TRANSMIT BEAMPATTERN DESIGN FOR MIMO RADAR WITH ONE-BIT DACS

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ABSTRACT

In this paper, we investigate the problem of transmit beampattern design for MIMO radar with one-bit DACs. Based on the Fast Fourier Transform (FFT), we propose an alternating minimization (AM) method to minimize the weighted squared-error (WSE) between the designed beampattern and a given one. In particular, we propose an approximation approach to achieve a high-quality solution with one-bit constraint. Finally, numerical simulations demonstrate that the effectiveness of the proposed method.

***Index Terms*—** MIMO radar, transmit beampattern design, one-bit DAC, alternating minimization method, approximation approach

1. INTRODUCTION

Multiple-input multiple-output (MIMO) system, as an emerging technology, has been widely studied in the fields of both wireless communication [1] and radar [2]–[7] during the last decades. In some application, in order to achieve unprecedented direction-of-arrival (DOA) estimation accuracy and spatial degrees of freedom (DOF), a MIMO radar system may be equipped with large-scale antennas (may be several hundreds or thousands of antennas). However, one challenge of the MIMO radar system with large-scale antennas is that the hardware cost and circuit power consumption will be very high if the system deploys conventional high-resolution digital-to-analog converters (DACs) and analog-to-digital converters (ADCs) for each antenna [8, 9]. Another challenge is that the low-complexity and efficient design scheme is essential to this MIMO system. A potential solution to address the above-mentioned challenges is to use the low-resolution quantizers (e.g., one-bit quantizer), which has been extensively studied in massive MIMO wireless communication recently [10]–[16].

On the other hand, the transmit beampattern design is an important problem in MIMO radar. There has been a large of literatures focusing on this problem [17]–[23]. For instance, in [17] the waveform covariance matrix $\boldsymbol{R}$ is optimized to approximate the desired transmit beampattern and minimize sidelobe level using the semidefinite quadratic programming (SQP) technique, and then a cyclic algorithm (CA) [18] is presented to synthesis the constant modulus or low peak-to-average-power ratio (PAPR) waveform [24], whose covariance matrix is close to the $\boldsymbol{R}$. In addition, in [23], the authors propose two "one-step" algorithms to design constant modulus waveforms directly based on the alternating direction method of multipliers (ADMM) method. However, all of these works merely focus on the transmit array with ideal ∞-bit quantizers and not on one-bit MIMO radar.

In this paper, we propose an low-complexity iteration method to solve the problem of the transmit beampattern design for a collocated MIMO radar with one-bit DACs. Using the criterion of the weighted squared-error (WSE) between the designed beampattern and a given one, we formulate the problem of the transmit beampattern design that consists of a nonconvex fourth-order objective and a set of nonconvex discrete constraints by considering uniformly sampling on the possible range of normalized spatial frequencies. Inspiring by the idea of the cyclic-algorithm-new (CAN) method in [25, 26], we handle with the resulting nonconvex problem by utilizing the alternating minimization (AM) framework based of the Fast Fourier Transform (FFT). In particular, to achieve a high-quality solution with one-bit constraint, we propose a continuous and differentiable function to approximate the one-bit quantizer. By doing so, the problem with discrete constraint is converted as an unconstrained minimization problem, which can be effectively solved by using the Limited-memory Broyden Fletcher Goldfarb and Shanno (L-BFGS) approach [27]. Numerical examples illustrated that the proposed algorithm outperform the state-of-the-art method [16] in terms of the matching performance.

2. PROBLEM FORMULATION

We consider a collocated MIMO radar with N transmit antennas, which are palced along an uniform linear array (ULA) with inter-element spacing of half wavelength. Each antenna emits a different waveform $s_n(l)$. Let $\boldsymbol{s}_n = [s_n(1), \cdots, s_n(L)]^T \in \mathbb{C}^{N\times 1}$ be the temporal waveform vector from the n-th antenna, where L is the number of discrete time samples, and $\boldsymbol{S} = \left[\boldsymbol{s}_1^T; \cdots; \boldsymbol{s}_N^T\right]$ be the space-time transmit waveform matrix. In this paper, we aim to design the waveform when one-bit DACs are used at each transmit antenna, as shown in Fig. 1, $2N$ one-bit DACs quantize the real and imaginary parts of the transmit waveform, respectively.

