Target Localization and Mutual Information Improvement for Cooperative MIMO Radar and MIMO Communication Systems

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***Abstract*—In this work, we study coexisting MIMO radar and MIMO communication systems, where the two systems work cooperatively. The radar shares its antenna positions and transmitted signals with the communication system. The communication system informs the radar about the antenna locations, as well as the statistics of the communication signals. Previous work has presented a performance gain for both the radar and communication systems in terms of target localization performance and mutual information. With the removal of the assumption about completely decoded communication signals at the radar receiver, this paper analyzes the performance metrics and shows that there is still a significant gain obtained through cooperation.**

***Index Terms*—Radar and communication, MIMO, mutual information (MI), Cramer-Rao bound (CRB).**

I. Introduction

With the development of new technologies, radar-communication integration becomes an inevitable trend in both civil and military fields [1], such as, intelligent transportation vehicles and unmanned combat aerial vehicles. Research on coexisting radar and communication systems as an important task of integration is of considerable interest [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12]. In [5], [6], the mutual interference between the two systems is reduced by the design of the transmitted waveform. In [7], [8], [9], the interference is eliminated by methods such as successive interference cancellation. However, both systems could benefit from each other. It has been shown that the communication signals can be exploited to improve the target detection [3] and delay estimation [4] performance of the radar with a single antenna.

We consider cooperative MIMO radar and MIMO communication systems, all with widely spaced transmit and receive stations [13]. Assume that the antenna positions of both systems are known to both systems. The radar signals are known to the two systems. The useful information about the communication signals is shared with the radar system. Thus, the radar system could achieve a better performance by using the target returns from the communication system in target localization, forming a hybrid active-passive MIMO radar network. Similarly, using shared radar knowledge, the communication system can estimate the unknown target parameters, and the communication signals reflected from the radar target can also be used to extract useful communication information.

For coexisting radar and communication systems, the radar system might employ one of two approaches for dealing with the communication signals. One is that the correctly decoded communication signals can be obtained at the radar system, and the other is that the statistics of the communication signals can be shared with the radar system [12]. In [13], we have shown that when the MIMO radar receivers can decode the MIMO communication signals, the radar can use the communication signals to improve the performance of parameter estimation. In this work, we discuss cooperative MIMO radar and MIMO communication systems under the second assumption (the communication system informs the radar about the statistics of the communication signals). It is shown that by cooperation, the radar target localization performance CRB can be reduced, and the communication MI can be increased, so there is still gain for the cooperative systems.

II. Hybrid Active-passive MIMO Radar

Consider a coexisting MIMO radar and MIMO communication systems consisting of M_C communication transmitters, N_C communication receivers, M_R radar transmitters, and N_R radar receivers, all widely spaced. The signal transmitted from the m'th ($m' = 1, ..., M_C$) communication transmitter and mth ($m = 1, ..., M_R$) radar transmitter are $\sqrt{E_{C,m'}} s_{C,m'}(kT_s)$ and $\sqrt{E_{R,m}} s_{R,m}(kT_s)$, respectively, where $E_{C,m'}$ and $E_{R,m}$ denote the transmit power, T_s the sampling period, and k ($k = 1, ..., K$) an index running over the different time samples. There is a target present at unknown position $[x, y]$. The signal received at the nth ($n = 1, ..., N_R$) radar receiver at time instant kT_s is modeled as follows

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$$
\begin{aligned}
r_{R,n}[k] =& \sum_{m=1}^{M_R} \sqrt{E_{R,m}}\zeta_{Rt,nm} s_{R,m}(kT_s - \tau_{Rt,nm}) \\
&+ \sum_{m=1}^{M_R} \sqrt{E_{R,m}} s_{R,m}(kT_s - \tau_{R,nm}) \\
&+ \sum_{m'=1}^{M_C} \sqrt{E_{C,m'}}\zeta_{Ct,nm'} s_{C,m'}(kT_s - \tau_{Ct,nm'}) \\
&+ \sum_{m'=1}^{M_C} \sqrt{E_{C,m'}} s_{C,m'}(kT_s - \tau_{C,nm'}) + w_{R,n}[k] \qquad (1)
\end{aligned}
$$

