MIMO RADAR TRANSMIT BEAMPATTERN SYNTHESIS VIA WAVEFORM DESIGN FOR TARGET LOCALIZATION

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ABSTRACT

The problem of transmit beampattern synthesis in multiple input multiple output (MIMO) radar for target localization is investigated in this paper. By appropriately designing the cross correlation matrix of the transmitted signal waveforms, we can focus the transmit energy into the sector(s) of interest where targets are likely to be located. The proposed energy focusing approach is capable of enhancing the intensity of signals which are reflected from the targets and hence the preferable performance of target localization can be attained. Comparing with the existing energy focusing techniques, our new method realizes a desired pattern via designing the waveform cross correlation matrix rather than the transmit weight vector. Moreover, it does not impose additional transmit power constraints or require a prescribed beampattern to be approximated. Simulation results demonstrate that an improvement of the target localization performance can be offered by the proposed MIMO radar transmit beampattern design technique compared with existing approaches.

Index Terms— Multiple-input multiple-output (MIMO) radar, waveform design, transmit beampattern synthesis, target localization.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) radar has been attracting increasing research interest during the past decades. It is well known that MIMO radar can be depicted as a radar system equipped with multiple transmit antennas and receive antennas. Moreover, compared with phased array radar which radiates the same signal waveform with phase shifts, MIMO radar emits multiple selective waveforms through the transmit elements. At the same time, the receive antennas can simultaneously capture the multiple waveforms reflected back from the target(s). By taking advantage of the waveform diversity, MIMO radar is able to provide better parameter identifiability, improved spatial resolution and enhanced flexibility in transmit beampattern design [1, 2].

As a fundamental radar application in practice, target localization in the context of MIMO radar has been extensively studied by extending classical direction finding approaches. For instance, the estimation of signal parameters via rotation invariance techniques (ESPRIT) and multiple signal classification (MUSIC) estimation approach in bistatic MIMO radar is combined in [3]. The resulting method has high computation complexity due to exhausted spatial spectrum search. An ESPRIT-based method with lower complexity is thus developed in [4] by applying a reduced-dimension transformation. However, these algorithms assume that the transmitted waveforms are perfectly orthogonal, which cannot be always guaranteed due to system imperfections or waveform specifications. Therefore, a pre-whitening approach was proposed recently in [5]. More recently in [6], a low-rank matrix completion technique is developed. Compared with [5], the method in [6] does not rely on the exact knowledge of the waveform correlation matrix, and hence, it offers considerably better robustness against uncertainties.

Although waveform orthogonality is desired in many applications, it limits the signal-to-noise ratio (SNR) at the receiver due to the omni-directional radiation at the transmitter. Because the transmit beampattern and performance of target detection and localization are directly dependent on the cross-correlation of the transmitted waveforms, the problem of waveform design has been extensively studied [7–15]. In particular, the signal cross-correlation is designed to minimize the difference between the true and desired transmit beampatterns in [15]. A constrained optimization problem is formulated and solved with interior-point methods. With such a designed cross-correlation matrix, the signal waveforms can then be synthesized. Concurrent with this work, a modified beampattern matching criterion is reported in [16], where a more computationally efficient algorithm, i.e., semi-definite quadratic programming (SQP) algorithm, for solving the signal design problem is outlined. Unlike these waveform design techniques, a transmit beamspace energy focusing technique is proposed in [17] under the assumption of perfect orthogonal waveforms. The transmitted beampattern is synthesized by designing a transmit beamspace weight matrix.

In this paper, we propose a flexible beampattern design method for MIMO radar so that the transmitted energy within

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the sector of interest can be guaranteed within a certain level, and meanwhile, the energy radiated elsewhere is minimized. By doing so, both the power response and the ripple of the mainlobe can be precisely controlled, and the response level of the sidelobe is low. After focusing the energy into the chosen sector, the directions of arrival (DOAs) are determined by exploiting a shift-invariant property as in ESPRIT. The effectiveness of the proposed transmit beampattern synthesis is validated by numerical examples. The performance of DOA estimation based on the designed transmit waveform correlation is examined by comparing with existing approaches.