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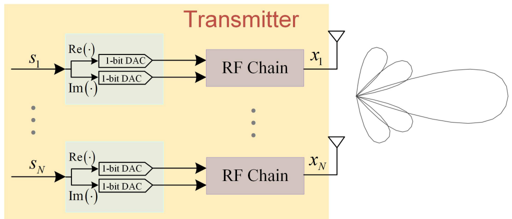

Fig. 1. A collocated MIMO radar system with one-bit DACs.

Then, the quantizer output $\boldsymbol{x}_n$ can be expressed as

$$\boldsymbol{x}_n = \mathcal{Q}_C\left(\boldsymbol{s}_n\right) \tag{1}$$

where $\mathcal{Q}_C(\cdot) = \mathcal{Q}(\cdot) + \jmath\mathcal{Q}(\cdot)$ represents the complex-valued element-wise quantization function, which consists of two one-bit real-valued quantizer $\mathcal{Q}(\cdot)$. In addition, the transmit signal $\boldsymbol{x}_n$ is required to meet the energy constraint, i.e.,

$$\sum_{n=1}^{N} \left\|\boldsymbol{x}_n\right\|^2 = E. \tag{2}$$

where E is the total power for the transmitter. Thus, the transmit alphabet $\mathcal{X}$ is $\left\{\pm\sqrt{\frac{E}{2NL}} \pm \jmath\sqrt{\frac{E}{2NL}}\right\}$, which can be viewed as QPSK signal.

Under the narrow-band assumption, the synthesized signal at the normalized spatial frequency $v = \sin\theta$ is given by

$$\boldsymbol{y}(v) = \boldsymbol{a}_t^\dagger(v)\boldsymbol{X} \tag{3}$$

where $\boldsymbol{X} = \left[\boldsymbol{x}_1^T; \cdots; \boldsymbol{x}_N^T\right]$ and $\boldsymbol{a}_t(v)$ is the transmit steering vector as

$$\boldsymbol{a}_t(v) = \left[1, e^{\jmath\pi v}, \cdots, e^{\jmath\pi(N-1)v}\right]^T \tag{4}$$

Hence, the power of the probing signal at direction v can be written as

$$P(v) = \boldsymbol{y}(v)\boldsymbol{y}^\dagger(v) = \boldsymbol{a}_t^\dagger(v)\boldsymbol{X}\boldsymbol{X}^\dagger\boldsymbol{a}_t(v) \tag{5}$$

Similar to the previous works [17, 21], the WSE between the designed beampattern $P(v)$ and a given one $d(v)$ is used as the figure of merit, which is given by

$$J(\boldsymbol{X}, \alpha) = \sum_{k=1}^{2K} \omega_k \left|\boldsymbol{a}_t^\dagger(v_k)\boldsymbol{X}\boldsymbol{X}^\dagger\boldsymbol{a}_t(v_k) - \alpha^2 d(v_k)\right|^2 \tag{6}$$

where α is a scaling parameter to be optimized. $\omega_k \geq 0$ is the weight for the k-th discrete spatial frequency v_k. In order to compute the $\boldsymbol{X}$ based on the FFT, here we assume that the possible range of normalized spatial frequencies $v \in [-1, 1)$ (i.e., $\theta \in [-90^\circ, 90^\circ)$) is uniformly sampled with $2K$ points. This means that v_k is expressed as

$$v_k = \frac{2(k-1)}{2K} - 1, k = 1, \cdots, 2K \tag{7}$$

Let $\boldsymbol{A}$ be the $N \times 2K$ steering matrix, as

$$\boldsymbol{A} = \left[\boldsymbol{a}_t(v_1), \boldsymbol{a}_t(v_2), \cdots, \boldsymbol{a}_t(v_{2K})\right] \tag{8}$$

and $\boldsymbol{F}$ be the $N \times 2K$ Inverse FFT (IFFT) matrix, as

$$\boldsymbol{F} = \left[\boldsymbol{f}_1, \boldsymbol{f}_2, \cdots, \boldsymbol{f}_{2K}\right] \tag{9}$$

where $\boldsymbol{f}_k (k = 1, \cdots, 2K)$ is defined as

$$\boldsymbol{f}_k = \left[1, e^{j\frac{2\pi(k-1)}{2K}}, \cdots, e^{j\frac{2\pi(k-1)(n-1)}{2K}}, \cdots, e^{j\frac{2\pi(k-1)(N-1)}{2K}}\right]^T \tag{10}$$

