_j(k) + \mathsf{n}_j(k) \in \mathbb{C}^M,\ k = 1, 2, \ldots, K$$

where $\mathsf{s}_j(k) = \begin{bmatrix} \mathsf{s}_{j1}(k) & \mathsf{s}_{j2}(k) & \cdots & \mathsf{s}_{jN_{\mathrm{a}}}(k) \end{bmatrix}^{\mathrm{T}}$ is the received signal in which $\mathsf{s}_{ji}(k)$, from the ith agent, is modeled as a stationary Gaussian process with zero mean and variance Λ. The steering vector $\boldsymbol{a}(\theta_{ji})$ is the array response to $\mathsf{s}_{ji}(k)$ with the incident angle θ_{ji}, and the steering matrix takes the form of $\boldsymbol{A}(\boldsymbol{\theta}_j) = \begin{bmatrix} \boldsymbol{a}(\theta_{j1}) & \boldsymbol{a}(\theta_{j2}) \cdots \boldsymbol{a}(\theta_{jN_{\mathrm{a}}}) \end{bmatrix}$ with the angle-of-arrival (AOA) vector $\boldsymbol{\theta}_j = \begin{bmatrix} \theta_{j1} & \theta_{j2} & \cdots & \theta_{jN_{\mathrm{a}}} \end{bmatrix}^{\mathrm{T}}$ [10]. The noise $\mathsf{n}_j(k)$ is modeled as the additive white noise with unknown variance σ_{n}^2 and K is the the snapshot number.

In order to reduce the communication overhead, we project the array signal from the M-dimensional element space to the N_j-dimensional beamspace in which $M \gg N_j$. The low-dimensional *beamspace signal* is represented by

$$\mathsf{r}_{j,\mathrm{bs}}(k) = \boldsymbol{B}_j^{\mathrm{H}}\mathsf{r}_j(k) \in \mathbb{C}^{N_j}$$

where $\boldsymbol{B}_j \in \mathbb{C}^{M \times N_j}$ is the beamspace matrix. After that, the beamspace signal is transmitted to the fusion center.

At the fusion center, encompassing all the beamspace signals leads to

$$\mathsf{r}_{\mathrm{bs}}(k) = \boldsymbol{B}^{\mathrm{H}}\boldsymbol{A}(\boldsymbol{\theta})\mathsf{s}(k) + \boldsymbol{B}^{\mathrm{H}}\mathsf{n}(k)$$

where $\mathsf{r}_{\mathrm{bs}}(k)$, $\boldsymbol{\theta}$, $\mathsf{s}(k)$ and $\mathsf{n}(k)$ are the stacking column vectors from different base station parameters $\mathsf{r}_{j,\mathrm{bs}}(k)$, $\boldsymbol{\theta}_j$, $\mathsf{s}_j(k)$ and $\mathsf{n}_j(k)$ respectively, and $\boldsymbol{A}(\boldsymbol{\theta})$ and $\boldsymbol{B}$ are block diagonal matrices constructed from different $\boldsymbol{A}(\boldsymbol{\theta}_j)$ and $\boldsymbol{B}_j$.

The concept of beamspace direct localization is estimating agent positions $\boldsymbol{p} = \begin{bmatrix} \boldsymbol{p}_1^{\mathrm{T}} & \boldsymbol{p}_2^{\mathrm{T}} & \cdots & \boldsymbol{p}_{N_{\mathrm{a}}}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$ directly from the beamspace signal $\mathsf{r}_{\mathrm{bs}}(k) (k = 1, 2, \ldots, K)$. As we focus on the beamspace design problem in this paper, the detailed description about beamspace direct localization algorithm is omitted due to space constraints.

