JOINT DESIGN FOR MIMO RADAR AND DOWNLINK COMMUNICATION SYSTEMS COEXISTENCE

Ziyang Cheng, Bin Liao‡, Zishu He*, Jun Li* and Julan Xie**

*School of Information & Communication Engineering, UESTC, Chengdu 611731, China
‡College of Information Engineering, Shenzhen University, Shenzhen 518060, China
Email: zeeyoungcheng@163.com

ABSTRACT

This paper focuses on the problem of coexistence between a colocated multiple-in multiple-out (MIMO) radar and downlink communication systems. We propose an iterative algorithm to minimize the Cramer-Rao Bound (CRB) on the direction of arrival (DOA) of a target, accounting for the energy and similarity constraints. More Specifically, at each iteration of this algorithm, the radar waveform is obtained with the aid of the alternating direction method of multipliers (ADMM) algorithm, and then the communication weights are obtained by exploiting the block successive upper-bound minimization method (BSUM). Finally, numerical results are presented to evaluate the effectiveness of the proposed algorithm.

Index Terms— MIMO radar, downlink communicaiton, spectrum sharing, CRB.

1. INTRODUCTION

Recently, co-existence between radar and communication systems is becoming one of the challenging issues in the field of both radar and communication [1, 2]. In fact, the ever-growing demand of operating frequency bands are required to ensure high Quality of Service (QoS) for wireless devices and great parameter estimation performance for radars. The traditional methods to address this co-existence problem is designing radar waveforms with a suitable frequency bands such that the interference produced from the radar to others devices are keep to acceptable levels [3]–[6].

However, in same applications, the spectrum of the wireless devices are widely separated in the frequency band, in this case, the dynamic spectrum design methods for radar waveforms are no longer applicable owing to the fact that the bandwidths of radar and devices are overlapped. Towards that end, many joint design schemes for the co-existence structure are recently proposed to achieve the performance improvements for both radar and communication systems. For example, in [7] and [8], the joint design of the radar subsampling matrix and communication code book is introduced for the co-existence of the multiple-in multiple-out (MIMO) matrix completion (MIMO-MC) radar and MIMO communication systems. [9] proposes a joint design of the MIMO radar transmit space-time code, its receive filter and the communication covariance matrix to maximize the radar signal-to-interference-plus-noise ratio (SINR) while subject to the communication rate constraint. For more approaches on this topic, readers are referred to [10]–[12].

In this paper, we consider the problem of joint design for the co-existence of a colocated MIMO radar and downlink communication systems. Since the symbol data from the communication systems can be viewed as the interference onto the radar receiver, and affect the performance of direction of arrival (DOA) estimation. To this end, we propose a joint design of radar waveform and communication transmit weights to minimize the Cramer-Rao Bound (CRB) on DOA of a target. Moreover, a similarity-like constraint is enforced on the communication weights to make a tradeoff between the achievable CRB value and communication quality. In order to deal with the resulting nonconvex problem with an implicit objective function, an iterative algorithm is devised. Concretely, at each iteration, the radar waveform is optimized based on the alternating direction method of multipliers (ADMM) algorithm [13], then the communication transmit weights are optimized in parallel by employing the block successive upper-bound minimization method (BSUM) [14, 15]. Finally, numerical simulations are carried out to assess the proposed algorithm in terms of the convergence and the communication transmit beampattern properties.

Notation: $(\cdot)^T$ and $(\cdot)^H$ represent the transpose and conjugate transpose operators, respectively. $\otimes$ and $\odot$ stand for the Kronecker and Hadamard product, respectively.

2. PROBLEM FORMULATION

As shown in Fig.1, a joint radar-communication (Rad-Com) coexistence system, which share the same frequency spectrum, involves a collocated MIMO Rad system and downlink Com systems. The Rad transmitter (Tx) and receiver (Rx) are equipped with half-wavelength spaced uniform linear arrays (ULAs) of $N_{R,t}$ and $N_{R,r}$ antennas, respectively. The downlink Com systems serve K single-antenna users, where each Com Tx is equipped with a half-wavelength spaced ULA of $N_{C,t}$ antennas. The symbol data from the Com systems can be viewed as the interference onto the Rad Rx. We supposed

This work was supported in part by the National Natural Science Foundation of China under Grants 61671139, 61771316 and 61871085, and in part by the Foundation of Guangdong Science and Technology Department under Grant 2017A030313341.