Thus, from (4) and (7)–(10), it is easy to verify that

$$\boldsymbol{A} = \left[\boldsymbol{f}_{K+1}, \cdots, \boldsymbol{f}_{2K}, \boldsymbol{f}_1, \cdots, \boldsymbol{f}_K\right] \tag{11}$$

To proceed, our goal is to minimize the WSE by jointly optimizing $\boldsymbol{X}$ and α. Thus, the problem of the waveform design for the transmit beampattern synthesis can be formulated as follows:

$$\begin{aligned} \min_{\boldsymbol{X},\alpha} \quad & \sum_{k=1}^{2K} \omega_k \left|\boldsymbol{a}_t^\dagger(v_k)\boldsymbol{X}\boldsymbol{X}^\dagger\boldsymbol{a}_t(v_k) - \alpha^2 d(v_k)\right|^2 \\ \text{s.t.} \quad & X(n,l) \in \mathcal{X},\ n = 1, \cdots, N;\ l = 1, \cdots, L \end{aligned} \tag{12}$$

It can be seen that since the objective function (12) is nonconvex and the constraint is discrete, the optimization problem is thus generally intractable to be solved directly [28]. One possible method is to simply project the achievable solutions of the SQP+CA and ADMM onto the one-bit set. However, these methods may result in some performance error. More importantly, the SQP+CA and ADMM will suffer from high computational complexity. The complexities of each iteration of these two methods are $\mathcal{O}\left(N^{3.5}\right) + \mathcal{O}\left(N^2L^2\right)$ and $\mathcal{O}\left(N^3L^3\right)$, respectively. Therefore, the implementation of the SQP+CA and ADMM may not be practical in MIMO radar system with large-scale antennas. To this end, we will devise a low-complexity algorithm inspired by the idea of the CAN method in [25, 26].

Based on the CAN, we modify the problem (12) by introducing L-dimensional auxiliary variables $\{\boldsymbol{z}_k\}_{k=1}^{2K}$,

$$\begin{aligned} \min_{\boldsymbol{X},\alpha,\{\boldsymbol{z}_k\}} \quad & \sum_{k=1}^{2K} \omega_k \left\|\boldsymbol{a}_t^\dagger(v_k)\boldsymbol{X} - \alpha\boldsymbol{z}_k^T\right\|^2 \\ \text{s.t.} \quad & \|\boldsymbol{z}_k\| = \sqrt{d(v_k)}, k = 1, \cdots, 2K \\ & X(n,l) \in \mathcal{X},\ n = 1, \cdots, N;\ l = 1, \cdots, L \end{aligned} \tag{13}$$

Notice that it is difficult to optimize problem (13) directly due to the couple of $\boldsymbol{X}$, α and $\{\boldsymbol{z}_k\}$. Towards that end, in what follows, we can tackle this problem by employing the alternating minimization (AM) framework.

3. PROPOSED AM TO SOLVE PROBLEM (13)

In this section, we adopt the AM framework to obtain an sub-optimal solution to problem (13).

3.1. Optimization of α

We first optimize α while keeping $\boldsymbol{X}$ and $\{\boldsymbol{z}_k\}$ fixed. Then, the sub-problem with respect to α is expressed as

$$\min_{\alpha} \sum_{k=1}^{2K} \omega_k \left\|\boldsymbol{a}_t^\dagger(v_k)\boldsymbol{X} - \alpha\boldsymbol{z}_k^T\right\|^2, \tag{14}$$

whose closed-form solution can be obtained by

$$\alpha = \frac{\sum_{k=1}^{2K} \omega_k \mathrm{Re}\left(\boldsymbol{a}_t^\dagger\left(v_k\right)\boldsymbol{X}\boldsymbol{z}_k^*\right)}{\sum_{k=1}^{2K}\omega_k\left\|\boldsymbol{z}_k^T\right\|^2} \tag{15}$$

It is noteworthy that the direct computation of α from (15) is inefficient. Therefore, in the following, we shall provide an efficient method to compute α.