2.2. Performance Metric

For any unbiased position estimator $\hat{\mathsf{p}}$ for $\boldsymbol{p}$, it satisfies

$$\mathbb{E}\{(\hat{\mathsf{p}} - \boldsymbol{p})^{\mathrm{T}}(\hat{\mathsf{p}} - \boldsymbol{p})\} \geq \underbrace{\mathrm{tr}\Big\{\boldsymbol{J}^{-1}(\boldsymbol{p}; \boldsymbol{B})\Big\}}_{\text{SPEB}}$$

[1] Since the direct localization algorithm views the emitter of each multi-path signal as the possible agent, the multi-path signal can be considered as a special case of multiple agents.

where $\mathrm{tr}\{\cdot\}$ denotes the trace operator and $\boldsymbol{J}(\boldsymbol{p}; \boldsymbol{B})$ is the Fisher information matrix (FIM) [10–12]. The squared position error bound (SPEB) is the variation of the CRB and states the fundamental of the localization system. Due to its tractability and asymptotical achievability in high SNR regimes [10–12], we adopt the SPEB as the performance metric in this paper. The purpose of the beamspace design is to minimize the SPEB over the beamspace matrix $\boldsymbol{B}$. In [13], we have derived the FIM of beamspace direct localization.

Proposition 1 *([13]) When the signal waveform is unknown, the FIM for the positions is*

$$\boldsymbol{J}(\boldsymbol{p}; \boldsymbol{B}) = \boldsymbol{T}^{\mathrm{T}}\boldsymbol{J}(\boldsymbol{\theta}; \boldsymbol{B})\boldsymbol{T}$$

where $\boldsymbol{T} = \partial\boldsymbol{\theta}/\partial\boldsymbol{p}$ is the Jacobian matrix for the transformation from $\boldsymbol{p}$ to $\boldsymbol{\theta}$ and

$$\boldsymbol{J}(\boldsymbol{\theta}; \boldsymbol{B}) =$$
$$\frac{2K}{\sigma_{\mathrm{n}}^2}\Big\{\big(\boldsymbol{D}^{\mathrm{H}}(\boldsymbol{\Pi}_{\boldsymbol{B}} - \boldsymbol{\Pi}_{\boldsymbol{\Pi}_{\boldsymbol{B}}\boldsymbol{A}})\boldsymbol{D}\big) \odot \big(\boldsymbol{\Lambda}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{B}\boldsymbol{R}_{\boldsymbol{B}}^{-1}\boldsymbol{B}^{\mathrm{H}}\boldsymbol{A}\boldsymbol{\Lambda}\big)^*\Big\}$$

in which $\odot$ denotes the Hadamard product, $\boldsymbol{R}_{\boldsymbol{B}}$ and $\boldsymbol{\Lambda}$ are the covariance matrices of the beamspace signal $\mathsf{r}_{\mathrm{bs}}(k)$ and the transmitted signal $\mathsf{s}(k)$, respectively, $\boldsymbol{\Pi}_{\boldsymbol{B}} = \boldsymbol{B}(\boldsymbol{B}^{\mathrm{H}}\boldsymbol{B})^{-1}\boldsymbol{B}^{\mathrm{H}}$ is the orthogonal projection matrix onto the column space of $\boldsymbol{B}$, and $\boldsymbol{D}$ is the derivative of the steering matrix $\boldsymbol{A}$, i.e. $[\boldsymbol{D}]_{j,k} = \partial[\boldsymbol{A}]_{j,k}/\partial[\boldsymbol{\theta}]_k$.

Corollary 1 *([13]) If beamspace matrices $\boldsymbol{B}$ and $\boldsymbol{B}'$ span the same column space, then $\boldsymbol{J}(\boldsymbol{\theta}; \boldsymbol{B}) = \boldsymbol{J}(\boldsymbol{\theta}; \boldsymbol{B}')$ and further $\boldsymbol{J}(\boldsymbol{p}; \boldsymbol{B}) = \boldsymbol{J}(\boldsymbol{p}; \boldsymbol{B}')$.*

Corollary 1 demonstrates that the function from $\boldsymbol{B}$ to the SPEB is not injective, and the minimum SPEB corresponds to many different beamspace matrices. To ensure the unique solution, we turn to optimize over the projection matrix $\boldsymbol{\Pi}_{\boldsymbol{B}}$, which is a compact representation of the space spanned by an entire equivalence class of element $\boldsymbol{B}$.