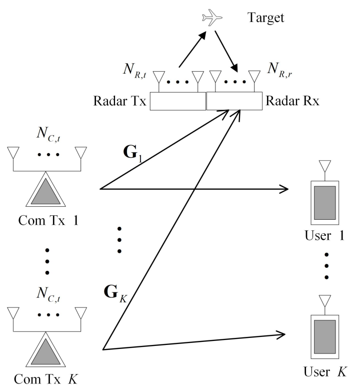


Fig. 1. Diagram of joint radar-communication co-existence system.

that the Rad and Com systems have the same symbol rate and achieve the synchronizations of the sampling time and initial phase [7]. The received matrix $\mathbf{Y}_r \in \mathbb{C}^{N_{R,r}\times L}$ at the Rad Rx with L samples can be expressed as

$$\mathbf{Y}_r = \underbrace{\xi \mathbf{a}_{R,r}(\theta_t)\mathbf{a}_{R,t}^T(\theta_t)\mathbf{S}}_{Rad\ Signal} + \underbrace{\sum_{k=1}^{K}\mathbf{G}_k\mathbf{w}_k\mathbf{x}_k^T}_{Com\ Interference} + \mathbf{N}_r \quad (1)$$

where

- ξ denotes the reflection coefficient, which depends on the target cross section, path loss and radar transmit power, etc.
- $\mathbf{a}_{R,t}(\theta_t)$ and $\mathbf{a}_{R,r}(\theta_t)$ are respectively Rad transmit and receive steering vectors with θ_t being the direction of the target with respect to (w.r.t.) the Rad system.
- $\mathbf{S} \in \mathbb{C}^{N_{R,t}\times L}$ are the space-time transmit waveform matrix from the Rad Tx.
- $\mathbf{G}_k \in \mathbb{C}^{N_{R,r}\times N_{C,t}}$ denotes the interference channel matrix from the Com Tx k to Rad Rx.
- $\mathbf{w}_k \in \mathbb{C}^{N_{C,t}}$ is the corresponding transmit weight vector for the k-th Com Tx.
- $\mathbf{x}_k = [x_k(1), x_k(2), \cdots, x_k(L)]^T$ denotes the the transmit symbol vector from the k-th Com Tx to the intended user k. Assume that the symbol streams $\{x_k(l)\}$ are statistically independent with distribution $\mathcal{CN}(0,1)$ [16].
- $\mathbf{N}_r$ denotes the additive noise of the Rad Rx, whose elements are supposed to be independent with distribution $\mathcal{CN}(0,\sigma_r^2)$.

Defining $\mathbf{y}_r = \mathrm{vec}(\mathbf{Y}_r)$, we have

$$\mathbf{y}_r = \xi(\mathbf{I}_L \otimes \mathbf{U}(\theta_t))\mathbf{s} + \sum_{k=1}^{K}(\mathbf{I}_L \otimes \mathbf{G}_k\mathbf{w}_k)\mathbf{x}_k + \mathbf{n}_r \quad (2)$$

where $\mathbf{U}(\theta_t) = \mathbf{a}_{R,r}(\theta_t)\mathbf{a}_{R,t}^T(\theta_t)$, $\mathbf{s} = \mathrm{vec}(\mathbf{S})$ and $\mathbf{n}_r = \mathrm{vec}(\mathbf{N}_r)$. Thus, the covariance matrix of interference plus noise in (2) has the form

$$\begin{aligned}\mathbf{Q} &= \sum_{k=1}^{K}(\mathbf{I}_L \otimes \mathbf{G}_k\mathbf{w}_k)E\left\{\mathbf{x}_k\mathbf{x}_k^H\right\}(\mathbf{I}_L \otimes \mathbf{G}_k\mathbf{w}_k)^H + \sigma_r^2\mathbf{I}_{N_{R,r}L} \\ &= \mathbf{I}_L \otimes \left(\sum_{k=1}^{K}\mathbf{G}_k\mathbf{w}_k\mathbf{w}_k^H\mathbf{G}_k^H\right) + \sigma_r^2\mathbf{I}_{N_{R,r}L}\end{aligned} \quad (3)$$