in which the first four terms correspond to the target reflected path and direct path signals contributed by the radar transmissions, and the target reflected path and direct path signals contributed by the communication transmissions, respectively. The $\tau_{Rt,nm}$, $\tau_{R,nm}$, $\tau_{Ct,nm'}$, and $\tau_{C,nm'}$ denote the corresponding time delays, $\zeta_{Rt,nm}$ and $\zeta_{Ct,nm'}$ denote the corresponding target reflection coefficients (assumed known possibly via preprocessing), and $w_{R,n}[k]$ represents the clutter-plus-noise. The overall radar received signal vector can be written as [13]

$$
\begin{aligned}
\mathbf{r}_R &= (r_{R,1}[1], r_{R,1}[2], \ldots, r_{R,N_R}[K])^\dagger \\
&= \mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_R\mathbf{s}_R + \mathbf{U}_{Ct}\mathbf{s}_{Ct} + \mathbf{U}_C\mathbf{s}_C + \mathbf{w}_R, \qquad (2)
\end{aligned}
$$

where "†" means transpose,

$$
\mathbf{U}_{Rt} = Diag\{\mathbf{u}_{Rt,1}^\dagger[1], \mathbf{u}_{Rt,1}^\dagger[2], \ldots, \mathbf{u}_{Rt,N_R}^\dagger[K]\}, \qquad (3)
$$

the operator $Diag\{\cdot\}$ denotes block diagonal, $\mathbf{u}_{Rt,n}[k] = (u_{Rt,n1}[k], \ldots, u_{Rt,nM_R}[k])^\dagger$, $u_{Rt,nm}[k] = \zeta_{Rt,nm}\sqrt{E_{R,m}}$, and

$$
\mathbf{s}_{Rt} = (\mathbf{s}_{Rt,1}[1]^\dagger, \mathbf{s}_{Rt,1}[2]^\dagger, \ldots, \mathbf{s}_{Rt,N_R}[K]^\dagger)^\dagger, \qquad (4)
$$

in which $\mathbf{s}_{Rt,n}[k] = [s_{R,1}(kT_s - \tau_{Rt,n1}), \ldots, s_{R,M_R}(kT_s - \tau_{Rt,nM_R})]^\dagger$. The terms $\mathbf{U}_R$, $\mathbf{s}_R$, $\mathbf{U}_{Ct}$, $\mathbf{s}_{Ct}$, $\mathbf{U}_C$, $\mathbf{s}_C$ in (2) are defined similarly from (1). The clutter-plus-noise vector $\mathbf{w}_R = [\mathbf{w}_{R,1}{}^\dagger, \ldots, \mathbf{w}_{R,N_R}{}^\dagger]^\dagger$ and $\mathbf{w}_{R,n} = (\mathbf{w}_{R,n}[1], \ldots, \mathbf{w}_{R,n}[K])^\dagger$, where $\mathbf{w}_R$ is assumed Gaussian distributed with zero mean and covariance matrix $\mathbf{Q}_R$.

A. CRB for Target Localization

Suppose the task of the radar system is to locate the target in a two dimensional space, so the parameters to be estimated are defined as $\boldsymbol{\theta} = [x, y]^\dagger$. Assume the communication signals $s_{C,m'}(kT_s)$ cannot be completely decoded at the radar receiver. Thanks to the cooperation, the statistics of the communication signal vector denoted by

$$
\mathbf{s} = [s_{C,1}(T_s), s_{C,2}(T_s), \ldots, s_{C,M_C}(KT_s)]^\dagger, \qquad (5)
$$

are shared from the communication system with the radar. Based on the radar received signal model in (2), the likelihood function conditioned on the given communication signal vector $\mathbf{s}$ can be computed

$$
\begin{aligned}
p(\mathbf{r}_R|\boldsymbol{\theta}, \mathbf{s}) =& \frac{1}{\pi^{KN_R}\det(\mathbf{Q_R})} \qquad (6) \\
&\times \exp\Big\{-(\mathbf{r}_R - \mathbf{U}_R\mathbf{s}_R - \mathbf{U}_{Rt}\mathbf{s}_{Rt} - \mathbf{U}_C\mathbf{s}_C - \mathbf{U}_{Ct}\mathbf{s}_{Ct})^H \\
&\times \mathbf{Q}_R^{-1}(\mathbf{r}_R - \mathbf{U}_R\mathbf{s}_R - \mathbf{U}_{Rt}\mathbf{s}_{Rt} - \mathbf{U}_C\mathbf{s}_C - \mathbf{U}_{Ct}\mathbf{s}_{Ct})\Big\},
\end{aligned}
$$