Define

$$\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2 \cdots, \boldsymbol{h}_L] = \boldsymbol{A}^\dagger \boldsymbol{X} \in \mathbb{C}^{2K\times L}, \tag{16}$$

and it is not difficult to varify that $\boldsymbol{h}_l, l = 1, \cdots, L$ can be obtained by the $2K$-point FFT of the l-th column of $\boldsymbol{X}$. Then, α can be recomputed as

$$\alpha = \frac{\mathrm{Re}\left(\tilde{\boldsymbol{h}}^T\tilde{\boldsymbol{z}}\right)}{\sum_{k=1}^{2K}\omega_k\left\|\boldsymbol{z}_k^T\right\|^2} \tag{17}$$

where

$$\tilde{\boldsymbol{h}}^T = \left[\tilde{\boldsymbol{h}}_1^T, \tilde{\boldsymbol{h}}_2^T, \cdots, \tilde{\boldsymbol{h}}_{2K}^T\right] \tag{18}$$

with $\tilde{\boldsymbol{h}}_k^T$ be the k-th row of $\boldsymbol{H}$ and

$$\tilde{\boldsymbol{z}} = \left[\omega_1\boldsymbol{z}_1^\dagger, \omega_2\boldsymbol{z}_2^\dagger, \cdots, \omega_{2K}\boldsymbol{z}_{2K}^\dagger\right]^T. \tag{19}$$

Hence, the calculation of α has a complexity of $\mathcal{O}\left(2KL\log(2K)\right)$, which is lower than $\mathcal{O}\left(4KNL\right)$ needed in the direct calculation from (15).

3.2. Optimizations of $\{\boldsymbol{z}_k\}_{k=1}^{2K}$

For the fixed α and $\boldsymbol{X}$, we can optimize $\boldsymbol{z}_k$ by solving the following problem:

$$\begin{aligned} &\min_{\boldsymbol{z}_k} \left\|\boldsymbol{a}_t^\dagger\left(v_k\right)\boldsymbol{X} - \alpha\boldsymbol{z}_k^T\right\|^2 \\ &\text{s.t.} \quad \|\boldsymbol{z}_k\| = \sqrt{d(v_k)} \end{aligned} \tag{20}$$

its closed-form solution can be obtained as

$$\boldsymbol{z}_k = \frac{\sqrt{d(v_k)}\boldsymbol{g}_k}{\|\boldsymbol{g}_k\|} \tag{21}$$

where $\boldsymbol{g}_k$ is defined as

$$\boldsymbol{g}_k^T = \boldsymbol{a}_t^\dagger\left(v_k\right)\boldsymbol{X} \tag{22}$$

Similarly, $\boldsymbol{g}_k^T$ is the k-the row of the $2K$-point FFT of $\boldsymbol{X}$.

3.3. Optimizations of $\boldsymbol{X}$

For the fixed α and $\{\boldsymbol{z}_k\}$, the optimization problem with respect to $\boldsymbol{X}$ is given by

$$\begin{aligned} &\min_{\boldsymbol{X}} \sum_{k=1}^{2K}\omega_k\left\|\boldsymbol{a}_t^\dagger\left(v_k\right)\boldsymbol{X} - \alpha\boldsymbol{z}_k^T\right\|^2 \\ &\text{s.t.} \quad X(n,l) \in \mathcal{X},\ n = 1, \cdots, N;\ l = 1, \cdots, L \end{aligned} \tag{23}$$

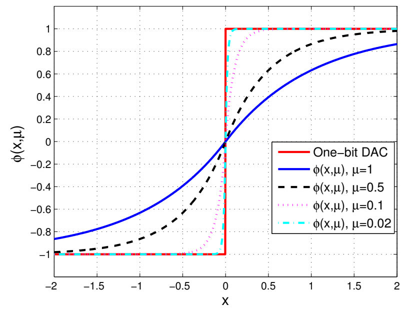

Fig. 2. Illustration of the approximation function of the one-bit quantizer. Assuming $\sqrt{E/2NL} = 1$.

Due to the discrete set $\mathcal{X}$, the optimal value of problem (23) can be obtained by using exhaustive search method. However, the total search number of this method is 4^{NL}. For example, for $N = 10, L = 6$, the search number is about 1.3×10^{36}, which is very hard to achieve on a regular computer. Therefore, we will propose a continuous and differentiable function to approximate the one-bit quantizer, such that the discrete constraint can be dropped.