In estimation theory and statistics, the CRB expresses a lower bound on the estimation accuracy. We now consider the CRB corresponding to the estimation of θ_t. Let $\boldsymbol{\mu}(\theta_t) = \xi\left(\mathbf{I}_L \otimes \mathbf{a}_{R,r}(\theta_t)\mathbf{a}_{R,t}^T(\theta_t)\right)\mathbf{s}$, then one gets the CRB on θ_t [17], as

$$\begin{aligned}\mathrm{CRB}(\theta_t) &= \left\{2\frac{\partial\boldsymbol{\mu}^H(\theta_t)}{\partial\theta_t}\mathbf{Q}^{-1}\frac{\partial\boldsymbol{\mu}(\theta_t)}{\partial\theta_t}\right\}^{-1} \\ &= \frac{1}{2|\xi|^2\mathbf{s}^H(\mathbf{I}_L \otimes \mathbf{A}(\theta_t))^H\mathbf{Q}^{-1}(\mathbf{I}_L \otimes \mathbf{A}(\theta_t))\mathbf{s}}\end{aligned} \quad (4)$$

where

$$\mathbf{A}(\theta_t) = \dot{\mathbf{a}}_{R,r}(\theta_t)\mathbf{a}_{R,t}^T(\theta_t) + \mathbf{a}_{R,r}(\theta_t)\dot{\mathbf{a}}_{R,t}^T(\theta_t) \quad (5)$$

with $\dot{\mathbf{a}}_{R,r}(\theta_t) = \frac{\partial\mathbf{a}_{R,r}(\theta_t)}{\partial\theta_t}$ and $\dot{\mathbf{a}}_{R,t}(\theta_t) = \frac{\partial\mathbf{a}_{R,t}(\theta_t)}{\partial\theta_t}$.

The goal of this paper is to jointly design the Rad waveform $\mathbf{s}$ and Com transmit weights $\{\mathbf{w}_k\}_{k=1}^K$, such that we minimize the achievable the direction of the target estimation accuracy for the Rad. Furthermore, in order to make the communication quality keep on a certain level, the designed transmit weights $\{\mathbf{w}_k\}_{k=1}^K$ should share the good transmit beampattern behavior of known weights $\{\mathbf{w}_{k,0}\}_{k=1}^K$. Thus, we introduce a similarity-like constraint [6] to allow a compromise between the Rad estimation performance and transmit beampattern property and consider the following constraint:

$$\|\mathbf{w_k} - \rho_k\mathbf{w}_{k,0}\| \le \gamma_{C,k}, k = 1,\cdots,K. \quad (6)$$

where $0 \le \gamma_{C,k} \le 1$ is a parameter to control the level of the similarity for the k-th weight, $\rho_k(|\rho_k| \le 1)$ is a scaling parameter.

With the optimization criterion of minimizing the CRB for the Rad, the problem of joint design of the Rad waveform $\mathbf{s}$ and Com weights $\{\mathbf{w}_k\}_{k=1}^K$ can be formulated as

$$\min_{\mathbf{s},\{\mathbf{w}_k\},\{\rho_k\}} \quad \mathrm{CRB}(\theta_t) \quad (7a)$$

$$\text{s.t.} \quad \|\mathbf{s}\|^2 \le P_R, \quad (7b)$$

$$\|\mathbf{s} - \mathbf{s}_0\|^2 \le \gamma_R^2, \quad (7c)$$

$$\|\mathbf{w}_k\|^2 \le 1,\ k = 1,\cdots,K, \quad (7d)$$

$$\|\mathbf{w}_k - \rho_k\mathbf{w}_{k,0}\|^2 \le \gamma_{C,k}^2,\ k = 1,\cdots,K. \quad (7e)$$

where P_R is the power constraint for the Rad waveform. The constraint (7c) is a similarity constraint [18], which make the designed $\mathbf{s}$ share the good pulse compression property of a reference waveform $\mathbf{s}_0$ with γ_R being a user-defined parameter to control the level of the similarity for the Rad waveform. (7d) and (7e) are the power constraint and similarity-like constraint for the k-th Com transmit weight.

It can be seen that the above optimization problem involves a nonconvex objective, nonhomogeneous inequality constraint (7c) and nonconvex inequality constraint (7d), and hence, it is NP-hard [19] and cannot be efficiently solved. To this end, in next section, an iterative algorithm is devised.