where $\det(\cdot)$ is the determinant operator. Therefore, the log-likelihood function conditioned on $\mathbf{s}$ is

$$
\begin{aligned}
\ln p(\mathbf{r}_R|\boldsymbol{\theta}, \mathbf{s}) \propto& -(\mathbf{r}_R - \mathbf{U}_R\mathbf{s}_R - \mathbf{U}_{Rt}\mathbf{s}_{Rt} - \mathbf{U}_C\mathbf{s}_C - \mathbf{U}_{Ct}\mathbf{s}_{Ct})^H\mathbf{Q}_R^{-1} \\
&\times (\mathbf{r}_R - \mathbf{U}_R\mathbf{s}_R - \mathbf{U}_{Rt}\mathbf{s}_{Rt} - \mathbf{U}_C\mathbf{s}_C - \mathbf{U}_{Ct}\mathbf{s}_{Ct})
\end{aligned}
$$

and the maximum-likelihood (ML) estimate of the target location $\boldsymbol{\theta}$ can be obtained

$$
\hat{\boldsymbol{\theta}}_{R,ML}|\mathbf{s} = \arg\max_{\boldsymbol{\theta}} \ln p(\mathbf{r}_R|\boldsymbol{\theta}, \mathbf{s}). \qquad (7)
$$

Next, we compute the CRB to characterize the best achievable estimation performance. Define an intermediate parameter vector $\boldsymbol{\psi} = [\tau_{Rt,11}, \ldots, \tau_{Rt,N_R M_R}, \tau_{Ct,11}, \ldots, \tau_{Rt,N_R M_C}]^\dagger$. The CRB conditioned on $\mathbf{s}$ for the estimate of $\boldsymbol{\theta}$ is [14]

$$
\mathbf{CRB}(\boldsymbol{\theta}|\mathbf{s}) = \left\{(\nabla_{\boldsymbol{\theta}}\boldsymbol{\psi}^\dagger)\mathbf{J}(\boldsymbol{\psi})(\nabla_{\boldsymbol{\theta}}\boldsymbol{\psi}^\dagger)^\dagger\right\}^{-1}, \qquad (8)
$$

where $\nabla_{\boldsymbol{\theta}}$ is the gradient operator, $\nabla_{\boldsymbol{\theta}}\boldsymbol{\psi}^\dagger = [\mathbf{D} \quad \mathbf{F}]$,

$$
\mathbf{D} = \begin{bmatrix} \frac{\partial\tau_{Rt,11}}{\partial x} & \cdots & \frac{\partial\tau_{Rt,N_R M_R}}{\partial x} \\ \frac{\partial\tau_{Rt,11}}{\partial y} & \cdots & \frac{\partial\tau_{Rt,N_R M_R}}{\partial y} \end{bmatrix},
$$

$$
\mathbf{F} = \begin{bmatrix} \frac{\partial\tau_{Ct,11}}{\partial x} & \cdots & \frac{\partial\tau_{Ct,N_R M_C}}{\partial x} \\ \frac{\partial\tau_{Ct,11}}{\partial y} & \cdots & \frac{\partial\tau_{Ct,N_R M_C}}{\partial y} \end{bmatrix},
$$

$$
[\mathbf{J}(\boldsymbol{\psi})]_{ij} = 2\Re\left\{\frac{\partial(\mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_{Ct}\mathbf{s}_{Ct})^H}{\partial\psi_i}\mathbf{Q}_R^{-1}\frac{\partial(\mathbf{U}_{Rt}\mathbf{s}_{Rt} + \mathbf{U}_{Ct}\mathbf{s}_{Ct})}{\partial\psi_j}\right\}
$$

for $i, j = 1, ..., N_R(M_R + M_C)$, and $\Re\{\cdot\}$ represents the real part.

