Group Sparsity Based Target Localization for Distributed Sensor Array Networks

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***Abstract*—The target localization problem for distributed sensor array networks where a sub-array is placed at each receiver is studied, and under the compressive sensing (CS) framework, a group sparsity based two-dimensional localization method is proposed. Instead of fusing the separately estimated angles of arrival (AOAs), it processes the information collected by all the receivers simultaneously to form the final target locations. Simulation results show that the proposed localization method provides a significant performance improvement compared with the commonly used maximum likelihood estimator (MLE).**

***Index Terms*—Distributed sensor array network, group sparsity, localization, angle of arrival, compressive sensing.**

I. Introduction

The angle of arrival (AOA) based target localization (also known as the bearing only localization), where the synchronization across distributed receivers is not required compared with the received signal strength (RSS) based localization [1] and the time of arrival (TOA) based localization [2], has attracted increasing attentions in distributed sensor array networks [3]–[5], and it has been widely applied in radar, sonar, massive MIMO, and wireless sensor networks [3]–[8].

In AOA based localization, the AOAs are obtained at independent distributed receivers using a sub-array, where traditional DOA estimation methods can be employed to acquire the AOA information, such as the subspace based methods including MUSIC [9], ESPRIT [10], and their extensions. On the other hand, the compressive sensing (CS) framework has been introduced for DOA estimation [11]–[14], with the ℓ_1-SVD method based on singular value decomposition (SVD) proposed in [15] and ℓ_1-SRACV based on a sparse representation of array covariance vectors presented in [16]. For wideband DOA estimation, apart from the typical methods such as the incoherent signal subspace method (ISSM) [17], the coherent signal subspace method (CSSM) [18], and the test of orthogonality of projected subspaces (TOPS) method [19], the group sparsity concept can be employed under the CS framework [20]–[22] by exploiting all the information across the frequencies of interest simultaneously to form a more effective solution.

These AOA measurements obtained from individual receivers are then fused together to localize the target with reference to the distributed sensor array network [5], [7], [23]. The maximum likelihood estimator (MLE) is the most commonly used fusion method, where the total errors of the AOA measurements is minimized under the least square sense through a direct grid search method [24], [25]. A number of iterative methods [26] and closed-form location estimators [8], [27] have been proposed for complexity reduction. However, all the aforementioned fusion methods employ the maximum likelihood (ML) criteria based on the AOA measurements processed independently at receivers.

In this paper, we focus on the target localization problem in a distributed sensor array network, and a linear sub-array is placed at each receiver. Although the targets are near-field compared to the entire network, they are far-field as observed from each linear sub-array at receivers. Instead of fusing preprocessed AOAs together for localization, we propose a novel two-dimensional group sparsity based localization method, where the collected information at all receivers are exploited simultaneously by applying the group sparsity concept under the CS framework to estimate the target locations directly, and a better performance can be achieved by the proposed solution compared with the commonly used MLE.

This paper is structured as follows. The distributed sensor array network is introduced in Section II, and the developed group sparsity based two-dimensional target localization method is proposed in Section III. Simulation results are provided in Section IV, and conclusions are drawn in Section V.

II. System Model

Consider a general distributed sensor array network as shown in Fig. 1, where there are M receivers with positions $\mathrm{U}_m(x_m, y_m)$, $m = 1, 2, \ldots, M$, and K targets located at $\mathrm{T}_k(x_{T_k}, y_{T_k})$, $k = 1, 2, \ldots, K$.

For each receiver, a linear sub-array with L_m sensors is employed, as shown in Fig. 2, and the set of sensor positions

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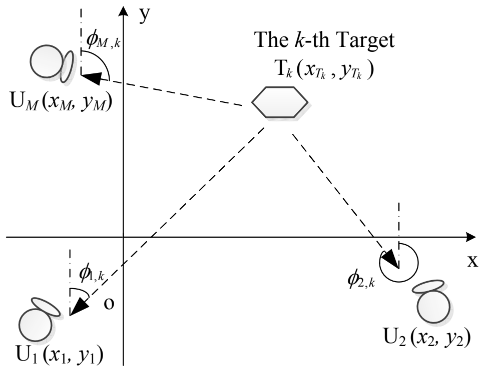


Fig. 1. A typical localization geometry for a distributed sensor array network.

