UNIFIED FRAMEWORK FOR MINIMAX MIMO TRANSMIT BEAMPATTERN MATCHING UNDER WAVEFORM CONSTRAINTS

Rui Zhou, Ziping Zhao, and Daniel P. Palomar

Department of Electronic and Computer Engineering,
The Hong Kong University of Science and Technology, Hong Kong.

ABSTRACT

Minimax multiple-input multiple-output (MIMO) transmit beampattern matching is a fundamental and important problem in many MIMO systems. The problem is formulated to minimize the maximum beampattern matching error as well as suppress the cross-correlation beampatterns while taking different practical waveform constraints into consideration. Due to the high nonconvexity of the problem, the traditional way for problem solving is a two-stage approach, where a waveform covariance matrix is firstly designed and then the waveforms are synthesized from the covariance matrix under a specific constraint. This approach is usually very time consuming and only results in suboptimal solutions. In this paper, a novel and unified one-stage approach is proposed to solve the minimax beampattern matching problem which is capable of considering multiple waveform constraints. Superior performance of the proposed approach over the classical approach is verified through numerical simulations.

Index Terms— MIMO systems, beampattern design, nonconvex optimization, majorization-minimization, primal dual

1. INTRODUCTION

Multiple-input multiple-output (MIMO) transmission is a promising technology for the fifth-generation millimeter-wave wireless communications systems [1] and the next-generation radar systems [2]. MIMO systems are flexible since they can achieve desired beampatterns for multiple transmission targets via waveform design techniques. Other advantages of MIMO systems are higher resolution property, better identifiability, usage of adaptive techniques, etc. [3].

In MIMO systems, transmit beampattern matching is a fundamental problem that consists of achieving a desired beampattern by designing the transmitting waveforms considering both relative phase and power allocation. Motivated by traditional finite impulse response filter design, the maximum error minimization (a.k.a. minimax) beampattern matching is natural and has attracted a lot of interest both in wireless communications [4] and radar [5]. Most of the literature focuses on the matching problem by using the least-squares (LS) criterion. Nevertheless, as criticized by [6], the matching result from the LS criterion spends too much effort in matching the boundary of the beampatterns instead of the other points. The minimax matching criterion is able to generate a flat mainlobe and is considered much better compared to the LS criterion.

Due to the highly nonconvex objective and waveform constraints in the minimax beampattern matching problem, it is difficult to solve directly. The classical method for problem solving is the *two-stage approach*, which is firstly optimizing a waveform covariance matrix and then synthesizing the waveforms from its covariance considering a waveform constraint. In [2], it was firstly proposed to approximate a desired transmit beampattern via waveform covariance optimization. A more advanced design method considering both beampattern matching and sidelobe suppression was studied in [5], where second order cone programming (SOCP) was applied.

The waveform synthesis method was studied for the scenario of coded binary phase shift keyed systems in [6]. In [7], an alternating minimization (AltMin) algorithm (a.k.a. cyclic algorithm) was proposed to synthesize waveforms under the general peak-to-average constraint. As admitted by the authors in [7], the synthesized waveforms by AltMin only suffice to be an approximation to the waveform covariance matrix. Also, imposing a constraint in the second stage can only generate a "suboptimal" solution to the overall design problem. Besides the optimality issue, more practical waveform constraints should be considered in practice. For example, the similarity constraint is important in controlling the designed waveforms to lie in the neighborhood of a reference one [8].

Recently, the *one-stage approach* has shown great computation efficiency and better performance in MIMO beampattern matching problem based on the LS criterion [9, 10]. In this paper, a novel and unified one-stage approach will be derived to solve the minimax MIMO transmit beampattern matching problem under multiple waveform constraints for the first time. The performance of the proposed method over the benchmarks is verified through numerical simulations.

2. MINIMAX TRANSMIT BEAMPATTERN MATCHING PROBLEM FORMULATION

A colocated MIMO system [3] with M transmit antennas in a uniform linear array (ULA) is considered. Each transmit antenna can emit a different waveform $x_m(n) \in \mathbb{C}$ with $m = 1, \ldots, M$, $n = 1, \ldots, N$, where N is the number

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of samples. Let $\mathbf{x}(n) = [x_1(n), \ldots, x_M(n)]^T$ be the nth sample of the M transmit waveforms. We define the probing waveform vector as $\mathbf{x} \triangleq [\mathbf{x}^T(1), \ldots, \mathbf{x}^T(N)]^T \in \mathbb{C}^{NM}$.

