GRIDLESS SUPER-RESOLUTION DOA ESTIMATION WITH UNKNOWN MUTUAL COUPLING

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ABSTRACT

In this paper, a gridless super-resolution direction-of-arrival (DOA) estimation method with unknown mutual coupling is proposed. A new clean steering vector is obtained based on the banded symmetric Toeplitz structure of the mutual coupling matrix (MCM). Further, atomic norms associated with the array structure are generated, which can provide a breakthrough in solving super-resolution estimation problem by directly working on the continuous parameter domain. Finally, a semidefinite programming (SDP) method is derived to solve this atomic norm minimization problem. Simulations are provided to verify the effectiveness of the propose method.

Index Terms— DOA estimation, mutual coupling, gridless sparse recovery, atomic norm minimization.

1. INTRODUCTION

DOA estimation of impinging signals is significant in many applications such as radar, sonar and wireless communications. Two issues that an efficient DOA estimation method ought to address are the resolution of two or more closely-spaced sources with high precision from very few snapshots and the mutual coupling effect between the antenna elements.

For the first issue, the algorithms based on sparse representation recovery (SSR) [1, 2] break through the performance of traditional DOA estimation algorithms [3]. But when the DOAs lie off the discretized grid, the SSR-based algorithms will suffer from basis mismatch problem. A perturbed sparse Bayesian learning-based algorithm is proposed to solve the DOA estimation for off-grid signals in [4, 5], but it costs high complexity.

One category of gridless sparse recovery methods, atomic norm minimization (ANM) [6, 7], provides a new perspective of super-resolution parameter estimation. Combined with the Caratheodory-Toeplitz theorem, [6] proposes an ANM framework for parameter estimation problem in single measurement vector (SMV) condition. In addition, the framework is also applied to the sparse spatial line spectrum estimation. [7] extends the model to the multiple measurement vectors (MMVs) case. Another category of gridless sparse recovery methods is based on covariance matching criteria (CMC). The sparse iterative covariance-based estimation (SPICE) method is proposed for gridless DOA estimation at single snapshot [8], while [9] aims at the DOA estimation for multiple snapshots.

The second issue is that the traditional DOA estimation methods will suffer severe performance degradation for they don't take the unknown mutual coupling effects between the sensors into consideration. On the one hand, the mutual coupling effects are eliminated with the auxiliary sensors [10, 11], thus the conventional MUSIC and ESPRIT methods are valid for angle estimation after such preprocessing. On the other hand, T. Svantesson points out that for a uniform linear array (ULA), the mutual coupling matrix (MCM) can be modelled as a banded symmetric Toeplitz matrix [12]. Algorithm explores the inherent structure of the received data is proposed for the first time in [13], in which the center of the received data is chosen for DOA estimation based on ESPRIT. In the further study, the special Toeplitz structure of the MCM of a ULA is also employed to parameterize the steering vector for joint estimation of DOAs and MCM in [14].

The latest algorithms tackling the DOA estimation with unknown mutual coupling exploit both the Toeplitz structure of mutual coupling matrix and the sparsity of the DOA in entire space spectrum. [15] firstly calibrates the array manifold by making use of the inherent mechanism in a sparse representation perspective. While [16] further exploits the Toeplitz structure of MCM and solves the block-sparsity based convex problem by second-order cone programming.

In this paper, an effective gridless DOA estimation method is proposed, dealing with both the super-resolution and the unknown mutual coupling problems. By constructing a new clean steering vector with the parameterizing method, we build a new atomic set and make full use of the gridless

This work is sponsored the National Natural Science Foundation of China under Grant (No. 61871282), and by the Civil Aerospace Foundation, and by Zhejiang Provincial Natural Science Foundation of China under Grant (No. LQ19F010002). Email: {*dkchenhua0714@hotmail.com.}

property to achieve high resolution. The ANM problem is transformed to be convex problem, and then solved using SDP. Both SMV and MMVs cases are considered to demonstrate the effectiveness of the proposed method.