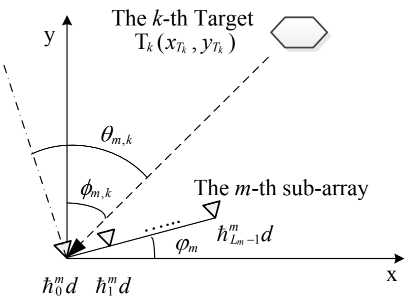


Fig. 2. A general array structure for the m-th sub-array carried by the corresponding receiver.

at each receiver is expressed as

$$\mathbb{S}_m = \{\hbar^m_l d,\ 0 \leq l \leq L_m - 1, l \in \mathbb{Z}\}\ , \tag{1}$$

where $\mathbb{Z}$ is the set of all integers, $\hbar^m_l d$ is the position of the l-th sensor, and the spacing between adjacent physical sensors $d \leq \lambda/2$ with λ being the signal wavelength.

We use $\phi_{m,k}$ to represent the angle measured between the direction from the k-th target to the m-th receiver and the y-axis, given by

$$\begin{aligned}\phi_{m,k} &= \arctan 2(\Delta x_{m,k}, \Delta y_{m,k}) \\ &= \begin{cases} \arctan(\frac{\Delta x_{m,k}}{\Delta y_{m,k}}), & \Delta y_{m,k} > 0, \\ \arctan(\frac{\Delta x_{m,k}}{\Delta y_{m,k}}) + \pi, & \Delta x_{m,k} \geq 0, \Delta y_{m,k} < 0, \\ \arctan(\frac{\Delta x_{m,k}}{\Delta y_{m,k}}) - \pi, & \Delta x_{m,k} < 0, \Delta y_{m,k} < 0, \\ +\frac{\pi}{2}, & \Delta x_{m,k} > 0, \Delta y_{m,k} = 0, \\ -\frac{\pi}{2}, & \Delta x_{m,k} < 0, \Delta y_{m,k} = 0, \\ \text{undefined}, & \Delta x_{m,k} = 0, \Delta y_{m,k} = 0, \end{cases}\end{aligned} \tag{2}$$

where $\Delta x_{m,k} = x_{T_k} - x_m$, and $\Delta y_{m,k} = y_{T_k} - y_m$. $\arctan 2(a, b) \in (-\pi, \pi]$ is the four-quadrant inverse tangent of a and b, with $\arctan(\frac{a}{b})$ returning the inverse tangent of $\frac{a}{b}$.

Define φ_m (measured between the end-fire direction of the linear sub-array and the x-axis) as the rotation angle of the m-th sub-array. Clearly, the incident angle of the received signal from the k-th target based on the m-th sub-array is $\theta_{m,k} = \phi_{m,k} + \varphi_m$. Then we denote $\mathbf{x}_m[i]$ as the $L_m \times 1$ observed signal vector at the m-th sub-array after sampling with a frequency f_s, and we have

$$\mathbf{x}_m[i] = \mathbf{A}_m(\boldsymbol{\theta}_m)\mathbf{s}_m[i] + \bar{\mathbf{n}}_m[i]\ , \tag{3}$$