2. SIGNAL MODEL

Let us consider a monostatic MIMO radar with M transmit and N receive antennas. Assume the K targets are located in $\theta_1, \cdots, \theta_K$, respectively. The $N \times 1$ complex data vector obtained from receive array can be expressed as

$$\mathbf{x}(t,\tau) = \sum_{k=1}^{K} \beta_k(\tau)\mathbf{a}^T(\theta_k)\mathbf{s}(t)\mathbf{b}(\theta_k) + \mathbf{z}(t,\tau) \tag{1}$$

where t and τ are fast time index and slow time index, respectively, $\beta_k(\tau)$ is the radar complex reflection coefficient of the kth target which is constant during the whole pulse but varies from one slow time to another. $\mathbf{z}(t,\tau)$ is an independent and identically distributed complex additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\sigma_n^2\mathbf{I}_N$. $\mathbf{a}(\theta_k)$ and $\mathbf{b}(\theta_k)$ denote the transmit and receive steering vector of the kth target, respectively. $(\cdot)^T$ and $\mathbf{I}_N$ denote transpose operation and $N \times N$ identity matrix, respectively.

In (1), $\mathbf{s}(t) = [s_1(t), \cdots, s_M(t)]^T$ denotes the transmitted complex waveform vector and the cross correlation matrix is

$$\mathbf{R}_s = \int_T \mathbf{s}(t)\mathbf{s}^H(t)dt = [\mathbf{r}_1, \cdots \mathbf{r}_M] \tag{2}$$

where $\mathbf{r}_m = \int_T \mathbf{s}(t)s_m^*(t)dt$ is the mth column of $\mathbf{R}_s$, T and $(\cdot)^*$ denotes the pulse width and operation of conjugate, respectively. In the case of orthogonal waveforms, $\mathbf{R}_s$ equals to a (scaled) identity matrix. However, the orthogonality condition is abandoned in this work to achieve a preferred transmit beampattern for target localization.

After matched filtering, the receive vector corresponding with mth waveform can be obtained as

$$\mathbf{x}_m(\tau) = \sum_{k=1}^{K} \beta_k(\tau)(\mathbf{a}^T(\theta_k)\mathbf{r}_m)\mathbf{b}(\theta_k) + \mathbf{n}_m(\tau) \tag{3}$$

where $\mathbf{n}_m(\tau) = \int_T \mathbf{z}(t,\tau)s_m^*(t)dt$ is the $N \times 1$ noise term. Then, stacking all the components $\mathbf{x}_1(\tau), \cdots, \mathbf{x}_M(\tau)$, yields the virtual data vector

$$\begin{aligned}\mathbf{y}(\tau) &= \sum_{k=1}^{K} \beta_k(\tau)(\mathbf{R}_s^T\mathbf{a}(\theta_k) \otimes \mathbf{b}(\theta_k)) + \mathbf{n}(\tau) \\ &= \mathbf{A}\boldsymbol{\beta}(\tau) + \mathbf{n}(\tau)\end{aligned} \tag{4}$$

where $\mathbf{A} = [\mathbf{R}_s^T\mathbf{a}(\theta_1) \otimes \mathbf{b}(\theta_1), \cdots, \mathbf{R}_s^T\mathbf{a}(\theta_K) \otimes \mathbf{b}(\theta_K)]$, $\boldsymbol{\beta} = [\beta_1(\tau), \cdots, \beta_K(\tau)]^T$, $\mathbf{n}(\tau) = [\mathbf{n}_1^T(\tau), \cdots, \mathbf{n}_M^T(\tau)]^T$. Recalling that $\mathbf{n}_m(\tau) = \int_T s_m^*(t)\mathbf{z}(t,\tau)dt$, we have $\mathbf{n}(\tau) = \int_T \mathbf{s}^*(t) \otimes \mathbf{z}(t,\tau)dt$ with

$$\mathbf{R}_n = E\{\mathbf{n}(\tau)\mathbf{n}^H(\tau)\} = \mathbf{R}_s^* \otimes \sigma_n^2\mathbf{I}_N \tag{5}$$

which indicates that the resulting data (4) is contaminated by colored noise. Nevertheless, given the designed cross correlation matrix $\mathbf{R}_s$, the colored noise will not impose much difficulty in parameter estimation. Instead, by appropriately designing $\mathbf{R}_s$, improved target localization performance can be achieved.