The signal at a target location with angle $\theta \in \Theta$ (Θ denotes the angle set) is represented by

$$\sum_{m=1}^{M} e^{-j\pi(m-1)\sin\theta} x_m(n) = \mathbf{a}^T(\theta)\,\mathbf{x}(n),\ n = 1, \ldots, N,$$

where $\mathbf{a}(\theta)$ is the transmit steering vector defined as $\mathbf{a}(\theta) \triangleq \left[1, e^{-j\pi\sin\theta}, \ldots, e^{-j\pi(M-1)\sin\theta}\right]^T$. Then the power for signal $\mathbf{x}$ at location θ also named the *transmit beampattern* is

$$\begin{aligned} P(\theta,\mathbf{x}) &= \textstyle\sum_{n=1}^{N} \left(\mathbf{a}^T(\theta)\,\mathbf{x}(n)\right)^* \left(\mathbf{a}^T(\theta)\,\mathbf{x}(n)\right) \\ &= \mathbf{x}^H \left(\mathbf{I}_N \otimes \mathbf{a}^*(\theta)\,\mathbf{a}^T(\theta)\right)\mathbf{x} \triangleq \mathbf{x}^H \mathbf{A}(\theta)\,\mathbf{x}, \end{aligned}$$

where $\mathbf{A}(\theta) \triangleq \mathbf{I}_N \otimes \left(\mathbf{a}^*(\theta)\,\mathbf{a}^T(\theta)\right)$. The *cross-correlation sidelobes* (a.k.a. *cross-correlation beampattern*) is given by

$$P_{cc}(\theta_i, \theta_j, \mathbf{x}) \quad = \mathbf{x}^H \mathbf{A}(\theta_i, \theta_j)\,\mathbf{x}\ ,$$

where $\mathbf{A}(\theta_i,\theta_j) \triangleq \mathbf{I}_N \otimes \left(\mathbf{a}^*(\theta_i)\,\mathbf{a}^T(\theta_j)\right)$ with $\theta_i \neq \theta_j \in \Theta_s$ which is the angle set of interest.

The objective of the transmit beampattern matching problem is to match a desired transmit beampattern $p(\theta)$ as well as suppress the cross-correlation sidelobes. The minimax matching objective can be formulated as follows[1]:

$$J(\alpha,\mathbf{x}) = \max_{\theta_i \in \Theta} \left|\alpha p(\theta_i) - P(\theta_i, \mathbf{x})\right|\ , \tag{1}$$

and the sidelobe suppression term is described as

$$E(\mathbf{x}) = \sum_{(\theta_i,\theta_j)\in\Theta_s, i\neq j} \left|P_{cc}(\theta_i,\theta_j,\mathbf{x})\right|^2\ . \tag{2}$$

Finally, the minimax MIMO transmit beampattern matching problem (Minimax-TxBM) is formulated as follows:

$$\begin{array}{ll} \underset{\alpha,\mathbf{x}}{\mathsf{minimize}} & J(\alpha,\mathbf{x}) + \mu E(\mathbf{x}) \\ \mathsf{subject\ to} & \mathbf{x} \in \mathcal{X} \triangleq \mathcal{X}_0 \cap (\cap_i \mathcal{X}_i)\,, \end{array} \tag{3}$$