3. SOLUTION TO THE OPTIMIZATION PROBLEM

Now, we show in this section how the designed variables $(\mathbf{s},\{\mathbf{w}_1,\cdots,\mathbf{w}_K\})$ is optimzed in a cyclical manner.

Following the results in [6], problem (7) can be equivalently recast as

$$\max_{\mathbf{s},\{\mathbf{w}_k\}} \quad \mathbf{s}^H(\mathbf{I}_L\otimes\mathbf{A}(\theta_t))^H\mathbf{Q}^{-1}(\mathbf{I}_L\otimes\mathbf{A}(\theta_t))\mathbf{s}, \tag{8a}$$

$$\text{s.t.} \quad \|\mathbf{s}\|^2\le P_R, \tag{8b}$$

$$\|\mathbf{s}-\mathbf{s}_0\|^2\le\gamma_R^2, \tag{8c}$$

$$\|\mathbf{w}_k\|^2\le 1,\ k=1,\cdots,K, \tag{8d}$$

$$\mathbf{w}_k^H\mathbf{B}_k\mathbf{w}_k\le\gamma_{C,k}^2, k=1,\cdots,K. \tag{8e}$$

where $\mathbf{B}_k=\mathbf{I}_{N_{C,t}}-\mathbf{w}_{k,0}\mathbf{w}_{k,0}^H$ is a positive semidefinite (PSD) matrix.

A. Optimization of $\mathbf{s}$

For a given $\left\{\mathbf{w}_1^{(n-1)},\cdots,\mathbf{w}_K^{(n-1)}\right\}$, the optimization problem w.r.t. $\mathbf{s}$ can be formulated as

$$\max_{\mathbf{s}} \quad \mathbf{s}^H\mathbf{\Xi}\mathbf{s}, \tag{9a}$$

$$\text{s.t.} \quad \|\mathbf{s}\|^2\le P_R, \tag{9b}$$

$$\|\mathbf{s}-\mathbf{s}_0\|^2\le\gamma_R^2, \tag{9c}$$

where $\mathbf{\Xi}\triangleq(\mathbf{I}_L\otimes\mathbf{A}(\theta_t))^H\mathbf{Q}^{-1}(\mathbf{I}_L\otimes\mathbf{A}(\theta_t))$. Notice that problem (9) can not be solved by the semidefinite programming (SDP) technology due to the nonhomogeneous inequality constraint (9c) [20]. In the following, we will handle with this problem based on the ADMM method [13] and [21].

Before proceeding, we introduce auxiliary variable $\mathbf{t}$ to convert the problem (9) as

$$\begin{aligned}\max_{\mathbf{s}} \quad & \mathbf{s}^H\mathbf{\Xi}\mathbf{s}\\ \text{s.t.} \quad & \|\mathbf{s}\|^2\le P_R\\ & \mathbf{s}-\mathbf{s}_0-\mathbf{t}=\mathbf{0},\ \ \|\mathbf{t}\|^2\le\gamma_R^2\end{aligned} \tag{10}$$

Placing the equality constraint $\mathbf{s}-\mathbf{s}_0-\mathbf{t}=\mathbf{0}$ into the augmented Lagrangian function (scaled form) of (10) [13], as

$$\mathcal{L}(\mathbf{s},\mathbf{t},\mathbf{y})=-\mathbf{s}^H\mathbf{\Xi}\mathbf{s}+\frac{\rho}{2}\|\mathbf{s}-\mathbf{s}_0-\mathbf{t}+\mathbf{y}\|^2 \tag{11}$$

where $\mathbf{y}$ is Lagrange multiplier and ρ is a penalty parameter.

Therefore, at the (m)-th iteration of the ADMM framework, we obtain $\{\mathbf{s},\mathbf{t},\mathbf{y}\}$ via the following steps:

(1) Determin $\mathbf{s}^{(m)}$ by

$$\mathbf{s}^{(m)}=\begin{cases}\dfrac{\sqrt{P_R}\bar{\mathbf{s}}^{(m)}}{\left\|\bar{\mathbf{s}}^{(m)}\right\|}, & \left\|\bar{\mathbf{s}}^{(m)}\right\|\ge\sqrt{P_R}\\ \bar{\mathbf{s}}^{(m)}, & \text{otherwise}\end{cases} \tag{12}$$

where $\bar{\mathbf{s}}^{(m)}=\frac{\rho}{2}\left(\frac{\rho}{2}\mathbf{I}_{N_{R,t}L}-\mathbf{\Xi}\right)^{-1}\left(\mathbf{s}_0+\mathbf{t}^{(m-1)}-\mathbf{y}^{(m-1)}\right)$.