Due to the user interference, the SPEB is intricate for the multi-user system. For the ease of analysing, we will discuss the single-user case in the rest of the paper, while the complicated multi-user case will be our future work. For the single-user scenario, minimizing the SPEB over $\boldsymbol{\Pi}_{\boldsymbol{B}}$ can be decoupled into N_{b} subproblems that minimizing the CRB for angle, $J^{-1}(\theta_j; \boldsymbol{\Pi}_{\boldsymbol{B}_j})$, over $\boldsymbol{\Pi}_{\boldsymbol{B}_j}$, for all $j \in \mathcal{N}_{\mathrm{b}}$. This is because the observations at each base station are independent. Once each optimal $\boldsymbol{B}_j$ is obtained, the block-diagonal optimal beamspace matrix $\boldsymbol{B}$ corresponding to the minimum SPEB can be constructed from different $\boldsymbol{B}_j$. Hence, for brevity, subscripts i, j are omitted in the sequel.

3. ROBUST BEAMSPACE DESIGN

3.1. Robust Formulation

Note that the CRB for angle is a function of θ, which indicates that minimizing $J^{-1}(\theta; \boldsymbol{\Pi}_{\boldsymbol{B}})$ over $\boldsymbol{\Pi}_{\boldsymbol{B}}$ needs precise

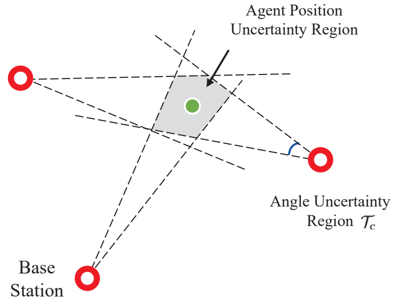

Fig. 1: Coarse position information with uncertainty region ($\theta \in \mathcal{T}_\mathrm{c}$).

agent position information. However, the precise agent position information is unavailable in practice. Instead, a region subjects to uncertainty can be inferred, as shown in Fig. 1.[2]

Suppose the actual angle θ lies in the linear set $\mathcal{T}_\mathrm{c}$. In the presence of parameter uncertainty, the goal of beamspace design is to provide the best worst-case localization performance. Then, the robust beamspace design problem is formulated as

$$\mathscr{P}_1: \quad \min_{\boldsymbol{X}} \quad \max_{\theta\in\mathcal{T}_\mathrm{c}} J^{-1}(\theta;\boldsymbol{X}) \qquad \text{s.t.} \quad \boldsymbol{X}\in\mathcal{X}_N$$

where

$$\mathcal{X}_N := \left\{\boldsymbol{B}\boldsymbol{B}^\mathrm{H} | \boldsymbol{B}^\mathrm{H}\boldsymbol{B} = \boldsymbol{I}_N, \boldsymbol{B}\in\mathbb{C}^{M\times N}, M \gg N\right\}$$

$$J^{-1}(\theta;\boldsymbol{X}) = \frac{1}{2K\sigma_\mathrm{s}^4}\frac{1+\sigma_\mathrm{s}^2\boldsymbol{a}^\mathrm{H}\boldsymbol{X}\boldsymbol{a}}{\boldsymbol{d}^\mathrm{H}\boldsymbol{X}\boldsymbol{d}\boldsymbol{a}^\mathrm{H}\boldsymbol{X}\boldsymbol{a} - \boldsymbol{d}^\mathrm{H}\boldsymbol{X}\boldsymbol{a}\boldsymbol{a}^\mathrm{H}\boldsymbol{X}\boldsymbol{d}}$$

in which $\boldsymbol{X}$ is the rank-N projection matrix, $\boldsymbol{a}$ and $\boldsymbol{d}$ are the steering vector and its first order derivative of θ, and σ_s^2 is the signal-to-noise-ratio (SNR), defined as $\sigma_\mathrm{s}^2 = \Lambda/\sigma_\mathrm{n}^2$.