Notations: $[\cdot]^H$ denotes the matrix and vector conjugate and transpose; $diag[\cdot]$ stands for the diagonalization operation of matrix blocks; $||\cdot||_p$ denotes the p-norm of a matrix; $|\cdot|$ is the absolute value of a scalar; $[\cdot]_{M\times N}$ indicates a matrix of M rows and N columns; $tr(\cdot)$ denotes the trace of a square matrix; $\mathbf{T}(\cdot)$ and $\mathbf{Toeplitz}(\cdot)$ create a Hermitian Toeplitz matrix and a banded symmetric Toeplitz matrix out of their inputs, respectively.

2. PROBLEM FORMULATION

2.1. The Data Model for SMV

Assume a ULA consists of M isotropic antenna elements with spacing d between adjacent sensors. N far-field narrowband signals $s_n(t)$, $n = 1, 2, \cdots, N$, are arriving at the array from directions $\theta_1, \theta_2, \cdots, \theta_N$, where t is the sample index, with $t = 1, 2, \cdots, T$.

For an ideal array without mutual coupling, the received vector of the array at one snapshot can be formulated as

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_M(t)]^T$ denotes the M received antenna signals. $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_N(t)]^T$ is the source signal vector. $\mathbf{n}(t) = [n_1(t), n_2(t), \cdots, n_M(t)]^T$ represents the independent and identically distributed additive white Gaussian noise vector with zero mean and variance σ_n^2. The array steering matrix is $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_N)]$, where $\mathbf{a}(\theta_n) = [1, \hat{\beta}(\theta_n), \cdots, \hat{\beta}(\theta_n)^{M-1}]^T$ denotes the nth signals ideal steering vector with $\hat{\beta}(\theta_n) = \exp(j\frac{2\pi d \sin\theta_n}{\lambda})$. In this paper, the spaced distance d is assumed to be equal to half of the wavelength λ, i.e., $d = \lambda/2$.

Define a mapping function $f_n = \frac{1+\sin(\theta_n)}{2} \in [0, 1]$ when the direction $\theta_n \in [-90^o, 90^o]$. In the following discussion, we mainly concentrate on the estimation of spatial frequency f_n, then θ_n can be obtained after a trivial transformation. Thus the array steering matrix $\mathbf{A}(\theta)$ can be rewritten as $\mathbf{A} = [\mathbf{a}(f_1), \mathbf{a}(f_2), \cdots, \mathbf{a}(f_N)]$, where $\mathbf{a}(f_n) = [1, \beta(f_n), \cdots, \beta(f_n)^{M-1}]^T$ and $\beta(f_n) = \exp(j2\pi f_n)$.

For practical realization, we take the mutual coupling effect among the neighbouring sensors into consideration. Then the received data model for SMV can be modified as

$$\mathbf{x}(t) = \mathbf{C}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

where $\mathbf{C}$ is the MCM. For ULAs, the MCM can be modelled as a $M \times M$ banded symmetric Toeplitz matrix as

$$\begin{aligned} &\mathbf{C} = \mathbf{Toeplitz}(\mathbf{c}), \\ &\mathbf{c} = [1, c_1, c_2, \ldots, c_p, \ldots, c_{P-1}, 0, \ldots, 0], \end{aligned} \tag{3}$$

where c_p denotes the mutual coupling coefficient between the mth and the $(m+p)$th sensor with $p = 0, 1, \cdots, P-1$, $m = 1, 2, \cdots, M$. And for the mth sensor, it will be disturbed by the mutual coupling effects coming from the $(m-P+1)$th, $\cdots$, $(m-1)$th, $(m+1)$th, $\cdots$, $(m+P-1)$th sensors, while the mutual coupling effects of the sensors far than P are too weak hence can neglected for the simplification of analysis.

2.2. The Data Model for MMVs

For multiple snapshots, the T subsequent observations considering the unknown mutual coupling can be modeled as

$$\mathbf{X} = \mathbf{C}\mathbf{A}\mathbf{S} + \mathbf{N} \in \mathbb{C}^{M\times T}, \tag{4}$$

where $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \cdots, \mathbf{x}(T)] \in \mathbb{C}^{M\times T}$, $\mathbf{S} = [\mathbf{s}(1), \mathbf{s}(2), \cdots, \mathbf{s}(T)] \in \mathbb{C}^{N\times T}$, $\mathbf{N} = [\mathbf{n}(1), \mathbf{n}(2), \cdots, \mathbf{n}(T)] \in \mathbb{C}^{M\times T}$ refer to the received matrix, transmitted matrix and noise matrix with dimension of $M \times T$ for MMVs, respectively. $\mathbf{A}$ and $\mathbf{C}$ are as defined in SMV case.