where $\mathbf{s}_m(t) = [s_{m,1}[i], s_{m,2}[i], \ldots, s_{m,K}[i]]^T$ is the signal vector with $s_{m,k}[i]$ representing the signal from the k-th target received at the m-th sub-array, and $\{\cdot\}^T$ denotes the transpose operation. $\bar{\mathbf{n}}_m[i]$ is the noise vector at the corresponding sub-array, while $\mathbf{A}_m(\boldsymbol{\theta}_m) = [\mathbf{a}_m(\theta_{m,1}), \mathbf{a}_m(\theta_{m,2}), \ldots, \mathbf{a}_m(\theta_{m,K})]$ is the steering matrix, with the steering vector $\mathbf{a}(\theta_{m,k}, t)$ corresponding to the k-th target expressed as

$$\mathbf{a}_m(\theta_{m,k}) = \left[e^{-j\frac{2\pi\hbar^m_0 d}{\lambda}\sin(\theta_{m,k})}, \ldots, e^{-j\frac{2\pi\hbar^m_{L_m-1}d}{\lambda}\sin(\theta_{m,k})}\right]^T. \tag{4}$$

III. Group Sparsity Based Two-Dimensional Target Localization

A. AOA Estimation Based on Sparse Representation of Array Covariance Vectors for a Single Sub-Array

Based on the signal model at each receiver in (3), we calculate the covariance matrix as

$$\begin{aligned}\mathbf{R}_{\mathbf{x}_m} &= \mathrm{E}\left\{\mathbf{x}_m[i]\mathbf{x}^H_m[i]\right\} \\ &= \mathbf{A}_m(\boldsymbol{\theta}_m)\mathbf{R}_{\mathbf{s}_m}\mathbf{A}^H_m(\boldsymbol{\theta}_m) + \sigma^2_m\mathbf{I}_{L_m}\ ,\end{aligned} \tag{5}$$

where $\mathrm{E}\{\cdot\}$ is the expectation operator, and $\{\cdot\}^H$ is the Hermitian transpose. σ^2_m represents the noise power at the m-th receiver, $\mathbf{R}_{\mathbf{s}_m} = \mathrm{E}\left\{\mathbf{s}_m[i]\mathbf{s}^H_m[i]\right\}$ is the covariance matrix of $\mathbf{s}_m[i]$, and $\mathbf{I}_{L_m}$ is the identity matrix with size of $L_m \times L_m$.

By defining $\mathbf{P}_m = \mathbf{R}_{\mathbf{s}_m}\mathbf{A}^H_m(\boldsymbol{\theta}_m)$, the covariance matrix can be rewritten as

$$\mathbf{R}_{\mathbf{x}_m} = \mathbf{A}_m(\boldsymbol{\theta}_m)\mathbf{P}_m + \sigma^2_m\mathbf{I}_{L_m}\ . \tag{6}$$

To estimate the AOA results under the CS framework, we first generate an overcomplete representation of the steering matrix based on a search grid of K_g ($K_g \gg K$) potential incident angles $\theta_{g,0}, \theta_{g,1}, \ldots, \theta_{g,K_g-1}$, given as

$$\mathbf{A}_m(\boldsymbol{\theta_g}) = \left[\mathbf{a}_m(\theta_{g,0}), \mathbf{a}_m(\theta_{g,1}) \ldots, \mathbf{a}_m(\theta_{g,K_g-1})\right]\ . \tag{7}$$

Then, we construct a $K_g \times L_m$ matrix $\mathbf{P}_{\mathbf{g},m}$ consisting of all entries to be estimated, and $\mathbf{p}^{k_g}_m$ is used to represent the k_g-th row vector of $\mathbf{P}_{\mathbf{g},m}$. Denote

$$\mathbf{p}_{\mathbf{g},m} = \left[\left\|\mathbf{p}^0_m\right\|_2, \left\|\mathbf{p}^1_m\right\|_2, \ldots, \left\|\mathbf{p}^{K_g-1}_m\right\|_2\right]^T\ , \tag{8}$$

where $\|\cdot\|_2$ is the ℓ_2 norm.