(2) Determin $\mathbf{t}^{(m)}$ by

$$\mathbf{t}^{(m)}=\begin{cases}\dfrac{\gamma_R\bar{\mathbf{t}}^{(m)}}{\left\|\bar{\mathbf{t}}^{(m)}\right\|}, & \left\|\bar{\mathbf{t}}^{(m)}\right\|\ge\gamma_R\\ \bar{\mathbf{t}}^{(m)}, & \text{otherwise}\end{cases} \tag{13}$$

where $\mathbf{t}^{(m)}=\mathbf{s}^{(m)}-\mathbf{s}_0+\mathbf{y}^{(m-1)}$.

(3) Determin $\mathbf{y}^{(m)}$ by

$$\mathbf{y}^{(m)}=\mathbf{y}^{(m-1)}+\mathbf{s}^{(m)}-\mathbf{s}_0+\mathbf{t}^{(m)}. \tag{14}$$

Steps (1)–(3) are repeated until $\left\|\mathbf{s}^{(m)}-\mathbf{s}^{(m-1)}\right\|\le\epsilon^{\text{ADMM}}$, where ϵ^{ADMM} is convergence parameter for the ADMM. Output $\mathbf{s}^{(n)}=\mathbf{s}^{(m)}$.

B. Optimization of $\{\mathbf{w}_k\}$

To proceed, for a given $\mathbf{s}^{(n)}$, we tackle the remaining K subproblems by defining $\mathbf{W}_k=\mathbf{w}_k\mathbf{w}_k^H$, then the relaxed problem w.r.t. $\mathbf{W}_k$ is cast as

$$\begin{aligned}\max_{\mathbf{W}_k} \quad & \text{Tr}\left(\mathbf{Q}^{-1}\mathbf{\Gamma}\right)\\ \text{s.t.} \quad & \text{Tr}\left(\mathbf{W}_k\right)\le 1,\\ & \text{Tr}\left(\mathbf{B}_k\mathbf{W}_k\right)\ \le\gamma_{C,k}^2\end{aligned} \tag{15}$$

where $\mathbf{\Gamma}=(\mathbf{I}_L\otimes\mathbf{A}(\theta_t))\mathbf{s}\mathbf{s}^H(\mathbf{I}_L\otimes\mathbf{A}(\theta_t))^H$.

Noticed that since the objective function in problem (15) is an implicit function w.r.t. $\mathbf{W}_k$, and thus it is hard to solve it directly. Towards that end, we shall tackle this problem by utilizing the BSUM method [14, 15].

The BSUM method updates the variables $\{\mathbf{W}_k\}$ by optimizing the surrogate functions of the original ones. The analysis in [14] shows that the BSUM can converge to a stationary solution as long as the surrogate function satisfies the Assumption 1 in [14].

Remark 1: Supposed that $\mathbf{\Gamma}$ is a PSD matrix, then the function $\text{Tr}\left(\mathbf{Q}^{-1}\mathbf{\Gamma}\right)$ is convex on $\mathbf{Q}$.

By using Remark 1 and the property of the convexity, the following inequality holds:

$$\text{Tr}\left(\mathbf{\Gamma}\mathbf{Q}^{-1}\right)\ge\text{Tr}\left\{\mathbf{\Gamma}\bar{\mathbf{Q}}^{-1}\right\}+\text{Tr}\left(\bar{\mathbf{D}}\left(\mathbf{Q}-\bar{\mathbf{Q}}\right)\right) \tag{16}$$

where

$$\bar{\mathbf{D}}=\nabla\,\text{Tr}\left(\mathbf{\Gamma}\mathbf{Q}^{-1}\right)\big|_{\mathbf{Q}=\bar{\mathbf{Q}}}=-\bar{\mathbf{Q}}^{-1}\mathbf{\Gamma}\bar{\mathbf{Q}}^{-1} \tag{17}$$

It is not difficult to verify that if the surrogate function $\mathcal{U}_k(\mathbf{W}_k)$ of $\text{Tr}\left(\mathbf{\Gamma}\mathbf{Q}^{-1}\right)$ is selected as $\text{Tr}\left(\bar{\mathbf{D}}\mathbf{Q}\right)+\text{const}$, then the $\mathcal{U}_k(\mathbf{W}_k)$ meets Assumption 1 in [14].