3.2. Algorithm Design

The difficulties of solving $\mathscr{P}_1$ are: 1) The objective function $J^{-1}(\theta;\boldsymbol{X})$ is not convex neither in $\boldsymbol{X}$ nor in θ; 2) the feasible set $\mathcal{X}_N$ is not convex as well. In order to solve it effectively, we relax problem $\mathscr{P}_1$ to a convex programming, and further prove that the solution of the relaxed problem converges to the optimal solution of the original problem.

First, for variable θ, we partition the uncertainty set $\mathcal{T}_\mathrm{c}$ uniformly into S parts, and let $\mathcal{T}_\mathrm{d} = \{\vartheta_1,\vartheta_2,\ldots,\vartheta_{S+1}\} \subseteq \mathcal{T}_\mathrm{c}$ approximate the continuous region. This give rise to

$$\mathscr{Q}_1: \quad \min_{\boldsymbol{X}} \quad t \qquad \text{s.t.} \quad J^{-1}(\vartheta_i;\boldsymbol{X}) \le t, \quad i=1,2,\ldots,S+1, \quad \boldsymbol{X}\in\mathcal{X}_N.$$

Second, for the rank-N projection matrix $\boldsymbol{X}$, we have the following proposition.

[2]The uncertainty region can be obtained, for example, from the previous time steps position knowledge or this moment local base station observations.

Proposition 2 *Given θ, the CRB $J^{-1}(\theta;\boldsymbol{X})$ is a strictly quasiconvex function of $\boldsymbol{X}$.*

For the strictly quasiconvex function, any local minimum must be the global minimum element [14]. In other words, for the optimization problem $\mathscr{Q}_1$, any local minimum must be a global minimum.

Third, for the non-convex feasible set $\mathcal{X}_N$, we relax it to its convex hull, *Fantope* [15]. Thus, $\mathscr{Q}_1$ is relaxed to

$$\mathscr{Q}_2: \quad \min_{\boldsymbol{X}} \quad t \qquad \text{s.t.} \quad J^{-1}(\vartheta_i;\boldsymbol{X}) \le t, \quad i=1,2,\ldots,S+1, \quad \boldsymbol{X}\in\mathcal{F}_N$$

where $\mathcal{F}_N := \{\boldsymbol{X}\in\mathbb{S}^M | \boldsymbol{0} \preceq \boldsymbol{X} \preceq \boldsymbol{I}_M, \mathrm{tr}\{\boldsymbol{X}\} = N\}$ and $\mathbb{S}^M$ is the set of $M\times M$ positive-semidefinite matrix. Here, $\mathscr{Q}_2$ is a quasiconvex optimization problem with a unique minimizer, and can be solved by reducing it to a series of convex feasibility problems [16].

Fourth, note that the optimal solution of $\mathscr{Q}_2$, denoted as $\boldsymbol{X}_S^{q_2}$, generally has rank$(\boldsymbol{X}_S^{q_2}) \ge N$ which does not meet the rank-N constraint. *Ky Fan's maximum principle* states that the nearest rank-N projection matrix for $\boldsymbol{X}_S^{q_2}$ is the one that is constructed from its N leading eigenvectors [17].

Moreover, one important problem after a sequence of relaxation steps is measuring the gaps among these optimal values of different optimization problems. For the relaxation from $\mathscr{P}_1$ to $\mathscr{Q}_1$, we have Proposition 3 as the performance guarantee, and for the relaxation from $\mathscr{Q}_1$ to $\mathscr{Q}_2$ and further to the spectral decomposition result, we have Proposition 4 as the performance guarantee.

Proposition 3 *The optimal value of $\mathscr{P}_1$ is bounded below and above, respectively, by*

$$f_\mathrm{q}(\boldsymbol{X}_S^{q_1}) \le f_\mathrm{p}(\boldsymbol{X}^*) \le f_\mathrm{p}(\boldsymbol{X}_S^{q_1})$$

where $\boldsymbol{X}^*$ and $\boldsymbol{X}_S^{q_1}$ are the optimal solution of $\mathscr{P}_1$ and $\mathscr{Q}_1$, respectively, and $f_\mathrm{p}(\boldsymbol{X}) = \max_{\theta\in\mathcal{T}_\mathrm{c}} J^{-1}(\theta;\boldsymbol{X})$ whereas $f_\mathrm{q}(\boldsymbol{X}) = \max_{\vartheta\in\mathcal{T}_\mathrm{d}} J^{-1}(\vartheta;\boldsymbol{X})$. In addition, $|f_\mathrm{p}(\boldsymbol{X}_S^{q_1}) - f_\mathrm{q}(\boldsymbol{X}_S^{q_1})|$ converges to zero log-linearly with increasing S.