3. THE PROPOSED METHOD

In this section, the proposed method for DOA estimation based on parameterization of the steering vector and atomic norm minimization will be introduced.

3.1. Parameterization of the Steering Vector with Mutual Coupling

By analysing the structure of mutual coupling matrix $\mathbf{C}$ in equation (3), we note that the center part of $\mathbf{C}$ is cyclic with the whole unknown mutual coupling coefficients. Inspired by [15], a selection matrix $\mathbf{F} = [\mathbf{0}_{[M-2(P-1)]\times(P-1)} \mathbf{I}_{M-2(P-1)} \mathbf{0}_{[M-2(P-1)]\times(P-1)}]$ can be constructed to truncate the received data as follows:

$$\begin{aligned} \bar{\mathbf{x}}(t) = \mathbf{F}\mathbf{x}(t) &= \mathbf{F}\mathbf{C}\mathbf{A}\mathbf{s}(t) + \mathbf{F}\mathbf{n}(t) \\ &= \bar{\mathbf{C}}\mathbf{A}\mathbf{s}(t) + \mathbf{F}\mathbf{n}(t), \end{aligned} \tag{5}$$

where $\bar{\mathbf{C}} = \mathbf{F}\mathbf{C}$ is the center part of the original MCM $\mathbf{C}$.

$$\bar{\mathbf{C}} = \begin{bmatrix} c_{P-1} & \cdots & c_1 & 1 & c_1 & \cdots & c_{P-1} & 0 & 0 \\ 0 & \ddots & \vdots & \ddots & \ddots & \ddots & \ddots & 0 & 0 \\ 0 & 0 & c_{P-1} & \cdots & c_1 & 1 & c_1 & \cdots & c_{P-1} \end{bmatrix}_{[M-2(P-1)]\times M} \tag{6}$$

By exploring the inherent structure of $\bar{\mathbf{C}}$, the truncated steering vector with mutual coupling can be modified as

$$\tilde{\mathbf{a}}(f) = \bar{\mathbf{C}}\mathbf{a}(f) = \mathrm{H}(f)\bar{\mathbf{a}}(f), \tag{7}$$

where $\bar{\mathbf{a}}(f) = [1, \beta(f), \beta(f)^2, \cdots, \beta(f)^{M-2P+1}]^T$ and $\mathrm{H}(f) = \sum_{l=1-P}^{P-1} c_{|l|}\beta(f)^{l+P-1}$.

Note that $\mathrm{H}(f)$ is a scalar parameter related to the mutual coupling coefficients and DOAs. It may take a zero value for some very specific cases. However, in general, it is not zero-valued and we assume $\mathrm{H}(f) \neq 0$, $f \in [0, 1]$ in the following discussion. Then, (5) can be further reformulated as

$$\bar{\mathbf{x}}(t) = \bar{\mathbf{A}}\mathbf{\Gamma}\mathbf{s}(t) + \mathbf{F}\mathbf{n}(t), \tag{8}$$

where $\mathbf{\Gamma} = \begin{bmatrix} \mathrm{H}(f_1) & & & 0 \\ & \mathrm{H}(f_2) & & \\ & & \ddots & \\ 0 & & & \mathrm{H}(f_N) \end{bmatrix} \in \mathbb{C}^{N\times N}$ and the new array manifold $\bar{\mathbf{A}} = [\bar{\mathbf{a}}(f_1), \bar{\mathbf{a}}(f_2), \cdots, \bar{\mathbf{a}}(f_N)]$.

Note that $\mathbf{\Gamma}$ is a diagonal matrix with $\mathrm{H}(f) \neq 0$, $f \in [0,1]$. Define $\bar{\mathbf{s}}(t) = \mathbf{\Gamma}\mathbf{s}(t)$, then we obtain

$$\bar{\mathbf{x}}(t) = \bar{\mathbf{A}}\bar{\mathbf{s}}(t) + \mathbf{F}\mathbf{n}(t). \tag{9}$$

Similarly, for the MMVs model, the reconstructed received matrix is

$$\bar{\mathbf{X}} = \bar{\mathbf{A}}\bar{\mathbf{S}} + \mathbf{F}\mathbf{N} \in \mathbb{C}^{[M-2(P-1)]\times T}, \tag{10}$$

where $\bar{\mathbf{S}} = \mathbf{\Gamma}\mathbf{S} \in \mathbb{C}^{N\times T}$.