where $\mu \geq 0$ and $\mathcal{X}$ generally denotes the waveform constraint with $\mathcal{X}_0 \triangleq \left\{\mathbf{x} \mid \sum_{n=1}^{N} |x_m(n)|^2 = c_e^2\right\}$ representing the *total transmit energy constraint*. Some other practical waveform constraints are also considered. *The constant modulus constraint* $\mathcal{X}_1 \triangleq \left\{\mathbf{x} \mid |x(l)| = c_d = \frac{c_e}{\sqrt{N}}\right\}$ for $l = 1, \ldots, MN$ is to prevent the non-linearity distortion of the power amplifier to maximize the efficiency of the transmitter. The peak-to-average ratio $\mathrm{PAR}(\mathbf{x}_m) = \frac{\max|x_m(n)|^2}{\sum_n |x_m(n)|^2/N}$ represents the peak signal power to its average power that is constrained to a small threshold, so that the analog-to-digital and digital-to-analog converters can have lower dynamic range and fewer linear power amplifiers are needed. Since $\mathcal{X}_0$ exists, the *PAR constraint* becomes $\mathcal{X}_2 \triangleq \{\mathbf{x} \mid |x(l)| \leq c_p\}$. *The similarity constraint* $\mathcal{X}_3 \triangleq \{\mathbf{x} \mid |\mathbf{x} - \mathbf{x}_{\mathrm{ref}}| \leq c_\epsilon\}$ is to allow the designed waveforms to lie in the neighborhood of a reference one $\mathbf{x}_{\mathrm{ref}}$ which already attains a good performance [11]. Problem (3) is a constrained nonconvex problem due to the nonconvex objective and the nonconvex constraints.

[1]Variable α is introduced since $p(\theta)$ is typically given in a "normalized form" and we want to approximate a scaled version of $p(\theta)$, not $p(\theta)$ itself.

3. MINIMAX-TXBM PROBLEM SOLVING

3.1. The Penalty Dual Decomposition Method

The penalty dual decomposition method [12] is a primal dual (PD) optimization method to handle a class of nonsmooth nonconvex optimization problems. A problem is given by

$$(\mathcal{P}) \quad \begin{array}{ll} \underset{\mathbf{x},\mathbf{y}}{\mathsf{minimize}} & f(\mathbf{x},\mathbf{y}) + \sum_{j=1}^{N_y} g(\mathbf{y}_j) \\ \mathsf{subject\ to} & \mathbf{h}_0(\mathbf{x},\mathbf{y}) = \mathbf{0} \\ & h_i(\mathbf{x}_i) \leq 0, \mathbf{x}_i \in \mathcal{X}_i,\ i = 1, \ldots, N_x, \end{array}$$

where $\mathbf{x} \triangleq \{\mathbf{x}_1, \ldots, \mathbf{x}_{N_x}\}$, $\mathbf{y} \triangleq \{\mathbf{y}_1, \ldots, \mathbf{y}_{N_y}\}$, f and h_i ($i = 0, \ldots, N_x$) denote smooth functions, g is nonsmooth, $\mathcal{X}_i$ denotes convex constraints and specifically $\mathbf{h}_0(\mathbf{x},\mathbf{y}) = \mathbf{0}$ denotes the variable coupling constraints. To solve ($\mathcal{P}$) to a stationary point, this PD method update the primal, dual, and penalty variables by optimizing the following augmented Lagrange function

$$\begin{aligned} \mathcal{L}(\mathbf{x},\mathbf{y};\boldsymbol{\lambda},\rho) \triangleq{}& f(\mathbf{x},\mathbf{y}) + \textstyle\sum_{j=1}^{N_y} g(\mathbf{y}_j) \\ &+ \boldsymbol{\lambda}^T \mathbf{h}_0(\mathbf{x},\mathbf{y}) + \tfrac{1}{2\rho}\|\mathbf{h}_0(\mathbf{x},\mathbf{y})\|^2, \end{aligned}$$

where $\boldsymbol{\lambda} = \left[\lambda_1, \ldots, \lambda_{|\mathbf{h}_0|}\right]$ is the dual variable and ρ is a penalty parameter corresponding to $\mathbf{h}_0$. The dual variable $\boldsymbol{\lambda}$ and penalty variable ρ are updated when certain conditions are satisfied [12]. Given $\boldsymbol{\lambda}$ and ρ, the augmented Lagrangian optimization problem for the primal variable updating is given as follows:

$$(\mathcal{P}_{\boldsymbol{\lambda},\rho}) \quad \begin{array}{ll} \underset{\mathbf{x},\mathbf{y}}{\mathsf{minimize}} & \mathcal{L}(\mathbf{x},\mathbf{y};\boldsymbol{\lambda},\rho) \\ \mathsf{subject\ to} & h_i(\mathbf{x}_i) \leq 0, \mathbf{x}_i \in \mathcal{X}_i,\ i = 1, \ldots, N_x. \end{array}$$

Problem $\mathcal{P}_{\boldsymbol{\lambda},\rho}$ is usually with multi-block variables. Block structures should be exploited for efficient problem solving and as a result the block majorization-minimization (BMM) method [13, 14] can be used. Note that incorporating BMM makes the algorithm in a double-loop manner. To capture the nature of the resulting algorithm, the overall algorithm will be named as PD-BMM for short in this paper.

