Adaptive Reduced-Dimensional Beamspace Beamformer Design by Analogue Beam Selection

Xiangrong Wang*, Elias Aboutanios†
*School of Electronic and Information Engineering, Beihang University, Beijing, China; xrwang@buaa.edu.cn
†School of Electrical Engineering, University of New South Wales, Sydney, Australia; elias@unsw.edu.au

***Abstract*—Adaptive beamforming of large antenna arrays is difficult to implement due to prohibitively high hardware cost and computational complexity. An antenna selection strategy was utilized to maximize the output signal-to-interference-plus-noise ratio (SINR) with fewer antennas by optimizing array configurations. However, antenna selection scheme exhibits high degradation in performance compared to the full array system. In this paper, we consider a reduced-dimensional beamspace beamformer, where analogue phase shifters adaptively synthesize a subset of orthogonal beams whose outputs are then processed in a beamspace beamformer. We examine the selection problem to adaptively identify the beams most relevant to achieving almost the full beamspace performance, especially in the generalized case without any prior information. Simulation results demonstrated that the beam selection enjoys the complexity advantages, while simultaneously enhancing the output SINR of antenna selection.**

***Index Terms*—beamspace beamformer, analogue phase shifter, adaptive beamforming, beam selection**

I. Introduction

Dimensionality reduction in array signal processing is critical in extensive applications such as radar, sonar, wireless communications, radio astronomy and satellite navigations, to list a few [1]–[6]. The main limitation of large arrays is typically not the number of employed sensors but the hardware cost associated with increased Radio-Frequency (RF) channels, which usually comprise expensive low-noise amplifiers, down converters, and AD/DA converters, and high computational complexity required for digital signal processing [7]–[10]. A promising approach of capturing a large aperture and satisfactory performance at a reduced hardware cost and complexity is to optimally select a subset of “best” antennas from the larger set of available antennas. This is referred to as the element-space adaptive digital beamformer. The performance of systems deploying antenna selection schemes has been shown to be significantly higher than that of systems using the same number of antennas without any guided selection [11]–[14]. However, antenna selection exhibits high degradation in performance compared to the full array system. For example, consider the case of single source and single interference, halving the number of antennas in the beamformer implies almost 3dB output SINR loss in the case of widely separated source and interference [11], [15].

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To compensate the performance degradation of antenna selection while simultaneously reducing complexity, we examine the design of reduced-dimensional beamspace beamformer that combines adaptive beam selection with beamspace processing. The beamspace processing was considered in airborne radar [17] for counteracting heterogeneous clutter, in colocated MIMO radar systems [18] for multiple target tracking and in phased-array weather radar [16] for interference rejection. A hybrid digital and analogue beamforming design was proposed in [19] to maximize spectral efficiency of large MIMO communication systems. A few beamspace transformation techniques which take out-of-sector interfering sources into account have been reported in the literature [20]–[22]. However, their applicability is limited to the case of precisely known interfering sources DOAs. Furthermore, these works assume a pre-designed uniform beam fan covering the interested angular region, which may not necessarily be an optimum reduced-dimensional beamspace design in terms of maximizing the output SINR. A new data-adaptive beamspace design technique was proposed in [23] to account for the case without any prior information, where a unitary beamspace transformation matrix was calculated by solving a relaxed non-convex optimization problem. In this work, we formulate the beamspace design into an output SINR maximization through beam selection and adopt a sequential convex programming algorithm to solve the NP-hard problem. The key idea of this technique is adaptively choosing a subset of optimum beams formed by analogue phase shifters to achieve almost full array performance. Importantly, we investigate beam selection from received data directly for MPDR beamformer when there is no prior information of interferences. We show that the utilization of beam selection possesses all of the advantages of antenna selection, while simultaneously overcoming the disadvantages of antenna selection in terms of performance degradation.

The rest of the paper is organized as follows: the mathematical model of the reduced-dimensional beamspace beamformer is given in section II. The optimum beamspace beamformer design by analogue beam selection is solved in section III. Simulation results are presented in section IV. Finally, conclusions are provided in section V.