Here we construct the approximation function as

$$\phi_\mu(x) = \begin{cases} \sqrt{\frac{E}{2NL}}\left(1 - e^{-x/\mu}\right), & x \geq 0 \\ \sqrt{\frac{E}{2NL}}\left(-1 + e^{x/\mu}\right), & x < 0 \end{cases} \tag{24}$$

where μ is a parameter to control the accuracy of the approximation. Without loss of generality, assuming $\sqrt{E/2NL} = 1$, Fig. 2 depicts the approximation function of the one-bit quantizer for different μ. It can be seen from the figure that as the parameter μ is close to 0, the approximation function $\phi_\mu(x)$ approaches the one-bit quantizer. In general, when $|x/\mu| = 10$, we have $|\phi_\mu(x)| = 1$. As a consequence, there are two reasons why we choose $\phi_\mu(x)$ as the approximation function of the one-bit quantizer, i) great approximation property and ii) its gradient is easy to calculate.

By exploiting $X(n,l) \doteq \phi_\mu\left(S(n,l)\right), n = 1, \cdots, N; l = 1, \cdots, L$, we can formulate the approximate problem of problem (23) as follows:

$$\begin{aligned} \min_{\boldsymbol{S}} &\sum_{k=1}^{2K}\omega_k\left\|\boldsymbol{a}_t^\dagger\left(v_k\right)\left(\phi_\mu\left(\boldsymbol{S}_R\right) + \jmath\phi_\mu\left(\boldsymbol{S}_I\right)\right) - \alpha\boldsymbol{z}_k^T\right\|^2 \\ &+ \eta\left(E - \|\phi_\mu\left(\boldsymbol{S}_R\right)\|_F^2 - \|\phi_\mu\left(\boldsymbol{S}_I\right)\|_F^2\right) \end{aligned} \tag{25}$$

where $\boldsymbol{S}_R = \mathrm{Re}\left(\boldsymbol{S}\right)$ and $\boldsymbol{S}_I = \mathrm{Im}\left(\boldsymbol{S}\right)$. η is the dual variable corresponding to the constraint $X(n,l) \in \mathcal{X}$, $n = 1, \cdots, N$; $l = 1, \cdots, L$.

It is noticed that the optimization problem (25) can be equivalently expressed as

$$\begin{aligned} \min_{\boldsymbol{S}} &\left\|\boldsymbol{\Omega}\boldsymbol{A}^\dagger\left(\phi_\mu\left(\boldsymbol{S}_R\right) + \jmath\phi_\mu\left(\boldsymbol{S}_I\right)\right) - \alpha\boldsymbol{\Omega}\boldsymbol{Z}^T\right\|_F^2 \\ &- \eta\left(\|\phi_\mu\left(\boldsymbol{S}_R\right)\|_F^2 + \|\phi_\mu\left(\boldsymbol{S}_I\right)\|_F^2\right) \end{aligned} \tag{26}$$

where $\boldsymbol{\Omega} = \operatorname{diag}\left(\sqrt{\omega_1}, \cdots, \sqrt{\omega_{2K}}\right)$ and the matrix $\boldsymbol{Z} \in \mathbb{C}^{L \times 2K}$ is given by

$$\boldsymbol{Z} = [\boldsymbol{z}_1, \boldsymbol{z}_2, \cdots, \boldsymbol{z}_{2K}] \tag{27}$$

Notice that the solution $\hat{\boldsymbol{S}}$ to unconstrained minimization problem (26) can be effectively obtained by using the Limited-memory Broyden Fletcher Goldfarb and Shanno (L-BFGS) approach (For details about this approach, please refer to [27]).

Finally, to make the constraint $X(n,l) \in \mathcal{X}$, $n = 1, \cdots, N$; $l = 1, \cdots, L$ be satisfied, the solution $\hat{\boldsymbol{S}}$ should be projected onto the set $\mathcal{X}^{NL}$. Thus, the update of $\boldsymbol{X}$ can be computed as

$$\hat{X}(n,l) = \operatorname{proj}_{\mathcal{X}}\left(\hat{S}(n,l)\right) \tag{28}$$

where $\operatorname{proj}_{\mathcal{X}}(\cdot)$ denotes the projection operation, which projects an input variable onto the set $\mathcal{X}$.