3. WAVEFORM CORRELATION MATRIX DESIGN FOR PATTERN SYNTHESIS

From (1), it is seen that at the time instant t, the signal at the azimuth angle θ can be denoted by $\mathbf{a}^T(\theta)\mathbf{s}(t)$. As a result, the transmit beampattern which characterizes power distribution in the spatial domain can be expressed as

$$P(\theta) = \int_T |\mathbf{a}^T(\theta)\mathbf{s}(t)|^2dt = \mathbf{a}^H(\theta)\mathbf{R}_s\mathbf{a}(\theta). \tag{6}$$

Let Θ denote the spatial sector(s) of interest where hypothetical targets are located, what we are interested in is how to synthesize the transmit signal waveform parameters (here, the cross correlation matrix $\mathbf{R}_s$) such that the transmit energy can be focused at Θ (or say the mainlobe of the transmit beampattern appears at Θ) to increase the SNR at the receive end.

As mentioned earlier, existing transmit beampattern design approaches may impose relatively restrictive transmit power constraints, such as uniform element power constraint and total power constraint. Obviously, this will consume the degree of freedom (DOF) for waveform correlation matrix design. In this section, we devise a more general waveform correlation matrix design technique which does not demand any transmit power constraints. More specifically, the transmit waveform cross correlation matrix is designed to satisfy the following two basic requirements: i) a fairly stable transmit gain is maintained in the sector of interest Θ, and ii) the amount of transmit energy which is inevitably radiated in the uninterested sectors is minimized for the purpose of energy focusing.

Keeping the above requirements in mind, the following optimization problem is formulated:

$$\begin{aligned}\min_{\mathbf{R_s}} \quad & \int_\Omega \mathbf{a}^H(\theta)\mathbf{R}_s\mathbf{a}(\theta)d\theta \\ s.t. \quad & q_0 - \delta \le \mathbf{a}^H(\theta)\mathbf{R}_s\mathbf{a}(\theta) \le q_0 + \delta, \ \theta \in \Theta \\ & \mathbf{R}_s \in \mathbb{S}^{M\times M}\end{aligned} \tag{7}$$

where Ω denotes the whole spatial domain and we have $\Omega = [-\pi/2, \pi/2]$ in this work, $\mathbb{S}^{M\times M}$ denotes the set of semidefinite matrices, q_0 is the power response of the transmit beampattern at Θ and δ stands for the response ripple. It is noticed that for given q_0 and δ, and discretizing the section of interest Θ, the problem (7) is recast to a linear program.

To facilitate tackling the optimization problem (7), the objective function with integration is rewritten as

$$\int_{\Omega} \mathbf{a}^H(\theta)\mathbf{R}_s\mathbf{a}(\theta)d\theta = \mathrm{trace}(\mathbf{R}_s\mathbf{V}) \tag{8}$$

where $\mathrm{trace}(\cdot)$ denotes the matrix trace, the integration $\mathbf{V} = \int_{\Omega} \mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta$ can be computed using numerical methods. In particular, when the transmit array is a uniform linear array with half-wavelength inter-element spacing, we have $\mathbf{a}(\theta) = [1, e^{j\pi\sin(\theta)}, \cdots, e^{j(M-1)\pi\sin(\theta)}]^T$ and the (m,n)th entry of $\mathbf{V}$, denoted as v_{mn}, is given by

$$\begin{aligned} v_{mn} &= \int_{\Omega} e^{j(m+n-2)\pi\sin(\theta)}d\theta \\ &= \int_{\Omega} \cos((m+n-2)\pi\sin\theta)d\theta \end{aligned} \tag{9}$$

where the imaginary part is vanished due to $\int_{\Omega}\sin((m+n-2)\pi\sin\theta)d\theta = 0$ for $\Omega = [-\pi/2, \pi/2]$. Hence, $\mathbf{V}$ in (8) is a real-valued matrix. Using the above notations and further defining $\mathbf{Q}(\theta) \triangleq \mathbf{a}(\theta)\mathbf{a}^H(\theta)$, the problem (7) becomes