II. Mathematical Formulation

Consider a uniform linear array (ULA) with N isotropic antennas placed at positions $nd, n = 0, \ldots, N-1$ with d denoting the unit inter-element spacing. Suppose that the desired source and Q interfering signals are impinging on the array from the direction ϕ_s and $\phi_{v,q}, q = 1, \ldots, Q$, respectively. The steering vectors of the source and interferences are,

$$\begin{aligned} \mathbf{s} &= [1, e^{jk_0 d\cos\phi_s}, \ldots, e^{jk_0(N-1)d\cos\phi_s}]^T, \\ \mathbf{v}_q &= [1, e^{jk_0 d\cos\phi_{v,q}}, \ldots, e^{jk_0(N-1)d\cos\phi_{v,q}}]^T, \end{aligned} \tag{1}$$

where $k_0 = 2\pi/\lambda$ is defined as the wavenumber with λ being the wavelength, and ^{T} denoting the transpose operation. The u-space electronic angle is defined as $u = \cos\phi \in [-1, 1]$. The received signal at time instant t is given by,

$$\mathbf{x}(t) = \mathbf{s}s(t) + \sum_{q=1}^{Q} \mathbf{v}_q v_q(t) + \mathbf{n}(t). \tag{2}$$

In the above equation, $s(t) \in \mathbb{C}$ and $v_q(t) \in \mathbb{C}$ are, respectively, the statistically independent source and interfering signals, and $\mathbf{n}(t) \in \mathbb{C}^N$ denotes the received white Gaussian noise vector with zero mean and covariance $\sigma_n^2\mathbf{I}$.

Denote the commonly-used $N \times N$ Butler matrix as $\mathbf{B}$, which can form a set of N orthogonal beams and thus transform the element-space processing to beam-space domain. The ith column of the matrix $\mathbf{B}$ is defined as,

$$\mathbf{b}_i = \frac{1}{\sqrt{N}}[1, e^{jdk_0u_i}, \ldots, e^{j(N-1)dk_0u_i}]^T, i = 1, \ldots, N$$

which is the steering vector pointing towards direction u_i in u-space and $\{u_i, i = 1, \ldots, N\}$ is a set of uniformly spaced grid points with an interval of $2/N$. Clearly, $\mathbf{B}$ is a unitary matrix, that is $\mathbf{B}^H\mathbf{B} = \mathbf{B}\mathbf{B}^H = \mathbf{I}$. The steering vectors of the source and interferences in beamspace domain can then be expressed as,

$$\tilde{\mathbf{s}} = \mathbf{B}^H\mathbf{s}, \quad \tilde{\mathbf{v}}_q = \mathbf{B}^H\mathbf{v}_q, q = 1, \ldots, N.$$

Assume that the phase shifters can continuously adjust their phases such that we can set the first grid point to coincide with the source direction, that is $u_1 = \cos\phi_s$, and consequently the other $N-1$ grid points can be uniformly spaced within the range $[-1 : u_s - 2/N]$ and $[u_s + 2/N : 1]$ with an interval of $2/N$. According to the orthogonality property among the N beams, we thus have that $\tilde{\mathbf{s}} = [1, 0, \ldots, 0]^T$.

The normalized interference plus noise covariance matrix in beamspace domain becomes,

$$\tilde{\mathbf{R}}_n = \frac{1}{\sigma_n^2}\mathbf{B}^H\mathbf{V}\mathbf{R}_j\mathbf{V}^H\mathbf{B} + \mathbf{I} = \frac{1}{\sigma_n^2}\tilde{\mathbf{V}}\mathbf{R}_j\tilde{\mathbf{V}}^H + \mathbf{I}, \tag{3}$$

where $\mathbf{R}_j = E\{\mathbf{v}(t)\mathbf{v}^H(t)\}$ denotes the interference cross-correlation matrix with $\mathbf{v}(t) = [v_1(t), \ldots, v_Q(t)]^T$ and array manifold matrix $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_Q]$. Additionally, $\tilde{\mathbf{V}} = \mathbf{B}^H\mathbf{V}$. The white noise property is preserved under the condition of $\mathbf{B}^H\mathbf{B} = \mathbf{I}$. The beamspace minimum variance distortionless response (MVDR) beamformer is then given by [24],

$$\mathbf{w}_{\text{MVDR}} = \eta\tilde{\mathbf{R}}_n^{-1}\tilde{\mathbf{s}} = \frac{1}{\tilde{\mathbf{s}}^H\tilde{\mathbf{R}}_n^{-1}\tilde{\mathbf{s}}}\tilde{\mathbf{R}}_n^{-1}\tilde{\mathbf{s}}. \tag{4}$$