3.2. Proposed Atomic Norm Minimization Method

3.2.1. Atomic Norm Minimization Method in SMV Model

According to (9), we can rewrite the noise-free measurement vector as follows

$$\bar{\mathbf{x}}_0(t) = \bar{\mathbf{A}}\bar{\mathbf{s}}(t) = \sum_{i=1}^{N} \bar{\mathbf{a}}(f_i)\bar{s}_i(t) = \sum_{i=1}^{N} v_i\bar{\mathbf{a}}(f_i)\phi_i \quad , \tag{11}$$

where $\bar{s}_i(t)$ denotes the ith element of $\bar{\mathbf{s}}(t)$. $v_i = |\bar{s}_i(t)| > 0$ and $\phi_i = v_i^{-1}\bar{s}_i(t)$ with $|\phi_i| = 1$.

In consistence with the ANM framework [6], which exploits the sparsity in the continuous parameter space by means of the atomic norm metric, we define the atoms

$$\hat{\mathbf{a}}(f,\phi) = \bar{\mathbf{a}}(f)\phi \quad , \tag{12}$$

where $f \in [0,1]$ and $\phi \in \mathbb{C}$ with $|\phi| = 1$.

With $\hat{\mathbf{a}}(f,\phi)$ being the atom, the continuous dictionary, also termed atomic set, is given by

$$\mathcal{A} = \{\hat{\mathbf{a}}(f,\phi) | f \in [0,1], \phi \in \mathbb{C}, |\phi| = 1\}, \tag{13}$$

and the atomic norm of $\bar{\mathbf{x}}_0$ is defined as

$$\| \bar{\mathbf{x}}_0\|_{\mathcal{A}} = \inf\{\sum_i v_i | \sum_{i=1}^{K} v_i\hat{\mathbf{a}}(f_i,\phi_i), v_i \geq 0\}. \tag{14}$$

Again following [6], the equivalent SDP formulation of (14) is

$$\begin{aligned} \min \quad & \tfrac{1}{2}(t+u_1) \\ \text{s.t.} \quad & \begin{bmatrix} t & \bar{\mathbf{x}}_0^{\mathrm{H}} \\ \bar{\mathbf{x}}_0 & \mathbf{T}(\mathbf{u}) \end{bmatrix} \succeq 0 \quad , \end{aligned} \tag{15}$$

where $t > 0$, $\mathbf{T}(\mathbf{u})$ is a Hermitian Toeplitz matrix with $\mathbf{u} \in \mathbb{C}^{M-2(P-1)}$ and u_1 is the first element of $\mathbf{u}$.

According to equation (9), we estimate $\hat{\mathbf{x}}(t)$ by solving

$$\min_{\hat{\mathbf{x}}} \quad \frac{1}{2} \| \hat{\mathbf{x}} - \bar{\mathbf{x}} \|_2^2 + \tau \| \hat{\mathbf{x}}\|_{\mathcal{A}} \quad , \tag{16}$$

where τ is a regularization parameter.

Then the corresponding SDP formulation of (16) is

$$\begin{aligned} \min_{\hat{\mathbf{x}},t,\mathbf{u}} \quad & \tfrac{1}{2} \| \hat{\mathbf{x}} - \bar{\mathbf{x}} \|_2^2 + \tfrac{\tau}{2}(t+u_1) \\ \text{s.t.} \quad & \begin{bmatrix} t & \hat{\mathbf{x}}^H \\ \hat{\mathbf{x}} & \mathbf{T}(\mathbf{u}) \end{bmatrix} \succeq 0 \quad . \end{aligned} \tag{17}$$

With (17) being solved, the solution $\mathbf{T}(\mathbf{u})$ is obtained. As the Caratheodory theorem [6] states that any positive semidefinite Toeplitz matrix can be represented by a unique Vandermonde decomposition, $\mathbf{T}(\mathbf{u})$ is decomposed as

$$\begin{aligned} \mathbf{T}(\mathbf{u}) &= \bar{\mathbf{A}}\mathbf{Z}\bar{\mathbf{A}}^H \\ &= \sum_{i=1}^{N} z_i\bar{\mathbf{a}}(f_i)\bar{\mathbf{a}}^H(f_i) \quad , \end{aligned} \tag{18}$$

where $\mathbf{Z} = diag(\mathbf{z}), \mathbf{z} = [z_1, z_2, \cdots, z_N]^T$ contains the coefficients $z_i > 0$ $(i = 1, \cdots, N)$ on its diagonal.