4. NUMERICAL RESULTS

In this section, we evaluate the performance of the proposed algorithm via numerical simulations. Unless otherwise specified, in all simulations, we assume an ULA with $N = 80$ transmit antennas, the inter-element spacing is half-wavelength. Each transmit pulse has $L = 160$ samples. The total power for the transmitter is $E = 1$. The possible range of normalized directions $v \in [-1, 1)$ (i.e., $\theta \in [-90°, 90°)$) is assumed to be uniformly sampled with $2K = 200$ points, and the weights for all grid points are $\omega_k = 1$, $k = 1, 2, \cdots, 2K$. As to the approximation parameter μ for the approximation function of the one-bit quantizer, we set $\mu = 0.01$. We assume the dual variable to be $\eta = 0.65$. The initial transmit waveform $\boldsymbol{S}^0$ is selected to be a random matrix, whose entries are assumed to obey complex-valued Gaussian distribution $\mathcal{CN}(0,1)$.

We consider a symmetric desired beampattern with two mainlobes. The desired beampattern is

$$d(v) = \begin{cases} \cos(3\pi v - \pi/2), & -\frac{2}{3} \le v < -\frac{1}{3} \\ \cos(3\pi v + \pi/2), & \frac{1}{3} \le v < \frac{2}{3} \\ 0, & \text{otherwise} \end{cases}$$

Fig. 3 shows the values of objective function in (24) versus the iteration number by using the proposed low-complexity AM method. For comparison purpose, we consider the following cases: the proposed approximation method with $\phi_\mu(\boldsymbol{S})$, the $\boldsymbol{S}$ followed by one-bit DACs, the optimal constant modulus (CM) waveform with ∞-bit DACs, and the optimal CM waveform with one-bit DACs (which is similar to the method in [16]). The achieved matching errors between the designed patterns and the desired one for the four cases are 13.43 dB, 13.29 dB, 3.65 dB and 16.25 dB, respectively. As expected, the one-bit quantization results in the larger matching error than the ideal ∞-bit quantization. The results from Fig.3 also reveal that the achieved $\boldsymbol{S}$ followed by one-bit DACs can provide nearly the same matching performance to the $\phi_\mu(\boldsymbol{S})$, the performance gap is just 0.14 dB. This implies

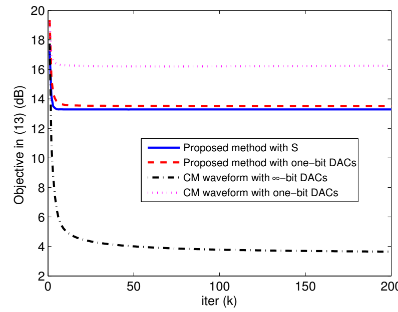

Fig. 3. The values of objective function in problem (13) versus the iteration number. $N = 80, L = 160, 2K = 200$.

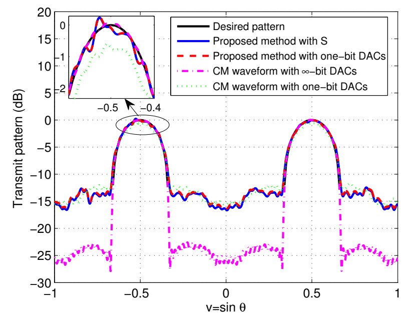

Fig. 4. The beampattern behaviors of the designed waveforms. $N = 80, L = 160, 2K = 200$.

that the proposed approximation method can achieve the excellent approximation property to the one-bit quantizer. More importantly, the achieved $\boldsymbol{S}$ followed by one-bit DACs outperforms than the optimal CM waveform with one-bit DACs.

Next, Fig. 4 compares the resulting transmit beampatterns of the four cases. The result shows that the beampatterns of the one-bit quantization will suffer from the worse sidelobe performance than that of the ∞-bit quantization. In addition, it is also observed that the beampattern of the achieved $\boldsymbol{S}$ followed by one-bit DACs is almost the same as that of the $\phi_\mu(\boldsymbol{S})$, and can obtain a better mainlobe matching performance and lower sidelobe level than that of the optimal CM waveform with one-bit DACs.

5. CONCLUSION

In this paper, the problem of transmit beampattern matching design for MIMO radar with one-bit DACs has been addressed. In order to tackle the resultant problem which involves a nonconvex objective function and a nonconvex one-bit constraint, an AM framework inspired by the CAN algorithm have been devised. Each primitive variable has be efficiently computed based on the FFT. More importantly, we have proposed an approximation method to obtain a high-quality solution with one-bit constraint. Finally, the numerical simulations have indicated that the proposed algorithm possesses a fast convergence and outperform the state-of-the-art methods in terms of the matching performance.

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