The array gain of the beamspace MVDR beamformer is then defined as the ratio between the output SINR and the input SNR, and given by [25]

$$\mathrm{G} = \tilde{\mathbf{s}}^{\mathrm{H}}\tilde{\mathbf{R}}_{\mathrm{n}}^{-1}\tilde{\mathbf{s}}. \tag{5}$$

Utilizing the matrix inversion lemma, we obtain the inverse beamspace interference-plus-noise covariance matrix as,

$$\tilde{\mathbf{R}}_n^{-1} = \mathbf{I} - \mathbf{B}^H\mathbf{V}(\mathbf{C}_j + \mathbf{V}^H\mathbf{V})^{-1}\mathbf{V}^H\mathbf{B}, \tag{6}$$

where $\mathbf{C}_j = \sigma_n^2\mathbf{R}_j^{-1}$. Substituting Eq. (6) into Eq. (5) yields,

$$\mathrm{G} = \tilde{\mathbf{s}}^{\mathrm{H}}\tilde{\mathbf{s}} - \tilde{\mathbf{s}}^{\mathrm{H}}\tilde{\mathbf{V}}(\mathbf{C}_{\mathrm{j}} + \tilde{\mathbf{V}}^{\mathrm{H}}\tilde{\mathbf{V}})^{-1}\tilde{\mathbf{V}}^{\mathrm{H}}\tilde{\mathbf{s}}. \tag{7}$$

III. Reduced-Dimensional Beamspace Beamformer Design by Beam Selection

Since the directional angles of incoming signals exhibit a sparse property in beamspace, the number of orthogonal beams required for transformation is much smaller than that of physical antennas. A subset $\mathcal{S}$ of K beams ($K < N$) is selected for beamspace transformation and referred to as a beam fan [25]. Clearly, the beamspace MVDR beamformer reduces the signal processing dimension from N to K, so that the computational complexity involved in the covariance inversion is dramatically alleviated from $O(N^3)$ to $O(K^3)$. Moreover, the number of snapshots required for a sufficiently accurate estimate of the covariance matrix $\tilde{\mathbf{R}}_n$ is also decreased. Given that the beam pointing towards the source must be selected, and thus the array gain in Eq. (7) is rewritten as

$$G = 1 - \tilde{\mathbf{s}}^H\tilde{\mathbf{V}}(\mathbf{C}_j + \tilde{\mathbf{V}}^H\tilde{\mathbf{V}})^{-1}\tilde{\mathbf{V}}^H\tilde{\mathbf{s}}. \tag{8}$$

We can observe from Eq. (8) that the array gain G is determined by the first entry of the matrix $\tilde{\mathbf{V}}(\mathbf{C}_j + \tilde{\mathbf{V}}^H\tilde{\mathbf{V}})^{-1}\tilde{\mathbf{V}}^H$, and clearly the subset $\mathcal{S}$ of selected beams affects the array gain. The design of optimum subset $\mathcal{S}$ by beam selection is delineated in the following two different cases.