Proposition 4 *The optimal value of $\mathscr{Q}_1$ is bounded below and above, respectively, by*

$$f_\mathrm{q}(\boldsymbol{X}_S^{q_2}) \le f_\mathrm{q}(\boldsymbol{X}_S^{q_1}) \le f_\mathrm{q}(\mathcal{P}_{\mathcal{X}_N}(\boldsymbol{X}_S^{q_2}))$$

where $\mathcal{P}_{\mathcal{X}_N}(\boldsymbol{X}_S^{q_2})$ is the nearest rank-N projection matrix for $\boldsymbol{X}_S^{q_2}$ according to the Ky Fan's maximum principle. Furthermore, with Proposition 3, the optimal value of $\mathscr{P}_1$ is bounded below and above, respectively, by

$$f_\mathrm{q}(\boldsymbol{X}_S^{q_2}) \le f_\mathrm{p}(\boldsymbol{X}^*) \le f_\mathrm{p}(\mathcal{P}_{\mathcal{X}_N}(\boldsymbol{X}_S^{q_2})).$$

Through solving $\mathscr{Q}_2$ by convex optimization and further performing spectral decomposition, the robust beamspace $\mathcal{P}_{\mathcal{X}_N}(\boldsymbol{X}_S^{q_2})$ with the worst-case performance guarantee is obtained. In addition, the robust beamspace matrix $\boldsymbol{B}$ is constructed by N leading eigenvectors of $\mathcal{P}_{\mathcal{X}_N}(\boldsymbol{X}_S^{q_2})$.

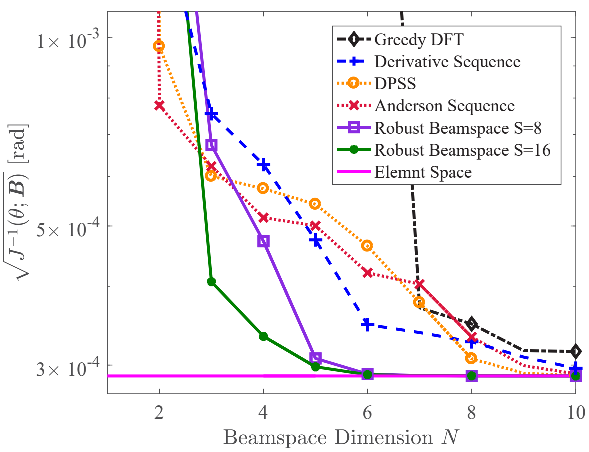

Fig. 2: Worst-case performance comparison among different beamspace schemes.

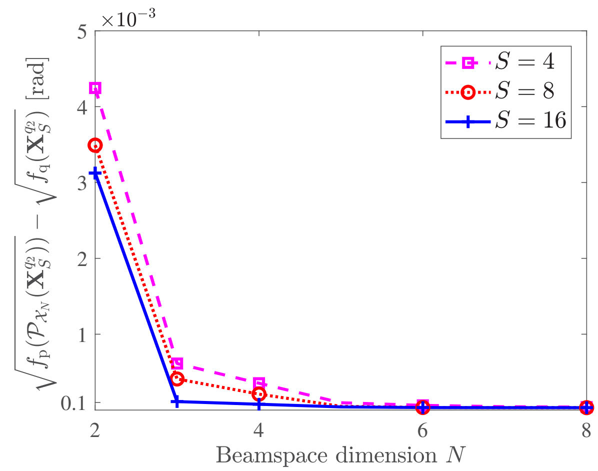

Fig. 4: The gap between $f_\mathrm{p}(\mathcal{P}_{\mathcal{X}_N}(\boldsymbol{X}_S^{q_2}))$ and $f_\mathrm{q}(\boldsymbol{X}_S^{q_2})$ changes along with the beamspace dimension N.