Therefore, from (3), the objective function in problem (15) can be replaced by

$$\mathcal{U}_k(\mathbf{W}_k)=\text{Tr}\left(\left(\sum_{l=1}^{L}\bar{\mathbf{D}}_{ll}\right)\mathbf{G}_k\mathbf{W}_k\mathbf{G}_k^H\right)+\text{const} \tag{18}$$

where

$$\bar{\mathbf{D}}=\begin{bmatrix}\bar{\mathbf{D}}_{11} & \cdots & \bar{\mathbf{D}}_{1L}\\ \vdots & \ddots & \vdots\\ \bar{\mathbf{D}}_{L1} & \cdots & \bar{\mathbf{D}}_{LL}\end{bmatrix}$$

As analyzed above, the BSUM method update $\mathbf{W}_k$ at the n-th iteration by solving

$$\begin{aligned} \max_{\mathbf{W}_k} \ & \mathrm{Tr}\left(\mathbf{M}^{(n-1)}\mathbf{W}_k\right) \\ \text{s.t.} \quad & \mathrm{Tr}\left(\mathbf{W}_k\right) \le 1, \\ & \mathrm{Tr}\left(\mathbf{B}_k\mathbf{W}_k\right) \ \le \gamma_{C,k}^2 \end{aligned} \tag{19}$$

where

$$\mathbf{M}^{(n-1)} = \mathbf{G}_k^H \left(\sum_{l=1}^{L} \mathbf{D}_{ll}^{(n-1)}\right) \mathbf{G}_k \tag{20}$$

with the matrix $\mathbf{D}_{ll}^{(n-1)}$ being obtained at the $(n-1)$-th iteration.

It is easily observed that the K subproblems (19) are SDP problems, which can be solved in parallel via the CVX toolbox [22], then followed by the randomization procedure to obtain the near-optimal solutions of $\{\mathbf{w}_k\}$.

4. NUMERICAL RESULTS

For the simulations, we consider a collocated MIMO Rad system with $N_{R,t} = 4$ transmit antennas and $N_{R,r} = 4$ receive antennas. We assume that a target is located at $\theta_t = 10°$ and its complex coefficient is unit. The downlink Com systems serve $K = 2$ users, and each Com Tx is equipped $N_{C,t} = 8$ antennas. We assume that the 2 users are located at $\theta_{C,1} = 0°$ and $\theta_{C,2} = -5°$ w.r.t. the corresponding Com Txs, respectively. The reference weights for the 2 Com Txs are respectively $\mathbf{w}_{1,0} = \mathbf{h} \odot \mathbf{a}_{C,t}(\theta_{C,1})$ and $\mathbf{w}_{2,0} = \mathbf{h} \odot \mathbf{a}_{C,t}(\theta_{C,2})$, where $\mathbf{a}_{C,t}(\theta)$ is a transmit steering vector and $\mathbf{h}$ is a Chebyshev window with 30 dB of ripple. As for the reference waveform the Rad, we choose the orthogonal linear frequency modulation (LFM) signal with the sample of $L = 16$, which is defined by [23]. The interference channel matrice $\mathbf{G}_1$ and $\mathbf{G}_2$ are assumed to have independent entries, distributed as $\mathcal{CN}(0, 1)$ [7]. The variance of the Gaussian white noise is $\sigma_r^2 = 0$ dB. Furthermore, for the ADMM method, we set $\mathbf{t}^{(0)} = \mathbf{0}$ and $\mathbf{y}^{(0)} = \mathbf{0}$, the penalty parameter $\rho = 200$ and the convergence parameter $\epsilon^{\mathrm{ADMM}} = 10^{-4}$.