A. Beam Selection with Prior Information of Interferences

Assume that we have some prior information of interfering signals, such as the number Q of interferences and their corresponding arrival angles. In such cases, an optimum subset of $K = Q + 1$ orthogonal beams is selected for beamspace transformation. Denote a beam selection vector $\mathbf{z} = [z_i, i = 1, \ldots, N] \in \{0, 1\}^N$ with "zero" entry for a discarded beam and "one" entry for a selected one. The diagonal matrix $\mathrm{D}(\mathbf{z})$ is the beam selection operator with the vector $\mathbf{z}$ populating along the diagonal. Proceeding from Eq. (7), the array gain of selected beamspace MVDR beamformer can be written as,

$$G(\mathbf{z}) = \tilde{\mathbf{s}}^H\mathrm{D}(\mathbf{z})\tilde{\mathbf{s}} - \tilde{\mathbf{s}}^H\mathrm{D}(\mathbf{z})\tilde{\mathbf{V}}(\mathbf{C}_j + \tilde{\mathbf{V}}^H\mathrm{D}(\mathbf{z})\tilde{\mathbf{V}})^{-1}\tilde{\mathbf{V}}^H\mathrm{D}(\mathbf{z})\tilde{\mathbf{s}},$$

As the first beam pointing towards the source must be selected, we need to select $K-1$ beams from the remaining $N-1$ candidates. Utilizing the Schur complement condition for positive semi-definiteness, the problem of optimum beamspace design by beam selection can be expressed in terms of linear matrix inequality (LMI). That is,

$$\begin{aligned} \max_{\mathbf{z},\gamma} \quad & \gamma, & (9)\\ \text{s.t.} \quad & \begin{bmatrix} \mathbf{C}_j + \tilde{\mathbf{V}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{V}} & \tilde{\mathbf{V}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{s}} \\ \tilde{\mathbf{s}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{V}} & \tilde{\mathbf{s}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{s}} - \gamma \end{bmatrix} \geq 0, \\ & \mathbf{1}^T \mathbf{z} = K-1, \quad \mathbf{z} \in \{0,1\}^{N-1}. \end{aligned}$$

To circumvent the boolean constraint of the selection vector $\mathbf{z}$ as per the third constraint in Eq. (9), we express $\mathbf{z} \in \{0,1\}^{N-1}$ as the difference of two convex sets, that is, $\mathbf{z}^T(\mathbf{z}-1)=0$, or equivalently, $\max_{\mathbf{z}} \ \mathbf{z}^T(\mathbf{z}-\mathbf{1})$ s.t. $0 \leq \mathbf{z} \leq 1$. As such, the objective function in Eq. (9) can be changed to $\gamma + \mu \mathbf{z}^T(\mathbf{z}-\mathbf{1})$ with the relaxed box constraints as follows,

$$\begin{aligned} \max_{\mathbf{z},\gamma} \quad & \gamma + \mu \mathbf{z}^T(\mathbf{z}-\mathbf{1}), & (10)\\ \text{s.t.} \quad & \begin{bmatrix} \mathbf{C}_j + \tilde{\mathbf{V}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{V}} & \tilde{\mathbf{V}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{s}} \\ \tilde{\mathbf{s}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{V}} & \tilde{\mathbf{s}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{s}} - \gamma \end{bmatrix} \geq 0, \\ & \mathbf{1}^T \mathbf{z} = K-1, \quad 0 \leq \mathbf{z} \leq 1, \end{aligned}$$

where μ is a trade-off parameter that controls the relative importance between the beam number and boolean property of the selection vector $\mathbf{z}$. The objective function in Eq. (10) becomes non-convex. A sequential convex programming (SCP) based on iteratively linearizing the second convex function of the objective is then utilized to reformualte the non-convex problem to a series of convex subproblems, each of which can be optimally solved using convex programming [26], [27]. The beam selection in the (k+1)th iteration can be formulated based on the solution $\mathbf{z}^{(k)}$ from the kth iteration as,

$$\begin{aligned} \max_{\mathbf{z},\gamma} \quad & \gamma + \mu[(2\mathbf{z}^{(k)}-\mathbf{1})^T\mathbf{z} - \mathbf{z}^{(k)T}\mathbf{z}^{(k)}], & (11)\\ \text{s.t.} \quad & \begin{bmatrix} \mathbf{C}_j + \tilde{\mathbf{V}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{V}} & \tilde{\mathbf{V}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{s}} \\ \tilde{\mathbf{s}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{V}} & \tilde{\mathbf{s}}^H \mathrm{D}(\mathbf{z})\tilde{\mathbf{s}} - \gamma \end{bmatrix} \geq 0, \\ & \mathbf{1}^T \mathbf{z} = K-1, \quad 0 \leq \mathbf{z} \leq 1. \end{aligned}$$

Note that the SCP is a local heuristic and its performance depends on the initial search point $\mathbf{z}^{(0)}$. It is, therefore, typical to initialize the algorithm with several feasible points and find the one with the maximum objective value over different runs.