Using the known statistics about the communication signal vector, one can take the expectation of the conditional $\mathbf{CRB}(\boldsymbol{\theta}|\mathbf{s})$ with respect to $\mathbf{s}$ to evaluate the average estimation performance, that is

$$
\mathbf{ECRB}(\boldsymbol{\theta}) = \mathbb{E}_{\mathbf{s}}\{\mathbf{CRB}(\boldsymbol{\theta}|\mathbf{s})\}, \qquad (9)
$$

which is the so-called expected CRB (ECRB) [15], [16] and $\mathbb{E}_{\mathbf{s}}\{\cdot\}$ is mathematical expectation operator.

III. MIMO Communication with Reflected Path

Under cooperation, the communication system can extract information not merely from the directly received communication signals, but also the signals reflected from the radar target. The signal received at the n'th communication receiver at time instant kT_s is given by

$$
\begin{aligned}
r_{C,n'}[k] =& \sum_{m'=1}^{M_C} \sqrt{E_{C,m'}}\zeta_{Ct,n'm'} s_{C,m'}(kT_s - \tilde{\tau}_{Ct,n'm'}) \\
&+ \sum_{m'=1}^{M_C} \sqrt{E_{C,m'}} s_{C,m'}(kT_s - \tilde{\tau}_{C,n'm'}) \\
&+ \sum_{m=1}^{M_R} \sqrt{E_{R,m}}\zeta_{Rt,n'm} s_{R,m}(kT_s - \tilde{\tau}_{Rt,n'm}) \\
&+ \sum_{m=1}^{M_R} \sqrt{E_{R,m}} s_{R,m}(kT_s - \tilde{\tau}_{R,n'm}) + w_{C,n'}[k] \qquad (10)
\end{aligned}
$$

in which the first four terms correspond to the target reflected path and direct path communication signals, and the target reflected path and direct path radar signals, respectively. The $\tilde{\tau}_{Ct,n'm'}$, $\tilde{\tau}_{C,n'm'}$, $\tilde{\tau}_{Rt,n'm}$, and $\tilde{\tau}_{R,n'm}$ represent the time delays associated with the four terms, $\zeta_{Ct,n'm'}$ and $\zeta_{Rt,n'm}$ denote the corresponding target reflection coefficients, and $w_{C,n'}[k]$ represents the clutter-plus-noise. The overall communication received signal vector can be written as [13]

$$\begin{aligned}\mathbf{r}_C &= [r_{C,1}[1], r_{C,1}[2], \ldots, r_{C,N_C}[K]]^\dagger \\ &= \tilde{\mathbf{U}}_{Ct}\tilde{\mathbf{s}}_{Ct} + \tilde{\mathbf{U}}_C\tilde{\mathbf{s}}_C + \tilde{\mathbf{U}}_{Rt}\tilde{\mathbf{s}}_{Rt} + \tilde{\mathbf{U}}_R\tilde{\mathbf{s}}_R + \mathbf{w}_C,\end{aligned} \tag{11}$$

where $\tilde{\mathbf{U}}_{Ct} = Diag\{\tilde{\mathbf{u}}^\dagger_{Ct,1}[1], \tilde{\mathbf{u}}^\dagger_{Ct,1}[2], \ldots, \tilde{\mathbf{u}}^\dagger_{Ct,N_C}[K]\}$, $\tilde{\mathbf{u}}_{Ct,n'}[k] = (\tilde{u}_{Ct,n'1}[k], \ldots, \tilde{u}_{Ct,n'M_C}[k])^\dagger$, $\tilde{u}_{Ct,n'm'}[k] = \zeta_{Ct,n'm'}\sqrt{E_{C,m'}}$, and

$$\tilde{\mathbf{s}}_{Ct} = (\tilde{\mathbf{s}}_{Ct,1}[1]^\dagger, \tilde{\mathbf{s}}_{Ct,1}[2]^\dagger, \ldots, \tilde{\mathbf{s}}_{Ct,N_C}[K]^\dagger)^\dagger, \tag{12}$$