The standard ESPRIT method can be applied to $\mathbf{T}(\mathbf{u})$ to obtain the final estimated spatial frequencies $f_i, i = 1, 2, \cdots, N$.

3.2.2. Atomic Norm Minimization Method in MMVs Model

For the multiple snapshots, the noise-free measurement matrix is

$$\bar{\mathbf{X}}_0 = \bar{\mathbf{A}}\bar{\mathbf{S}} = \sum_{i=1}^{N} \bar{\mathbf{a}}(f_i)\bar{\mathbf{s}}_i^T = \sum_{i=1}^{N} r_i\bar{\mathbf{a}}(f_i)\mathbf{\Phi}_i \quad , \tag{19}$$

where $\bar{\mathbf{s}}_i^T$ is the ith row of $\bar{\mathbf{S}}$, $r_i = \| \bar{\mathbf{s}}_i^T\|_2 > 0$ and $\mathbf{\Phi}_i = r_i^{-1}\bar{\mathbf{s}}_i^T$ with $\| \mathbf{\Phi}_i\|_2 = 1$.

The atom of MMVs model can be written as

$$\hat{\mathbf{a}}_M(f,\mathbf{\Phi}) = \bar{\mathbf{a}}(f)\mathbf{\Phi} \quad , \tag{20}$$

where $f \in [0,1], \mathbf{\Phi} \in \mathbb{C}^{1\times \mathrm{T}}, \| \mathbf{\Phi}\|_2 = 1$.

Then the set of atoms is

$$\mathcal{A} = \{\hat{\mathbf{a}}_M(f,\mathbf{\Phi}) | f \in [0,1], \mathbf{\Phi} \in \mathbb{C}^{1\times \mathrm{T}}, \| \mathbf{\Phi}\|_2 = 1\}, \tag{21}$$

and the atomic norm of $\bar{\mathbf{X}}_0$ is

$$\| \bar{\mathbf{X}}_0\|_{\mathcal{A}} = \inf\{\sum_k r_k | \sum_{i=1}^{K} r_k\hat{\mathbf{a}}_M(f_i,\mathbf{\Phi}_i), r_k \geq 0\}. \tag{22}$$

The corresponding SDP formulation of (22) is

$$\begin{aligned} \min_{\mathbf{W},\mathbf{u}} \quad & \tfrac{1}{2\sqrt{M}}(tr(\mathbf{W}) + tr(\mathbf{T}(\mathbf{u}))) \\ \text{s.t.} \quad & \begin{bmatrix} \mathbf{W} & \bar{\mathbf{X}}_0^{\mathrm{H}} \\ \bar{\mathbf{X}}_0 & \mathbf{T}(\mathbf{u}) \end{bmatrix} \succeq 0 \quad , \end{aligned} \tag{23}$$

where $\mathbf{W} \in \mathbb{C}^{T\times T}$.

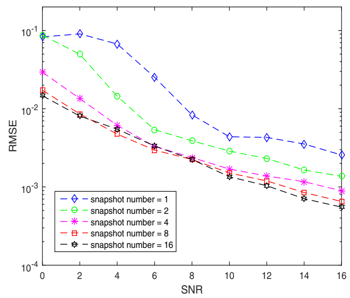

Fig. 1: The RMSE of SMV and MMVs versus SNR.

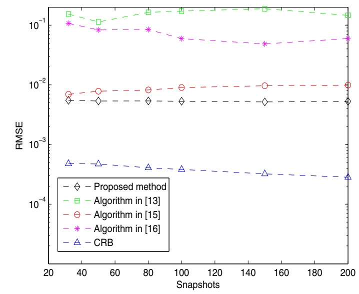

Fig. 2: The RMSEs versus snapshot number.