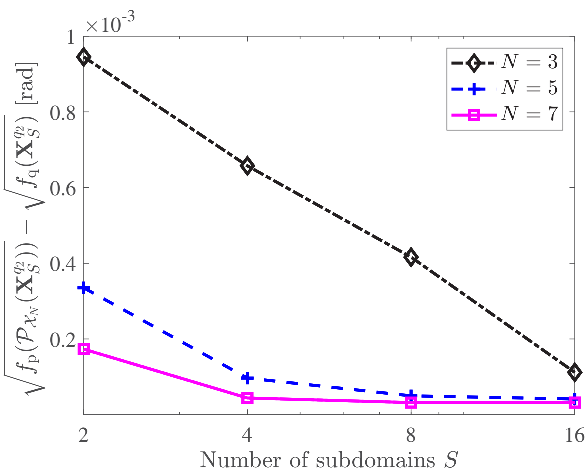

Fig. 3: The gap between $f_\mathrm{p}(\mathcal{P}_{\mathcal{X}_N}(\boldsymbol{X}_S^{q_2}))$ and $f_\mathrm{q}(\boldsymbol{X}_S^{q_2})$ changes along with the number of subdomains S.

4. NUMERICAL RESULTS

In this section, we will verify the effectiveness and robustness of the proposed beamspace design algorithm. The performance is evaluated by the worst-case CRB in the uncertainty region, and the parameter is settled as follows: the base station is equipped with a uniform linear array with $M = 128$ antennas, the SNR is 0 dB, the snapshot number is 5 and the sector-of-interest $\mathcal{T}_\mathrm{c}$ is $[27°, 33°]$.

We adopt various subdomain numbers $S = 2, 4, 8, 16$ to construct the robust beamspace and compare their performances with other existing works: the conventional DFT in [4], the DPSS in [8], the work by Anderson [9] and the derivative sequence in [13]. Fig. 2 shows the superiority of the proposed robust beamspace over the others. When the beamspace dimension N is equal to 3, the proposed robust beamspace has a performance improvement by more than 30%. This manifests the necessity of robust beamspace design to guarantee the localization performance under uncertainty. Moreover, when $N \geq 6$, it can nearly achieve the performance limit, the element space CRB. This means that there is no need to entirely transmit the 128-dimensional signal to the fusion center. Instead, a comparable localization accuracy can be obtained with little communication overhead.

Fig. 3 and Fig. 4 show that the robust beamspace design problem $\mathscr{P}_1$ can be near-optimally solved by the developed algorithm. Fig. 3 depicts that the performance gap, between the upper and lower bound of the robust problem $\mathscr{P}_1$, converges to zero with the increasing subdomain number S. Note that $\boldsymbol{X}_S^{q_1}$ is unavailable, the log-linear convergence rate of $|f_\mathrm{p}(\boldsymbol{X}_S^{q_1}) - f_\mathrm{q}(\boldsymbol{X}_S^{q_1})|$ on S cannot be numerical verified. Fig. 4 illustrates that the performance gap between the developed algorithm and the lower bound of original problem vanishes with the increasing beamspace dimension N. These results validate the effectiveness and robustness of our developed algorithm.

5. CONCLUSION

This paper proposed a robust beamspace design technique for direct localization system to reduce array signal dimension. In the presence of parameter uncertainty, we formulated the beamspace design problem as a robust optimization to guarantee the worst-case localization performance. Since the problem is non-convex, we proposed a convex relaxation and proved that the solution of the relaxed problem asymptotically converges to that of the original problem. Simulation results showed that the proposed robust beamspace scheme significantly outperforms other existing beamspace schemes. Our results can serve as guidelines for localization networks to achieve high-accuracy positioning with orders of magnitude lower signal dimension.

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