in which $\tilde{\mathbf{s}}_{Ct,n'}[k] = [s_{C,1}(kT_s - \tilde{\tau}_{Ct,n'1}), \ldots, s_{C,M_C}(kT_s - \tilde{\tau}_{Ct,n'M_C})]^\dagger$. The $\tilde{\mathbf{U}}_C$, $\tilde{\mathbf{s}}_C$, $\tilde{\mathbf{U}}_{Rt}$, $\tilde{\mathbf{s}}_{Rt}$, $\tilde{\mathbf{U}}_R$, and $\tilde{\mathbf{s}}_R$ in (11) are defined similarly. The clutter-plus-noise vector at the communication receivers $\mathbf{w}_C = [\mathbf{w}_{C,1}{}^\dagger, \ldots, \mathbf{w}_{C,N_C}{}^\dagger]^\dagger$ is assumed to have a zero-mean Gaussian distribution with covariance matrix $\mathbf{Q}_C$, where $\mathbf{w}_{C,n'} = (\mathbf{w}_{C,n'}[1], \ldots, \mathbf{w}_{C,n'}[K])^\dagger$.

A. Mutual Information

From the signal model in (11), the likelihood function conditioned on $\mathbf{s}$ can be calculated

$$\begin{aligned}p(\mathbf{r}_C|\boldsymbol{\theta}, \mathbf{s}) =& \frac{1}{\pi^{KN_C}\det(\mathbf{Q_C})} \\ &\times \exp\left\{-(\mathbf{r}_C - \tilde{\mathbf{U}}_{Ct}\tilde{\mathbf{s}}_{Ct} - \tilde{\mathbf{U}}_C\tilde{\mathbf{s}}_C - \tilde{\mathbf{U}}_{Rt}\tilde{\mathbf{s}}_{Rt} - \tilde{\mathbf{U}}_R\tilde{\mathbf{s}}_R)^H\right. \\ &\left.\times\mathbf{Q}_C^{-1}\left(\mathbf{r}_C - \tilde{\mathbf{U}}_{Ct}\tilde{\mathbf{s}}_{Ct} - \tilde{\mathbf{U}}_C\tilde{\mathbf{s}}_C - \tilde{\mathbf{U}}_{Rt}\tilde{\mathbf{s}}_{Rt} - \tilde{\mathbf{U}}_R\tilde{\mathbf{s}}_R\right)\right\},\end{aligned} \tag{13}$$

and the ML estimate of the target position can be computed

$$\hat{\boldsymbol{\theta}}_{C,ML}|\mathbf{s} = \arg\max_{\boldsymbol{\theta}} \ln p(\mathbf{r}_C|\boldsymbol{\theta}, \mathbf{s}). \tag{14}$$

Using $\hat{\boldsymbol{\theta}}_{C,ML}|\mathbf{s}$ and the known antenna positions, we can obtain the estimated time delays due to target reflection associated with the communication transmissions

$$\hat{\tau}_{Ct,n'm'} = \tilde{\tau}_{Ct,n'm'} + n_{Ct,n'm'}, \tag{15}$$

and the radar transmissions

$$\hat{\tau}_{Rt,n'm} = \tilde{\tau}_{Rt,n'm} + n_{Rt,n'm}, \tag{16}$$

where $n_{Ct,n'm'}$ and $n_{Rt,n'm}$ are the estimation errors assumed to be Gaussian distributed [8].

Replace $\tilde{\tau}_{Rt,n'm}$ with $\hat{\tau}_{Rt,n'm}$ in $\tilde{\mathbf{s}}_{Rt}$ and the estimate $\hat{\mathbf{s}}_{Rt}$ of $\tilde{\mathbf{s}}_{Rt}$ can be obtained, which can be employed to eliminate the radar target return from $\mathbf{r}_C$ in (11). With shared radar signals and antennas position, the direct path received signals from the radar transmission can also be eliminated. Similarly, the unknown $\tilde{\tau}_{Ct,n'm'}$ in $\tilde{\mathbf{s}}_{Ct}$ can be replaced with $\hat{\tau}_{Ct,n'm'}$. To simplify analysis, the noise $\tilde{n}_{Ct,n'm'}$ is assumed small enough to be ignored [13]. Therefore, after eliminating these terms contributed by radar from (11), the received signal vector at the communication receivers becomes