$$\begin{aligned} \min_{\mathbf{R_s}} \quad & \mathrm{trace}(\mathbf{R}_s\mathbf{V}) \\ s.t. \quad & |\mathrm{trace}(\mathbf{R}_s\mathbf{Q}(\theta_i)) - q_0| \leq \delta, \;\; \theta_i \in \Theta \\ & \mathbf{R}_s \in \mathbb{S}^{M\times M} \end{aligned} \tag{10}$$

where θ_i represents the ith sampling point of Θ. The problem can be simply tackled by efficient convex optimization solvers such as CVX. Denote by $\mathbf{R}_{s\star}$ the optimal solution to (10), the problem of DOA estimation on this basis is discussed in the sequel. It should be mentioned that the waveform sequence $\mathbf{s}(t)$ has to be synthesized such that the correlation meets the above design. However, due to space limitation, this will be skipped in this paper.

4. DOA ESTIMATION BASED ON THE DESIGNED WAVEFORM CROSS CORRELATION

We now proceed to the discussion of DOA estimation based on the designed waveform correlation $\mathbf{R}_{s\star}$, which results in a preferred transmit beampattern that has a mainlobe (and thus focus the transmit power) at the target directions. However, as mentioned earlier, whenever the transmitted signal waveforms are nonorthogonal, the radiation at the transmitter is directional due to the waveform correlations, which result in correlations among the match-filtered noises, i.e., colored noise. A simple manner to tackle the colored noise is pre-whitening the data (4) before applying high-resolution algorithms. Unlike the MUSIC method [5], we employ the ESPRIT algorithm in this work as in [6] to considerably reduce the computational complexity. However, since the waveform cross correlation matrix is available from the above design, no special treatments such as low-rank matrix completion as in [6] are required to deal with the colored noise.

Assume the receive array employs a uniform linear array and with half-wavelength inter-element spacing, i.e., $\mathbf{b}(\theta) = [1, e^{j\pi\sin(\theta)}, \cdots, e^{j(N-1)\pi\sin(\theta)}]^T$. Following [5], the pre-whitened data is expressed as

$$\mathbf{y}_\star(\tau) = \mathbf{R}_{n\star}^{-\frac{1}{2}}\mathbf{A}\boldsymbol{\beta}(\tau) + \mathbf{R}_{n\star}^{-\frac{1}{2}}\mathbf{n}(\tau) = \mathbf{A}_\star\boldsymbol{\beta}(\tau) + \mathbf{R}_{n\star}^{-\frac{1}{2}}\mathbf{n}(\tau) \tag{11}$$

where the new terms with subscript $\star$ are defined as

$$\mathbf{R}_{n\star} = \mathbf{R}_{s\star}^* \otimes \mathbf{I}_N \tag{12}$$

$$\mathbf{A}_\star = [(\mathbf{R}_{s\star}^T)^{\frac{1}{2}}\mathbf{a}(\theta_1)\otimes\mathbf{b}(\theta_1), \cdots, (\mathbf{R}_{s\star}^T)^{\frac{1}{2}}\mathbf{a}(\theta_K)\otimes\mathbf{b}(\theta_K)] \tag{13}$$

The corresponding covariance matrix is thus given by $\mathbf{R}_\star = E[\mathbf{y}_\star(\tau)\mathbf{y}_\star^T(\tau)]$, whose eigendecomposition can be written as

$$\mathbf{R}_\star = \mathbf{E}_s\boldsymbol{\Lambda}_s\mathbf{E}_s^H + \mathbf{E}_n\boldsymbol{\Lambda}_n\mathbf{E}_n^H \tag{14}$$

where the $MN \times K$ complex matrix $\mathbf{E}_s$ contains the eigenvectors (signal subspace) corresponding the K largest eigenvalues and the $K \times K$ diagonal matrix $\boldsymbol{\Lambda}_s$ contains these large eigenvalues. Similarly, the $MN \times (MN - K)$ matrix $\mathbf{E}_n$ contains the eigenvectors (noise subspace) corresponding the $MN - K$ smallest eigenvalues and the $(MN-K)\times(MN-K)$ diagonal matrix $\boldsymbol{\Lambda}_n$ contains these small eigenvalues.