B. Generalized Beam Selection without Prior Information

Generally, the receiver has no prior information of interferences. We can either conduct estimation before beam selection as per section III-A or perform the optimum beam selection based on the received data directly. When there are T snapshots of received data, $\mathbf{x}(t), t=1,\ldots,T$, the maximum likelihood estimate of the data covariance matrix is,

$$\mathbf{R}_x = \sigma_s^2 \mathbf{v}_s \mathbf{v}_s^H + \mathbf{V}\mathbf{R}_j\mathbf{V}^H + \sigma_n^2 \mathbf{I} \approx \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}(t)\mathbf{x}^H(t), \quad (12)$$

where $\sigma_s^2 = E\{|s(t)|^2\}$. The covariance matrix of the received data in the beamspace domain is expressed as,

$$\begin{aligned} \tilde{\mathbf{R}}_x &= \mathbf{B}^H(\sigma_s^2 \mathbf{v}_s \mathbf{v}_s^H + \mathbf{V}\mathbf{R}_j\mathbf{V}^H + \sigma_n^2 \mathbf{I})\mathbf{B}, & (13)\\ &= \sigma_s^2 \mathbf{E}_1 + \sigma_n^2 \tilde{\mathbf{R}}_n, \end{aligned}$$

where the first entry of $\mathbf{E}_1 \in \{0,1\}^{K\times K}$ is one and others zero, as $\mathbf{B}$ contains N orthonormal beams.

The minimum power distortionless response (MPDR) beamformer, that is $\mathbf{w}_{\mathrm{MPDR}} = \eta \tilde{\mathbf{R}}_x^{-1}\tilde{\mathbf{s}}$, is employed in this case. Utilizing Eq. (13), the array gain can be expressed as,

$$\begin{aligned} \mathrm{G} &= \frac{\mathbf{w}_{\mathrm{MPDR}}^H \tilde{\mathbf{s}}\tilde{\mathbf{s}}^H \mathbf{w}_{\mathrm{MPDR}}}{\mathbf{w}_{\mathrm{MPDR}}^H \tilde{\mathbf{R}}_n \mathbf{w}_{\mathrm{MPDR}}}, & (14)\\ &= \frac{\sigma_n^2(\tilde{\mathbf{s}}^H \tilde{\mathbf{R}}_x^{-1}\tilde{\mathbf{s}})^2}{\tilde{\mathbf{s}}^H \tilde{\mathbf{R}}_x^{-1}[\tilde{\mathbf{R}}_x - \sigma_s^2 \mathbf{E}_1]\tilde{\mathbf{R}}_x^{-1}\tilde{\mathbf{s}}}, \\ &= \frac{\sigma_n^2}{1/(\tilde{\mathbf{s}}^H \tilde{\mathbf{R}}_x^{-1}\tilde{\mathbf{s}}) - \sigma_s^2}. \end{aligned}$$

We can observe from Eq. (14) that maximizing the array gain is equivalent to maximizing the value of $\tilde{\mathbf{s}}^H \tilde{\mathbf{R}}_x^{-1}\tilde{\mathbf{s}}$, which is the first entry of the inverse data covariance matrix $\tilde{\mathbf{R}}_x^{-1} = [\mathbf{B}^H \mathbf{R}_x \mathbf{B}]^{-1}$. According to the special structure of the matrix $\tilde{\mathbf{R}}_x$ as shown in Eq. (13), we further have that