$$\begin{aligned}\mathbf{r}'_C &= \mathbf{r}_C - \tilde{\mathbf{U}}_{Rt}\hat{\mathbf{s}}_{Rt} - \tilde{\mathbf{U}}_R\tilde{\mathbf{s}}_R \\ &= \tilde{\mathbf{U}}_{Ct}\tilde{\mathbf{s}}_{Ct} + \tilde{\mathbf{U}}_C\tilde{\mathbf{s}}_C + \tilde{\mathbf{V}}_{Rt}\tilde{\mathbf{n}}_{Rt} + \mathbf{w}_C,\end{aligned} \tag{17}$$

where $\tilde{\mathbf{V}}_{Rt} = Diag\{\tilde{\mathbf{V}}_{Rt,1}, ..., \tilde{\mathbf{V}}_{Rt,N_C}\}$, $\tilde{\mathbf{V}}_{Rt,n'} = (\tilde{\mathbf{v}}_{Rt,n'}[1], ..., \tilde{\mathbf{v}}_{Rt,n'M_R}[K])^\dagger$, $\tilde{\mathbf{v}}_{Rt,n'}[k] = (\tilde{v}_{Rt,n'1}[k], ..., \tilde{v}_{Rt,n'M_R}[k])^\dagger$,

$$\tilde{v}_{Rt,n'm}[k] = \sqrt{E_{Rm}}\zeta_{Rt,n'm}s^{(1)}_{R,m}(kT_s - \tilde{\tau}_{Rt,n'm}), \tag{18}$$

$s^{(1)}_{R,m}(t) = \partial s_{R,m}(t)/\partial t$ denotes the derivative of $s_{R,m}(t)$ with respect to t, and $\tilde{\mathbf{n}}_{Rt} = [\tilde{\mathbf{n}}^\dagger_{Rt,1}, ..., \tilde{\mathbf{n}}^\dagger_{Rt,N_C}]^\dagger$ with $\tilde{\mathbf{n}}_{Rt,n'} = [\tilde{n}_{Rt,n'1}, ..., \tilde{n}_{Rt,n'M_R}]^\dagger$. In the calculation of (17), the following approximation is adopted [8]

$$\begin{aligned}s_{R,m}(kT_s - \tilde{\tau}_{Rt,n'm}) &- s_{R,m}(kT_s - \tilde{\tau}_{Rt,n'm} - n_{Rt,n'm}) \\ &\approx s^{(1)}_{R,m}(kT_s - \tilde{\tau}_{Rt,n'm})n_{Rt,n'm}.\end{aligned} \tag{19}$$

Since both the $\tilde{\mathbf{s}}_{Ct}$ and $\tilde{\mathbf{s}}_C$ in the resulting measurement vector $\mathbf{r}'_C$ in (17) are contributed by the communication transmissions that contain useful information to be communicated, the mutual information is computed as follows [17]

$$\begin{aligned}I(\mathbf{r}'_C, \tilde{\mathbf{s}}_{Ct}, \tilde{\mathbf{s}}_C) &= H(\mathbf{r}'_C) - H(\mathbf{r}'_C|\tilde{\mathbf{s}}_{Ct}, \tilde{\mathbf{s}}_C) \\ &= \log\det\left(\tilde{\mathbf{U}}_C\tilde{\mathbf{S}}_c\tilde{\mathbf{U}}_C^H + \tilde{\mathbf{U}}_C\tilde{\mathbf{S}}_{ct}\tilde{\mathbf{U}}_{Ct}^H + \tilde{\mathbf{U}}_{Ct}\tilde{\mathbf{S}}_{ct}^H\tilde{\mathbf{U}}_C\right. \\ &\quad\left.+\tilde{\mathbf{U}}_{Ct}\tilde{\mathbf{S}}_{ctt}\tilde{\mathbf{U}}_{Ct}^H + \tilde{\mathbf{V}}_{Rt}\mathbf{Q}_{Rt}\tilde{\mathbf{V}}_{Rt}^H + \mathbf{Q}_C\right) \\ &\quad - \log\det\left(\tilde{\mathbf{V}}_{Rt}\mathbf{Q}_{Rt}\tilde{\mathbf{V}}_{Rt}^H + \mathbf{Q}_C\right) \\ &= \log\det\left\{\mathbf{I} + \left(\tilde{\mathbf{V}}_{Rt}\mathbf{Q}_{Rt}\tilde{\mathbf{V}}_{Rt}^H + \mathbf{Q}_C\right)^{-1}\left(\tilde{\mathbf{U}}_C\tilde{\mathbf{S}}_c\tilde{\mathbf{U}}_C^H\right.\right. \\ &\quad\left.\left.+\tilde{\mathbf{U}}_C\tilde{\mathbf{S}}_{ct}\tilde{\mathbf{U}}_{Ct}^H + \tilde{\mathbf{U}}_{Ct}\tilde{\mathbf{S}}_{ct}^H\tilde{\mathbf{U}}_C + \tilde{\mathbf{U}}_{Ct}\tilde{\mathbf{S}}_{ctt}\tilde{\mathbf{U}}_{Ct}^H\right)\right\},\end{aligned} \tag{20}$$