3.2. Minimax-TxBM via PD-BMM Method

First, we rewrite problem (3) in the following equivalent form

$$\begin{array}{ll} \underset{\alpha,\mathbf{t},\mathbf{x}}{\mathsf{minimize}} & \max_{\{i|\theta_i\in\Theta\}} |t_i| + \mu E(\mathbf{x}) \\ \mathsf{subject\ to} & t_i = \alpha p(\theta_i) - P(\theta_i,\mathbf{x}),\ \theta_i \in \Theta \\ & \mathbf{x} \in \mathcal{X} \triangleq \mathcal{X}_0 \cap (\cap_i \mathcal{X}_i)\,, \end{array} \tag{4}$$

where by defining $\mathbf{t} \triangleq \left[t_1, \ldots, t_{|\Theta|}\right]^T$, $\max_{\{i|\theta_i\in\Theta\}} |t_i|$ in the objective can be compactly rewritten as $\|\mathbf{t}\|_\infty$. Then the augmented Lagrangian for Problem (4) can be obtained as

$$\begin{aligned} \mathcal{L}(\alpha,\mathbf{t},\mathbf{x};\boldsymbol{\lambda},\rho) ={}& \|\mathbf{t}\|_\infty + \mu E(\mathbf{x}) \\ &+ \tfrac{1}{2\rho}\textstyle\sum_{i=1}^{|\Theta|}(t_i - \alpha p(\theta_i) + P(\theta_i,\mathbf{x}) + \rho\lambda_i)^2 + \mathit{const}. \end{aligned}$$

The dual variable $\boldsymbol{\lambda}$ and the penalty variable ρ can be easily updated in each iteration according to the rules given in [12]. In the following, we will mainly discuss how to update the primal variables (i.e., α, $\mathbf{t}$, and $\mathbf{x}$) by solving $\mathcal{L}(\alpha, \mathbf{t}, \mathbf{x}; \boldsymbol{\lambda}, \rho)$ based on the BMM method [13]. The problem at the kth iteration is readily given as follows:

$$\begin{array}{ll} \underset{\alpha, \mathbf{t}, \mathbf{x}}{\text{minimize}} & \mathcal{L}(\alpha, \mathbf{t}, \mathbf{x}; \boldsymbol{\lambda}^{(k)}, \rho^{(k)}) \\ \text{subject to} & \mathbf{x} \in \mathcal{X} \triangleq \mathcal{X}_0 \cap (\cap_i \mathcal{X}_i). \end{array} \tag{5}$$

Then given iterates $(\alpha^{(k)}, \mathbf{t}^{(k)}, \mathbf{x}^{(k)})$, the inner loop (denoted by superscript $(r|k)$) variable update rules to get $(\alpha^{(k+1)}, \mathbf{t}^{(k+1)}, \mathbf{x}^{(k+1)})$ are given in the following.

3.3. Solving The α-Subproblem in PD-BMM

The augmented Lagrangian $\mathcal{L}(\alpha, \mathbf{t}, \mathbf{x}; \boldsymbol{\lambda}, \rho)$ w.r.t. variable α is a typical convex quadratic function which is written as

$$\mathcal{L}(\alpha, \mathbf{t}^{(r|k)}, \mathbf{x}^{(r|k)}; \boldsymbol{\lambda}^{(k)}, \rho^{(k)}) = \frac{1}{2\rho} \sum_{i=1}^{|\Theta|} (t_i^{(r|k)} + P\left(\theta_i, \mathbf{x}^{(r|k)}\right) + \rho^{(k)} \lambda_i^{(k)} - \alpha p(\theta_i))^2 + const.$$

Minimizing the above function w.r.t. α admits the following simple closed-form solution

$$\alpha^{(r+1|k)} = \frac{\sum_{i=1}^{|\Theta|} p(\theta_i) (t_i^{(r|k)} + P\left(\theta_i, \mathbf{x}^{(r|k)}\right) + \rho^{(k)} \lambda_i^{(k)})}{\sum_{i=1}^{|\Theta|} p^2(\theta_i)}. \tag{6}$$