$$\begin{aligned} \tilde{\mathbf{s}}^H \tilde{\mathbf{R}}_x^{-1}\tilde{\mathbf{s}} &= \frac{|\sigma_n^2 \tilde{\mathbf{R}}_n|}{\sigma_n^2 |\tilde{\mathbf{R}}_x|}\tilde{\mathbf{s}}^H \tilde{\mathbf{R}}_n^{-1}\tilde{\mathbf{s}}, & (15)\\ &= \frac{|\tilde{\mathbf{R}}_n|}{\sigma_n^2 |(\sigma_s^2/\sigma_n^2)\mathbf{E}_1 + \tilde{\mathbf{R}}_n|}\tilde{\mathbf{s}}^H \tilde{\mathbf{R}}_n^{-1}\tilde{\mathbf{s}}, \\ &= \frac{1}{\sigma_n^2/(\tilde{\mathbf{s}}^T \tilde{\mathbf{R}}_n^{-1}\tilde{\mathbf{s}}) + \sigma_s^2}. \end{aligned}$$

Proceeding to substitute Eq. (15) into Eq. (14) yields $\mathrm{G} = \tilde{\mathbf{s}}^{\mathrm{T}}\tilde{\mathbf{R}}_{\mathrm{n}}^{-1}\tilde{\mathbf{s}}$. Therefore, the array gain of the beamspace MPDR beamformer remains the same as that of beamspace MVDR beamformer when a subset of orthogonal beams including the one pointing towards the source is selected.

To solve the beam selection using the received data directly, implementing eigenvalue decomposition to the received data covariance matrix $\mathbf{R}_x$ yields,

$$\mathbf{R}_x = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H \approx \bar{\mathbf{U}}\bar{\mathbf{\Lambda}}\bar{\mathbf{U}}^H + \bar{\sigma}_n^2 \mathbf{I}, \quad (16)$$

where $\mathbf{\Lambda}$ is a diagonal matrix with N ordered eigenvalues $\lambda_1 \geq \ldots \geq \lambda_N$ populating along the diagonal and $\mathbf{U} = [\mathbf{u}_1, \ldots, \mathbf{u}_N]$ is the corresponding eigenvector matrix. Assume that the first K eigenvalues are significantly larger than the remaining ones, the noise power is then estimated by

$$\bar{\sigma}_n^2 = \frac{1}{N-K}\sum_{k=K+1}^{N} \lambda_k. \quad (17)$$

Additionally, the first K eigenvalues are redefined as $\bar{\lambda}_k = \lambda_k - \bar{\sigma}_n^2, k=1,\ldots,K$ and $\bar{\mathbf{U}} = [\mathbf{u}_1, \ldots, \mathbf{u}_K]$, which yields the approximation of right-hand side in Eq. (16). The optimum beam selection without prior information of interferences can then be formulated as,

$$\begin{aligned} \max_{\mathbf{z},\gamma} \quad & \gamma, \\ \text{s.t.} \quad & \begin{bmatrix} \tilde{\boldsymbol{\Lambda}} + \tilde{\mathbf{U}}^H \mathrm{D}(\mathbf{z}) \tilde{\mathbf{U}} & \tilde{\mathbf{U}}^H \mathrm{D}(\mathbf{z}) \tilde{\mathbf{s}} \\ \tilde{\mathbf{s}}^H \mathrm{D}(\mathbf{z}) \tilde{\mathbf{U}} & \tilde{\mathbf{s}}^H \mathrm{D}(\mathbf{z}) \tilde{\mathbf{s}} - \gamma \end{bmatrix} \succeq 0, \\ & \mathbf{1}^T \mathbf{z} = K - 1, \quad \mathbf{z} \in \{0,1\}^{N-1}, \end{aligned} \tag{18}$$

where $\tilde{\mathbf{U}} = \mathbf{B}^H \bar{\mathbf{U}}$ and $\tilde{\boldsymbol{\Lambda}} = \sigma_n^2 \bar{\boldsymbol{\Lambda}}^{-1}$. The SCP can then be exploited to solve the beam selection problem in this case.

IV. Simulations

In this section, simulation results are presented to validate the proposed beamspace beamformer design.