Based on (10) and (23), the final SDP problem for MMVs model is

$$\begin{array}{ll} \min\limits_{\hat{\mathbf{X}},\mathbf{W},\mathbf{u}} & \frac{1}{2} \parallel \hat{\mathbf{X}} - \bar{\mathbf{X}} \parallel_2^2 + \frac{\tau}{2\sqrt{M}}(tr(\mathbf{W}) + tr(\mathbf{T}(\mathbf{u}))) \\ \text{s.t.} & \begin{bmatrix} \mathbf{W} & \hat{\mathbf{X}}^{\mathrm{H}} \\ \hat{\mathbf{X}} & \mathbf{T}(\mathbf{u}) \end{bmatrix} \succeq 0 \end{array} . \quad (24)$$

After obtaining the optimal solution $\mathbf{T}(\mathbf{u})$, again the standard ESPRIT method is utilized to estimate the DOAs.

4. SIMULATION RESULTS

In this section, some simulations are demonstrated to verify the performance of the proposed method. For all simulations, a ULA consists of $M = 15$ sensors with three far-field narrowband sources arriving at directions f_1, f_2, f_3 is employed. The mutual coupling coefficients are c_1, c_2, c_3 with $P = 4$. The Root Mean Square Error (RMSE) is adopted as a performance metric.

In the first set of simulation, $f_1 = 0.1, f_2 = 0.4, f_3 = 0.7$ and $c_1 = 0.4864 - 0.4776j, c_2 = 0.2325 + 0.1914j, c_3 = 0.1163 - 0.1089j$. Fig. 1 compares the performance between the SMV model and the MMVs model with 100 Monte-Carlo simulations. It can be seen that the SMV model has lower accuracy than the MMVs model. However, as the snapshots increase, the performance improvements are less noticeable but more time-consuming.

To explore the super-resolution performance of the proposed gridless method, the second set of simulation is performed in the scenario that $f_1 = 0.15, f_2 = 0.19, f_3 = 0.80$ and $c_1 = 0.5844 - 0.5476j, c_2 = 0.2625 + 0.1414j, c_3 = 0.1163 - 0.1289j$. Besides, the SSR based methods in [15, 16] and the ESPRIT algorithm in [13] are also provided as comparisons. When SNR is fixed at 10dB, the RMSEs with 100 Monte-Carlo simulations for the super-resolution test versus snapshots is shown in Fig. 2. From the result we can see that the proposed method has a consistently reasonable resolution in the super-resolution test. No need for reticence, the resolution of the method in [15] is pretty close to that of the proposed method. But it ought to be pointed out that the result still implies that the proposed method are quite suitable for parameter estimation with fewer snapshots, which is significant in the practical array signal processing.

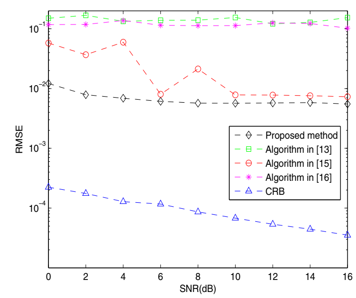

Fig. 3: The RMSEs versus SNR.

In the last set of experiment, further simulation is demonstrated to confirm the super-resolution performance of the proposed method versus SNR. The relevant parameter settings are the same with the second group of experiment, except for the snapshot number is equal to 16 while the SNR varies from 0dB to 16dB. As shown in Fig. 3, the proposed method has a stable resolution in the simulation and it has achieved the best performance in the lower SNR.

5. CONCLUSIONS

In this paper, an effective gridless DOA estimation method simultaneously considering about the super-resolution problem and the mutual coupling effect is proposed. The proposed method exploits the inherent structure of the received data and the MCM. Following the steps of the ANM framework, a new steering vector is formed by truncating the original received data and then assimilating the unknown mutual coupling coefficients into the signal part. Finally, an equivalent SDP problem is derived to complete the DOA estimation. Given that the ANM is a parameter estimation method in the continuous domain, the proposed method achieves the super-resolution and demonstrates a superior performance over existing methods in the case of low SNR and fewer snapshots, which is verified by simulation results. In the future study, the property (such as the noncircularity [17]) of the signal can be further exploited to enhance the performance of the proposed method.

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