Fig. 2 depicts the values of objective function in (7) versus the iteration number with a fixed $\gamma_R = 0.3$ but different $\gamma_C = 0.01, 0.1, 0.2, 0.3$. Besides, for comparison purpose, the cases of only optimization of $\mathbf{s}$ with $\gamma_R = 0.3$ and of only optimization of $\mathbf{w}_1$ and $\mathbf{w}_2$ with $\gamma_C = 0.3$ are considered. It is seen from Fig. 2 that the proposed algorithm can converge to a limit value. Moreover, the joint optimization of $\mathbf{s}$, $\mathbf{w}_1$ and $\mathbf{w}_2$ outperforms "Only optim. $\mathbf{s}$" or "Only optim. $\mathbf{w}_1$ and $\mathbf{w}_2$" in terms of the estimation performance. This observation agrees with our expectations. Additionally, Fig. 2 also shows that the achievable CRB values decrease with γ_C. This is owing to the fact that the higher the γ_C is, the more degrees of freedom are available to design.

Next, Fig. 3 displays convergence properties for the proposed algorithm with a fixed $\gamma_C = 0.2$ but different $\gamma_R = 0.01, 0.1, 0.2, 0.3$. As expected, the result show that the values of the objective function decrease with the γ_R.

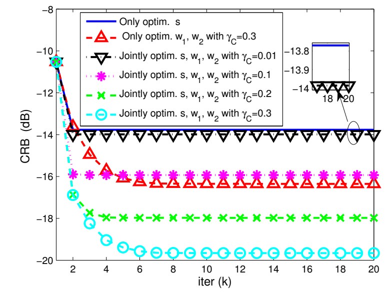


Fig. 2. The values of objective function versus the iteration number with $\gamma_R = 0.3$.

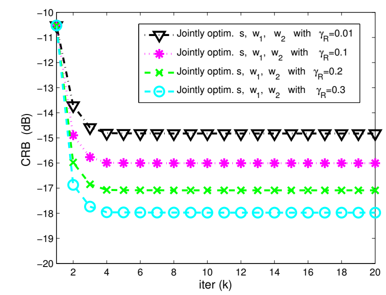
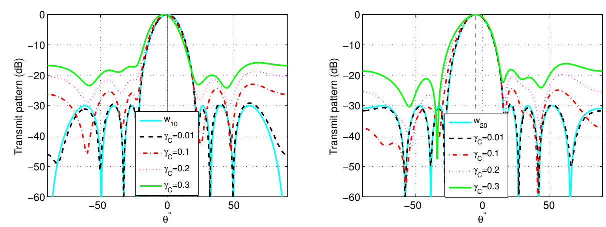


Fig. 3. The values of objective function versus the iteration number with $\gamma_C = 0.2$.



(a) (b)

Fig. 4. The transmit beampattern behaviors for communications with $\gamma_R = 0.3$. (a) $\mathbf{w}_1$, (b) $\mathbf{w}_2$.

Finally, the achievable transmit beampattern properties of the designed $\mathbf{w}_1$ and $\mathbf{w}_2$ are respectively compared to that of the reference transmit weights $\mathbf{w}_{1,0}$ and $\mathbf{w}_{2,0}$ in Fig. 4. For illustration purpose, we take $\mathbf{w}_1$ as an example to illustrate the beampattern behaviors of the different designed weights. It is observed from Fig. 4 that the smaller the γ_C is, the better the transmit beampattern behavior will be achieved.

5. CONCLUSION

In this paper, we have considered a co-existence system of MIMO radar and downlink communication systems. In order to improve the DOA estimation performance, we have proposed a joint design of the radar waveform and communication transmit weights to minimize the CRB on the direction of a target. The resultant nonconvex problem is tackled by alternating optimization method. Results have revealed that the proposed algorithm is able to realize a compromise between the DOA estimation performance for radar and the beampattern behaviors for the communication systems.

6. REFERENCES

[1] H. Griffiths, L Cohen, S Watts, et al., “Radar spectrum engineering and management: technical and regulatory issues,” *Proc. IEEE*, vol. 103, no. 1, pp. 85–102, Jan. 2014.

[2] F. Hessar and S. Roy, “Spectrum sharing between a surveillance radar and secondary Wi-Fi networks,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 52, no. 3, pp. 1434–1448, Jun. 2016.

[3] A. Aubry, A. De Maio, M. Piezzo, et al., “Radar waveform design in a spectrally crowded environment via non-convex quadratic optimization,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 50, no. 2, pp. 1138–1152, April 2014.

[4] A. Aubry, A. De Maio, Y. Huang, et al., “A new radar waveform design algorithm with improved feasibility for spectral coexistence,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 51, no. 2, pp. 1029–1038, April 2015.