which is adopted as a performance metric for the communication system, where $H(\cdot)$ denotes differential entropy, $\mathbf{Q}_{Rt} = \mathbb{E}\{\tilde{\mathbf{n}}_{Rt}\tilde{\mathbf{n}}_{Rt}^H\}$, $\tilde{\mathbf{S}}_C = \mathbb{E}\{\tilde{\mathbf{s}}_C\tilde{\mathbf{s}}_C^H\}$, $\tilde{\mathbf{S}}_{ct} = \mathbb{E}\{\tilde{\mathbf{s}}_C\tilde{\mathbf{s}}_{Ct}^H\}$, $\tilde{\mathbf{S}}_{ctt} = \mathbb{E}\{\tilde{\mathbf{s}}_{Ct}\tilde{\mathbf{s}}_{Ct}^H\}$, and $\mathbf{I}$ is the identity matrix.

IV. Simulation Results

In this section, the performance of the cooperative MIMO radar and MIMO communication systems is investigated via numerical results. Assume each of the radar and communication transmit and receive stations are located 70 km away from the origin of the coordinate system. The MIMO radar system has $M_R = 2$ transmitters and $N_R = 3$ receivers, and the transmitted waveforms are frequency spread single Gaussian pulse signals

$$s_{R,m}(t) = (2/T^2)^{(1/4)}\exp(-\pi t^2/T^2)e^{j2\pi m f_\Delta t}, \tag{21}$$

where f_Δ is the frequency offset between adjacent radar transmit signals and T the pulsewidth. Set $f_\Delta = 250Hz$ and $T = 0.01$. The power used for each radar transmit signals are identical so that $E_{R,1} = ... = E_{R,M_R} = E_R$. The covariance matrix of the additive Gaussian noise $\mathbf{w}_R$ in (2) is assumed to be $\mathbf{Q}_R = \sigma_w^2\mathbf{I}$. In the cooperative system, the antenna locations and transmitted signals $s_{R,m}(t)$ are shared to the communication system.

Assume the MIMO communication system has $M_C = 2$ transmitters and $N_C = 3$ receivers. The communication signals can be written as

$$s_{C,m'}(t) = z_{Cm'}(t)p_{T'}(t)e^{j2\pi m\Delta f t}, \tag{22}$$

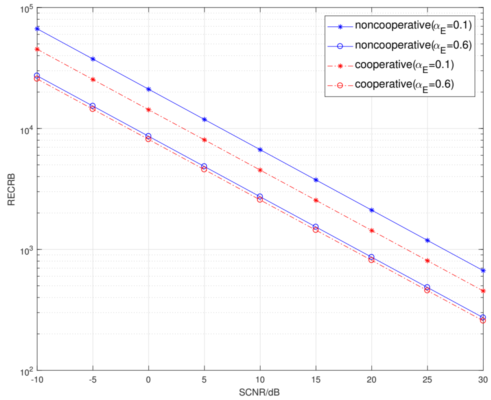

Fig. 1: Target localization RECRB versus SCNR for coexisting MIMO radar and MIMO communication systems.