Since the equivalent transmit steering vector $\mathbf{R}_s^T\mathbf{a}(\theta)$ loses the a shift-invariant property due to the waveform correlations, the receive array is thus divided for this purpose. In specific, the receive array is divided into two subarrays which consist of the first and last $N-1$ elements, respectively. $\mathbf{A}_\star$ is thus divided into two submatrices $\mathbf{A}_\star^{(1)}$ and $\mathbf{A}_\star^{(2)}$ as

$$\mathbf{A}_\star^{(\kappa)} = [(\mathbf{R}_{s\star}^T)^{\frac{1}{2}}\mathbf{a}(\theta_1)\otimes\mathbf{b}^{(\kappa)}(\theta_1), \cdots, (\mathbf{R}_{s\star}^T)^{\frac{1}{2}}\mathbf{a}(\theta_K)\otimes\mathbf{b}^{(\kappa)}(\theta_K)] \tag{15}$$

where $\kappa = 1, 2$, $\mathbf{b}^{(1)}(\theta)$ and $\mathbf{b}^{(2)}(\theta)$ denote the first and last $N-1$ rows of $\mathbf{b}(\theta)$, respectively. In the same way, we divide $\mathbf{E}_s$ into submatrices $\mathbf{E}_s^{(1)}$ and $\mathbf{E}_s^{(2)}$, and get $\mathbf{E}_s^{(\kappa)} = \mathbf{A}_\star^{(\kappa)}\mathbf{T}$. By applying the shift-invariant property of the receive array, i.e., $\mathbf{b}^{(2)}(\theta) = e^{j\pi\sin(\theta)}\mathbf{b}^{(1)}(\theta)$, we have $\mathbf{A}_\star^{(2)} = \mathbf{A}_\star^{(1)}\boldsymbol{\Phi}$ with $\boldsymbol{\Phi} = \mathrm{diag}\{e^{j\pi\sin\theta_1}, \cdots, e^{j\pi\sin\theta_K}\}$ containing the angle information to be determined.

With the above identities, it can be readily derived that $\mathbf{E}_s^{(2)} = \mathbf{E}_s^{(1)}\boldsymbol{\Psi}$ with $\boldsymbol{\Psi} \triangleq \mathbf{T}^{-1}\boldsymbol{\Phi}\mathbf{T}$. Obviously, $\boldsymbol{\Psi}$ is similar to $\boldsymbol{\Phi}$ (having the same eigenvalues) and can be obtained as

$$\boldsymbol{\Psi} = (\mathbf{E}_s^{(1)H}\mathbf{E}_s^{(1)})^{-1}\mathbf{E}_s^{(1)H}\mathbf{E}_s^{(2)}. \tag{16}$$

As a consequence, the DOAs $\theta_1, \cdots, \theta_K$ can be determined from the eigenvalues of $\boldsymbol{\Psi}$ as

$$\theta_k = \arcsin(\pi^{-1}\arg(\zeta_k)) \tag{17}$$

where ζ_k denotes the kth eigenvalue. It is seen that after designing the waveform correlation matrix $\mathbf{R}_{s\star}$, the computations in parameter estimation mainly come from the eigendecompstion of $\mathbf{R}_\star$, which is $\mathcal{O}(M^3N^3)$.

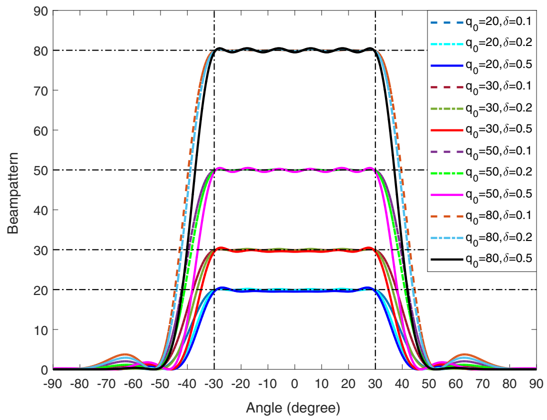

Fig. 1. Illustration of the synthesized transmit beampatterns.