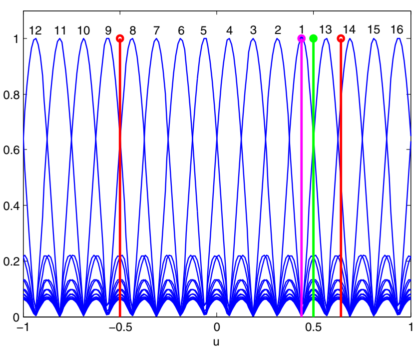

Fig. 1: The set of 16 orthogonal beams, where the source is indicated with filled pink/green circle. The interferences are indicated with hollow red circles.

Consider a ULA comprising 16 antennas, each connected with one analog phase shifter which is capable of continuously adjusting its phase within the range $[-\pi, \pi]$. The desired source is assumed to come from $\phi_s = 64°$, as indicated by the pink line in Fig. 1. A set of 16 orthogonal beams is formed to cover the full angular space with one beam pointing exactly towards the source. There are two interferences arriving from $\phi_{v,1} = 50°, \phi_{v,2} = 120°$ with INR being 10dB and 30dB, respectively. The number K of selected beams is changing from 1 to 16 and the optimum subset of beams is obtained for each number. We implement beam selection for both beamspace MVDR beamformer with the assumption of prior information and beamspace MPDR beamformer from the received data directly. The number of snapshots is changing among $T = 1000, 10000$ and 100000. The array gain corresponding to the optimum subset of K beams is plotted in Figs. 2 versus different numbers K. We can see that three beams including the one pointing towards the source are sufficient to guarantee an array gain equivalent to the full system. Furthermore, the performance of the MPDR beamformer approaches that of the MVDR beamformer with an increasing number of snapshots. The optimum subset of K selected beams are listed in Table. I. Comparing Table. I with Fig. 1, we arrive at an intuitive conclusion that when the source arrival angle aligns exactly with one beam, all other

TABLE I: The indices of K selected beams.

1/1							
1/1	8/13						
1/1	8/13	14/14					
1/1	8/2	13/13	14/14				
1/1	8/2	9/13	13/14	14/15			
1/1	8/2	9/3	13/13	14/14	15/15		
1/1	7/2	8/3	9/13	13/14	14/15	15/16	
1/1	2/2	7/3	8/4	9/13	13/14	14/15	15/16

The black numbers correspond to continuous beam selection, while red numbers for discrete beam selection. $K = 1 \sim 8$.

$K-1$ beams are selected around the interferences for enhanced suppression in beamspace domain.

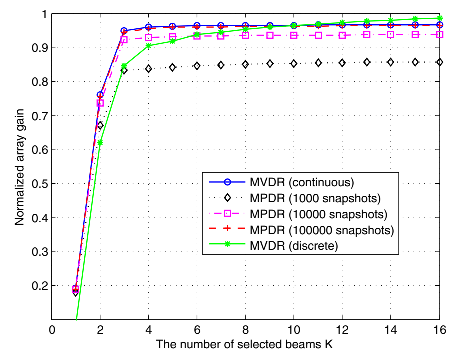

Fig. 2: Array gain versus the number of selected beams for both beamspace MVDR and MPDR beamformers.

Finally, we investigate the case of discrete beam selection. That is, the phase shifters are not allowed to adjust their phases arbitrarily, while constrained to a set of fixed values instead. Assume that the set of fixed orthogonal beams that can be synthesized remains the same as Fig. 1. The arrival angle of the source is changed to 60°, as indicated by the green line in Fig. 1, and no beams can point towards the source exactly. The array gain versus different numbers K is plotted in Fig. 2 and the optimum subset of selected beams are listed in Table I. We observe that three beams are not sufficient to prevent performance loss in this case. When the source arrival angle deviates from the beam positions, most of the K beams around the source should be selected for source amplification while very few beams are allocated to interference estimation.

V. Conclusions

In this paper, we investigated the beam selection problem for reduced-dimensional beamspace beamformer design. A subset of orthogonal beams formed by the analogue phase shifters was adaptively selected for large array dimensionality reduction and beamspace processing. The number of beams required to achieve optimum SINR as full beamspace processing, was as few as the number of signals in the array FoV, thus reducing the hardware cost and computational complexity significantly. The answers to optimum beam selection carried an interesting interpretation and insight, that is the interplay between source amplification and interference suppression.

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