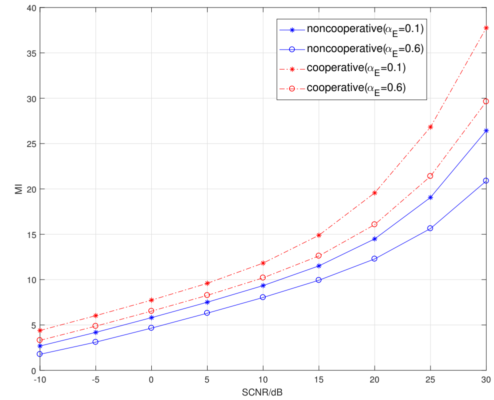

Fig. 2: MI versus SCNR for coexisting MIMO radar and MIMO communication systems.

in which $z_{Cm'}(t)$ are assumed spatially white Gaussian with autocorrelation $\mathbb{E}\{z_{C,m'}(kT_s)z_{C,m'}(k'T_s)\} = 0.9^{|(k-k')T_s|}$ [18], $p_{T'}(t)$ is a rectangular pulse with unit amplitude and width T', Δf is the frequency spacing between two transmit signals, and T' the pulsewidth. Let $\Delta f = 125$ Hz and $T' = 0.01$. Each of the communication signals has identical power $E_{C,1} = ... = E_{C,M_C} = E_C$. The noise $\mathbf{w}_C$ in (11) has covariance matrix $\mathbf{Q}_C = \sigma_w^2\mathbf{I}$. From (22), the statistics of the communication signal vector $\mathbf{s}$ in (5) can be obtained, which, along with the communication antenna positions are sent to the radar for cooperation.

Suppose a target is present at $(50, 30)$m. Denote the total transmit power of the coexisting radar and communication systems by E and the percentage of power assigned to radar by α_E, thus $M_R E_R = E\alpha_E$ and $M_C E_C = E(1-\alpha_E)$. Define the signal to clutter-plus-noise (SCNR) as SCNR$= 10\log_{10}(E/\sigma_w^2)$.

Assuming the estimation accuracy for the target position on the horizontal and vertical axes have equal weights, define an overall estimation performance metric as the averaged root ECRB (RECRB), as given by RECRB $= \left(\sqrt{\mathbf{ECRB}_{1,1}} + \sqrt{\mathbf{ECRB}_{2,2}}\right)/2$. Fig. 1 plots the RECRB for the estimation of target position versus SCNR for the two cases, with (cooperative) and without (noncooperative) cooperation under different α_E. We see that the performance of the cooperative case is always better than that for the noncooperative case, which shows that cooperation can improve the parameter estimation performance for the radar task. As per (2), the cooperative case considers a hybrid active-passive MIMO radar, while the noncooperative case only considers an active MIMO radar. As α_E increases, the performance gain decreases, and when $\alpha_E = 0.6$, there is almost no performance gain because the active radar is dominant and only the statistics of the passive radar signal are known.

For the same coexisting systems, under the same parameter settings, in Fig. 2 the MI is plotted versus SCNR for the cooperative case and noncooperative case under different α_E. It is observed that the MI for the noncooperative case is always worse than that for the cooperative case, which shows significant performance gains in terms of the MI for the communication task. As shown in (11), the cooperative case considers the MIMO communication with reflected path, while the noncooperative case considers the MIMO communication without reflected path. Taken together, Figs. 1 and 2 show that by cooperation, both radar system and communication system can achieve performance gains. There is a trade-off between communication efficiency and radar estimation performance for power-constrained systems because of the desire for lower CRB for the radar system and larger MI for the communication system.

V. Conclusions

In this paper, the coexistence of a MIMO radar and a MIMO communication system is studied in a cooperative way. The assumption that the correctly decoded communication signals at the radar receiver is removed as opposed to the shared statistics of the communication signals. For the radar, target localization using target reflected radar signals and target reflected communication signals has been analyzed and the localization CRB has been provided. On the communication side, the direct path communication signals and target reflected communication signals are exploited to extract the useful information and the corresponding MI is derived to quantify communication efficiency. Numerical results have shown that, there is still a significant gain for both the radar and communication systems. A tradeoff between the radar estimation performance (lower RCRB expected) and communication efficiency (larger MI expected) has been found.

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