The AOA estimation method based on sparse representation of array covariance vectors (ℓ_1-SRACV) [16] can be expressed as

$$\begin{aligned}\min_{\mathbf{P}_{\mathbf{g},m}, \sigma^2_m} \quad & \left\|\mathbf{p}^\circ_{\mathbf{g},m}\right\|_1 \\ \text{subject to} \quad & \left\|\mathbf{R}_{\mathbf{x}_m} - \mathbf{A}_m(\boldsymbol{\theta_g})\mathbf{P}_{\mathbf{g},m} - \sigma^2_m\mathbf{I}_{L_m}\right\|_F \leq \varepsilon\ ,\end{aligned} \tag{9}$$

where $\|\cdot\|_1$ is the ℓ_1 norm, $\|\cdot\|_F$ is the Frobenius norm, $\mathbf{p}^\circ_{\mathbf{g},m} = [\mathbf{p}^T_{\mathbf{g},m}, \sigma^2_m]^T$, and ε is the allowable error bound.

It is noted the entries in $\mathbf{p}_{\mathbf{g},m}$ are the corresponding AOA estimation results over the K_g search grids, while the noise power σ_m^2 is also considered as unknown variable to be estimated. It is also noted that the AOA results obtained from solving the optimization problem in (9), i.e., $\tilde{\boldsymbol{\theta}}_m$, can be converted to the predefined Cartesian coordinate system by $\tilde{\boldsymbol{\phi}}_m = \tilde{\boldsymbol{\theta}}_m - \varphi_m$.

B. Proposed Method Employing the Group Sparsity Concept

For each target located at $T_k(x_{T_k}, y_{T_k})$, a unique incident angle $\theta_{m,k}$ can be obtained and triangulation can be applied for localization. To form a more effective solution, we propose a group sparsity based two-dimensional localization method, referred to as GS-Localization, where a common sparse structure across all the receivers is enforced and therefore the information collected by all sub-arrays can be processed as a whole.

Without loss of generality, we assume that the area of interest in the predefined Cartesian coordinate system is a square shape. It is divided into K_xK_y grids with K_x and K_y being the number of grid points along the x-axis and the y-axis, respectively. $G(x_{k_x}, y_{k_y})$ ($k_x = 0, 1, \ldots, K_x - 1$ and $k_y = 0, 1, \ldots, K_y - 1$) is used to represent the location of the (k_x, k_y)-th search grid, and the angle $\theta_{g,m}(k_x, k_y)$ corresponding to the grid $G(x_{k_x}, y_{k_y})$ and the m-th sub-array is obtained by

$$\theta_{g,m}(k_x, k_y) = \arctan 2(\Delta x_{m,k_x}, \Delta y_{m,k_y}) + \varphi_m \ , \quad (10)$$

where

$$\begin{aligned} \Delta x_{m,k_x} &= x_{k_x} - x_m \ , \\ \Delta y_{m,k_y} &= y_{k_y} - y_m \ . \end{aligned} \quad (11)$$

Then for each sub-array, we can obtain its array model under the CS framework in the two-dimensional case, expressed as

$$\mathbf{R}_{\mathbf{x}_m} = \mathbf{A}_m(\tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m})\tilde{\mathbf{P}}_{\mathbf{g},m} + \sigma_m^2 \mathbf{I}_{L_m} \ , \quad (12)$$

where $\tilde{\mathbf{P}}_{\mathbf{g},m}$ is a $K_xK_y \times L_m$ matrix consisting of all variables to be estimated, and its each row vector corresponds to the grid at the same row in $\tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m}$. $\tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m}$ is a $K_xK_y \times 1$ column vector including incident angles at all potential grids, given by

$$\begin{aligned} \tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m} = \big[&\theta_{g,m}(0,0), \theta_{g,m}(0,1), \ldots, \theta_{g,m}(0, K_y - 1), \\ &\theta_{g,m}(1,0), \theta_{g,m}(1,1), \ldots, \theta_{g,m}(1, K_y - 1), \\ &\ldots\ldots \\ &\theta_{g,m}(K_x - 1, 0), \ldots, \theta_{g,m}(K_x - 1, K_y - 1)\big]^T . \end{aligned} \quad (13)$$

Based on the array output models, target localization can be performed by forcing the common sparsity in the area of interest among all receivers due to the uniqueness of the incident angle group for each grid $G(x_{k_x}, y_{k_y})$.