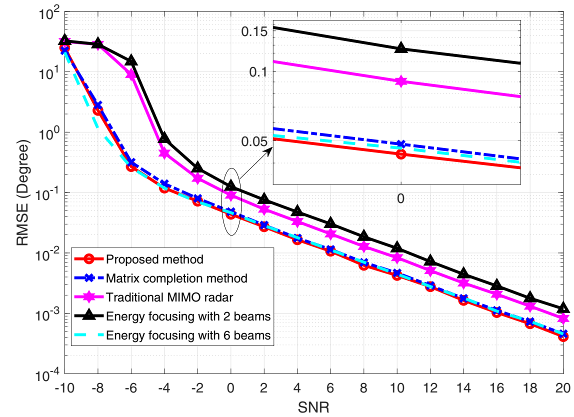

Fig. 2. Comparison of DOA estimation RMSE versus SNR.

5. SIMULATION RESULTS

In our simulations, both the transmit and receive arrays are assumed to be uniform linear arrays with half-wavelength inter-element spacing and $M = 10$ and $N = 10$. Two ($K = 2$) targets locate at $-8°$ and $8°$, respectively. The sector of interest is set as $\Theta = [-30°, 30°]$. The radar reflection coefficient $\{\beta_k(\tau)\}_{k=1}^{K}$ are assumed to be zero-mean Gaussian distribution with covariance σ_β^2. The total transmit energy is $E = 5$ and the covariance matrix is obtained by 1000 snapshots. The performance measures (RMSE and probability of resolution, the sources are considered as resolved if the bias of each DOA estimation is less than $0.25°$) are computed through 500 independent experiments.

Fig. 1 shows the synthesized transmit beampatterns with different mainlobe power levels versus azimuth angle. The sampling stepsize of the spatial sectors is $0.5°$. It is observed that the proposed method achieves a fairly uniform energy distribution in the sector of interest with different mainlobe powers q_0. Meanwhile, the sidelobe is controlled within a low level. This implies that most of the transmitted energy can be focused into the selected sector. This is of great importance for localization in MIMO radar. It is worth emphasizing that q_0 should be properly selected to achieve the most expected transmitted beampattern. This will be further exploited later in a full version of this work.

We now examine the parameter estimation performance of the proposed method based on the designed waveform correlation matrix, by comparing with existing methods including the traditional MIMO radar with orthogonal waveforms, the energy focusing method with 2 and 6 beams [17] and the recent matrix completion based method [6]. Fig. 2 presents the RMSE of DOA estimation versus SNR. It is seen that the proposed method performs similarly to the matrix completion based method and energy focusing method with 6 beams, and outperforms the other methods under the considered scenario. Fig. 3 depicts the probability of resolution versus SNR.

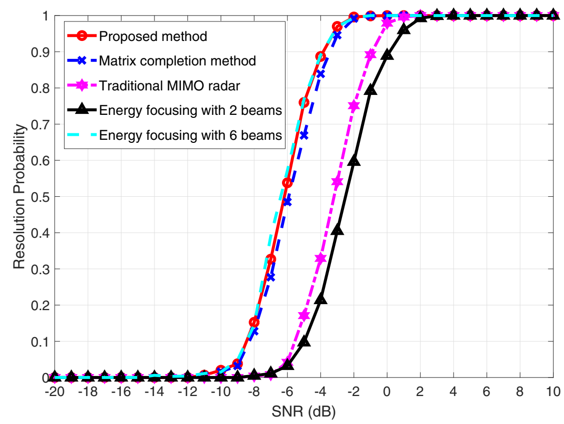

Fig. 3. Comparison of the resolution probability versus SNR.

Again, the excellent performance of the proposed method is observed. It is worth pointing out that the energy focusing method with 2 beams does not perform satisfactorily in this scenario, mainly due to the fact that the transmit beampattern is not properly synthesized when the sector of interest Θ is large but the number of beams are insufficient.

6. CONCLUSION

In this paper, a new energy focusing technique in MIMO radar for the multiple targets localization is proposed. For the problem of transmit beampattern synthesis, by carefully designing the waveform cross correlation matrix, the resulting beampattern can maintain a fairly flat power response within the sector of interest and minimize the amount of energy radiated elsewhere. With the designed waveform correlation, the angle parameters of the targets can be determined by pre-whitening the data and exploiting the shift-invariance property. Simulation results illustrate that an improved performance can be achieved by the proposed method.

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