[5] Z. Cheng, B. Liao, Z. He, et al., “Spectrally compatiblewaveform design for MIMO radar in the presence of multiple targets,” *IEEE Trans. Signal Process.*, vol. 66, no. 13, pp. 3543–3555, Jul. 2018.

[6] A. Aubry, V. Carotenuto, and A. De Maio, “Forcing multiple spectral compatibility constraints in radar waveforms,” *IEEE Signal Process. Lett.*, vol. 23, no. 4, pp. 483–487, April 2016.

[7] B. Li, A. P. Petropulu, and W. Trappe, “Optimum codesign for spectrum sharing between matrix completion based MIMO radars and a MIMO communication system,” *IEEE Trans. Signal Process.*, vol. 64, no. 17, pp. 4562–4575, Sep. 2016.

[8] B. Li and A. P. Petropulu, “Joint transmit designs for coexistence of MIMO wireless communications and sparse sensing radars in clutter,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 53, no. 6, pp. 2846–2864, Dec. 2017.

[9] L. Zheng, M. Lops, X. Wang, et al., “Joint design of overlaid communication systems and pulsed radars,” *IEEE Trans. Signal Process.*, vol. 66, no. 1, pp. 139–154, Jan. 2018.

[10] J. Qian, M. Lops, L. Zheng, et al., “Joint system design for coexistence of MIMO radar and MIMO communication,” *IEEE Trans. Trans. Signal Process.*, vol. 66, no. 13, pp. 3504–3519, Jul. 2018.

[11] F. Liu, C. Masouros, A. Li, et al., “MU-MIMO communications with MIMO radar: from co-Existence to joint transmission,” *IEEE Trans. Wireless Commun.*, vol. 17, no. 4, pp. 2755–2770, Apr. 2018.

[12] C. Shi, F. Wang, M. Sellathurai, et al., “Power minimization-based robust OFDM radar waveform design for radar and communication systems in coexistence,” *IEEE Trans. Signal Process.*, vol. 66, no. 5, pp. 1316–1330, Mar, 2018.

[13] S. Boyd, N. Parikh, E. Chua, et al., “Distributed optimization and statistical learning via the alternating direction method of multipliers,” *Found, Trend. Mach. Learn.*, vol. 3, no. 1, pp. 1–122, 2011.

[14] M. Razaviyayn, M. Hong, and Z. Q. Luo, “A unified convergence analysis of block successive minimization methods for nonsmooth optimization,” *SIAM J. Optim.*, vol. 23, no. 2, pp. 1126–1153, 2012.

[15] M. Hong, M. Razaviyayn, Z. Q. Luo, et al., “A unified algorithmic framework for block-structured optimization involving big data: With applications in machine learning and signal processing,” *IEEE Signal Process. Mag.*, vol. 33, no. 1, pp. 57–77, 2016.

[16] Q. Shi, M. Razaviyayn, M. Hong, et al., “ SINR constrained beamforming for a MIMO multi-user downlink system: algorithms and convergence analysis,” *IEEE Trans. Signal Process.*, vol. 64, no. 11, pp. 2920–2933, 2016.

[17] H. L. Van Trees, *Optimum Array Processing. Part IV of Detection, Estimation and Modulation Theory*. New York: Wiley, 2002.

[18] Z. Cheng, Z. He, B. Liao, et al., “MIMO Radar Waveform Design With PAPR and Similarity Constraints,” *IEEE Trans. Signal Process.*, vol. 66, no. 4, pp. 968–981, 2018.

[19] S. Boyd and L. Vandenberghe, *Convex optimization*, Cambridge, U.K.: Cambridge Univ. Press, 2004.

[20] Z. Q. Luo, W. Ma, A. So, et al., “Semidefinite relaxation of quadratic optimization problems,” *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.

[21] Z. Cheng, C. Han, B. Liao, et al., “Communication-aware waveform design for MIMO radar with good transmit beampattern,” *IEEE Trans. Signal Process.*, vol. 66, no. 21, pp. 15549–5562, Nov, 2018.

[22] M. Grant and S. Boyd, CVX: Matlab software for disciplined convex programming, version 2.1, Feb. 2016. [Online]Available: http://cvxr.com/cvx.

[23] G. Cui, H. Li and M. Rangaswamy, “MIMO radar waveform design with constant modulus and similarity constraints,” *IEEE Trans. Signal Process.*, vol. 62, no. 2, pp. 343–353, 2014.