By vectorizing the signal covariance matrix $\mathbf{R}_{\mathbf{x}_m}$ in (6), we obtain

$$\tilde{\mathbf{z}}_m = \mathrm{vec}\left\{\mathbf{R}_{\mathbf{x}_m}\right\} = \mathrm{vec}\left\{\mathbf{A}_m(\boldsymbol{\theta}_m)\mathbf{P}_m + \sigma_m^2 \mathbf{I}_{L_m}\right\} \ . \quad (14)$$

Then, based on the array signal model under the CS framework in (12), we generate a $K_xK_yL \times 1$ column vector $\tilde{\mathbf{b}}_{\mathbf{g}}$ and a $K_xK_y \times L$ matrix $\tilde{\mathbf{U}}_{\mathbf{g}}$ as

$$\begin{aligned} \tilde{\mathbf{b}}_{\mathbf{g}} &= \left[\tilde{\mathbf{b}}_{\mathbf{g},1}^T, \tilde{\mathbf{b}}_{\mathbf{g},2}^T, \ldots, \tilde{\mathbf{b}}_{\mathbf{g},M}^T\right]^T \ , \\ \tilde{\mathbf{U}}_{\mathbf{g}} &= \left[\tilde{\mathbf{P}}_{\mathbf{g},1}, \tilde{\mathbf{P}}_{\mathbf{g},2}, \ldots, \tilde{\mathbf{P}}_{\mathbf{g},M}\right] \ , \end{aligned} \quad (15)$$

where

$$\tilde{\mathbf{b}}_{\mathbf{g},m} = \mathrm{vec}\left\{\mathbf{A}_m(\tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m})\tilde{\mathbf{P}}_{\mathbf{g},m} + \sigma_m^2 \mathbf{I}_{L_m}\right\} \ , \quad (16)$$

and row vector $\tilde{\mathbf{u}}_{\mathbf{g}}^{k_g}$, $0 \le k_g \le K_xK_y - 1$, is the k_g-th row of $\tilde{\mathbf{U}}_{\mathbf{g}}$.

All the elements in $\tilde{\mathbf{u}}_{\mathbf{g}}^{k_g}$ with $k_g = k_x \cdot K_x + k_y$ are associated with the same grid $G(x_{k_x}, y_{k_y})$, sharing the same spatial support in the Cartesian coordinate system, and therefore the group sparsity concept can be applied for target localization.

Finally, the group sparsity based two-dimensional target localization method (GS-Localization) is formulated as

$$\begin{aligned} \min_{\tilde{\mathbf{U}}_{\mathbf{g}}, \sigma_m^2} \quad & \left\|\tilde{\mathbf{u}}_{\mathbf{g}}^{\circ}\right\|_1 \\ \text{subject to} \quad & \left\|\tilde{\mathbf{z}} - \tilde{\mathbf{b}}_{\mathbf{g}}\right\|_2 \le \varepsilon \ , \end{aligned} \quad (17)$$

where the matrix $\tilde{\mathbf{U}}_{\mathbf{g}}$ as well as all the noise terms are considered as unknown variables to be estimated, and

$$\begin{aligned} \tilde{\mathbf{z}} &= \left[\tilde{\mathbf{z}}_1^T, \tilde{\mathbf{z}}_2^T, \ldots, \tilde{\mathbf{z}}_M^T\right]^T \ , \\ \tilde{\mathbf{u}}_{\mathbf{g}}^{\circ} &= \left[\left\|\tilde{\mathbf{u}}_{\mathbf{g}}^0\right\|_2, \left\|\tilde{\mathbf{u}}_{\mathbf{g}}^1\right\|_2, \ldots, \left\|\tilde{\mathbf{u}}_{\mathbf{g}}^{K_xK_y-1}\right\|_2, \tilde{\sigma}_{\bar{n}}^2\right]^T \ , \end{aligned} \quad (18)$$

with

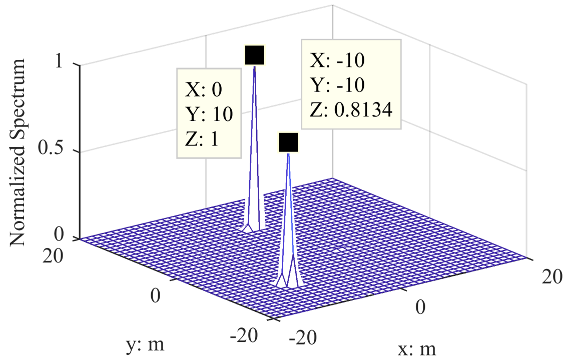
$$\tilde{\sigma}_{\bar{n}}^2 = \left\| \left[\sigma_1^2, \sigma_2^2, \ldots, \sigma_M^2\right] \right\|_2 \ . \quad (19)$$

*R*emark 1: The first K_xK_y elements of the column vector $\tilde{\mathbf{u}}_{\mathbf{g}}^{\circ}$ are the corresponding localization results over the predefined search grids, and the optimization problem in (17) can be solved using the CVX package [28], [29]. Compared with the MLE where the AOA measurements separately estimated at receivers are combined together to obtain the final locations under the least square sense, the information acquired by all the sub-arrays in the distributed sensor array network can be processed jointly in our proposed method, and therefore improved performance can be achieved.

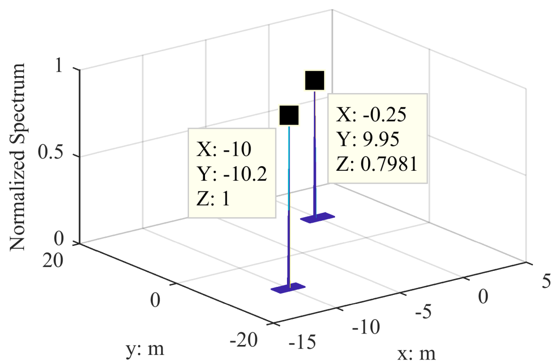
*R*emark 2: Furthermore, to reduce the computational complexity, a grid refining strategy can be employed, where in the first step, a search grid with a large step size is employed in the GS-Localization method to find a coarse position estimation of the targets, i.e., $\tilde{T}_k(\tilde{x}_{T_k}, \tilde{y}_{T_k})$, in the area of interest, followed by a refined search grid covering much smaller areas around the positions $\tilde{T}_k(\tilde{x}_{T_k}, \tilde{y}_{T_k})$ while a small step size is applied.

IV. Simulation Results

A distributed sensor array network consisting of $M = 3$ receivers is considered, where a uniform linear sub-array with $L_m = 4$ ($m = 1, 2, 3, 4$) sensors is equipped at each receiver, and the spacing between adjacent physical sensors is set as $d = \lambda/2$. The three receivers are placed at locations $\mathrm{U}_1(10, -40)$,



(a) Localization results for the first step.



(b) Localization results for the second step.

Fig. 3. Localization results obtained by the proposed GS-Localization method.

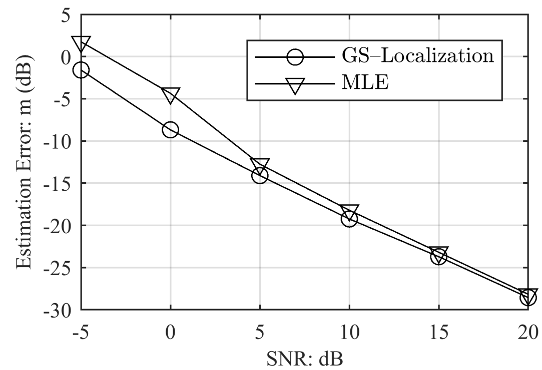


Fig. 4. RMSE results versus input SNRs.

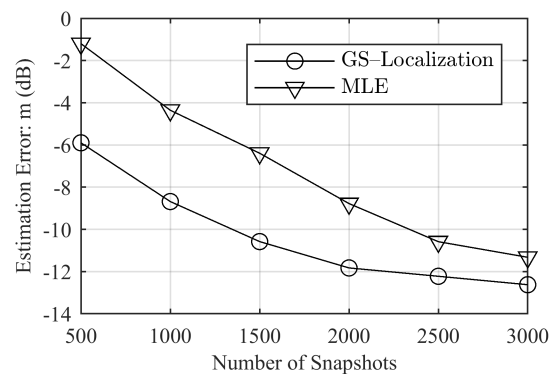


Fig. 5. RMSE results versus the number of snapshots.

$\mathrm{U}_2(30, 10)$, and $\mathrm{U}_3(-80, 90)$, while their rotation angles φ_m are $5°$, $100°$, and $-115°$, respectively. There are $K = 2$ targets located at $T_1(-10, -10)$ and $T_2(0, 10)$. Here all the location coordinates are measured in meters. The allowable error bound ε is chosen to give the best estimation results through trial-and-error in every experiment, and the MLE with direct grid search method is employed as a comparison.

The square area of $-20\text{m} \leq x \leq 20\text{m}$ and $-20\text{m} \leq y \leq 20\text{m}$ is of interest for this target localization problem. In the first step, 1m is the step size for search grid generation within the entire area of interest, and $\tilde{T}_k(\tilde{x}_{T_k}, \tilde{y}_{T_k})$, $k = 1, 2, \ldots, K$ are the estimated locations. Then, a smaller step size of 0.05 m is utilized to generate the search grid in a refined area of $\tilde{x}_{T_k} - 1 \leq x \leq \tilde{x}_{T_k} + 1$ m and $\tilde{y}_{T_k} - 1 \leq y \leq \tilde{y}_{T_k} + 1$ m for localization with high accuracy.

For the first set of simulations, the input signal to noise ratio (SNR) is set to 0 dB and the number of snapshots involved is 1000. Figs. 3(a) and 3(b) give the target localization results obtained by the proposed GS-Localization method in two steps, where the two peaks resolved in the predefined Cartesian coordinate system represent the estimated locations of the targets. Obviously, the proposed method can localize the targets effectively, with the estimated results close to the actual target locations.

For the second set of simulations, we compare the root mean square error (RMSE) results of the MLE and the proposed GS-Localization method with respect to the input SNRs, as shown in Fig. 4, where the number of snapshots is fixed at 1000. Both methods are capable of localizing the targets over a wide range of input SNRs, with a better performance achieved by the proposed GS-Localization method especially for low input SNRs.

Finally, as shown in Fig. 5, we analyze the RMSE results of different localization methods versus the number of snapshots. It is clear that the proposed method outperforms the MLE consistently by a big margin, which again verifies the superior performance the proposed group sparsity based solution due to joint simultaneous exploitation of the data acquired by all receivers.

V. Conclusions

In this paper, the target localization problem for distributed sensor array networks has been studied. Unlike previous solutions where the separately obtained angle of arrival estimates were fused under the least square case, a group sparsity based two-dimensional target localization method was proposed, where target locations are obtained directly by jointly processing the received signals across all sub-arrays. It has been shown by simulations that this proposed method works effectively over a wide range of input SNRs and number of snapshots, and it outperforms the existing MLE consistently.

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