3.4. Solving The $\mathbf{t}$-Subproblem in PD-BMM

The augmented Lagrangian $\mathcal{L}(\alpha, \mathbf{t}, \mathbf{x}; \boldsymbol{\lambda}, \rho)$ for variable $\mathbf{t}$ is

$$\begin{aligned} &\mathcal{L}(\alpha^{(r+1|k)}, \mathbf{t}, \mathbf{x}^{(r|k)}; \boldsymbol{\lambda}^{(k)}, \rho^{(k)}) \\ &\quad = \tfrac{1}{2\rho} \|\mathbf{t} - \mathbf{h}^{(r|k)}\|_2^2 + \|\mathbf{t}\|_\infty + const. \end{aligned}$$

where $\mathbf{h}^{(r|k)}$ is a vector with its ith element $h_i^{(r|k)} \triangleq \alpha^{(r+1|k)} p(\theta_i) - P(\theta_i, \mathbf{x}^{(r|k)}) - \rho^{(k)} \lambda_i^{(k)}$.

Then the $\mathbf{t}$-subproblem is given in the following form

$$\underset{\mathbf{t}}{\text{minimize}} \quad \frac{1}{2} \|\mathbf{t} - \mathbf{h}^{(r|k)}\|_2^2 + \rho \|\mathbf{t}\|_\infty, \tag{7}$$

which is a convex variational problem and can be efficiently solved by an analytical solution.

Lemma 1. *[15] The optimal solution to Problem (7) is*

$$\mathbf{t}^\star = \mathbf{h}^{(r|k)} - \tilde{\mathbf{t}}^\star,$$

where $\tilde{\mathbf{t}}^\star$ is the optimal solution of the conjugate of Problem (7) (According to the Moreau decomposition rule, the Fenchel conjugate of $\rho \|\mathbf{t}\|_\infty$ is the indicator function for the constraint $\|\tilde{\mathbf{t}}\|_1 \leq \rho$.) which is given as follows:

$$\begin{array}{ll} \underset{\tilde{\mathbf{t}}}{\text{minimize}} & \frac{1}{2} \|\tilde{\mathbf{t}} - \mathbf{h}^{(r|k)}\|_2^2 \\ \text{subject to} & \|\tilde{\mathbf{t}}\|_1 \leq \rho. \end{array} \tag{8}$$

Problem (8) is the classical projection onto the ℓ_1-ball problem [16, 17]. Analytical solutions can be easily obtained by the water-filling procedure.

Lemma 2. *[16] Problem (8) has the water-filling solution.*

> ***if*** $||\mathbf{h}||_1 \leq \rho$ ***then***
> $\quad \tilde{\mathbf{t}} = \mathbf{h}$, ***return*** $\mathbf{z}$
> ***else***
> $\quad \mathbf{a} = \operatorname{sign}(\mathbf{h})$ *and* $\mathbf{b} = \operatorname{abs}(\mathbf{h})$
> $\quad$ *Sort* $\mathbf{b}$ *in order:* $b_{(1)} \geq b_{(2)} \geq \cdots \geq b_{(N)}$
> $\quad \nu = \arg \max_{1 \leq j \leq |\Theta|} \left\{ b_{(j)} - \frac{1}{j} \left(\sum_{i=1}^{j} b_{(i)} - \rho \right) > 0 \right\}$
> $\quad \gamma = \frac{1}{\nu} \left(\sum_{i=1}^{\rho} b_{(i)} - \rho \right)$
> $\quad \tilde{t}_j = a_j \max\{b_j - \gamma, 0\}$, $1 \leq j \leq |\Theta|$, ***return*** $\tilde{\mathbf{t}}$
> ***end if***

Finally, the $\mathbf{t}$-subproblem can be easily solved based on the above results.

3.5. Solving The $\mathbf{x}$-Subproblem in PD-BMM

The augmented Lagrangian w.r.t. $\mathbf{x}$ can be written as

$$\begin{aligned} &\mathcal{L}(\alpha^{(r+1|k)}, \mathbf{t}^{(r+1|k)}, \mathbf{x}; \boldsymbol{\lambda}^{(k)}, \rho^{(k)}) \\ &\quad = \tfrac{1}{2\rho} \sum_{i=1}^{|\Theta|} (P(\theta_i, \mathbf{x}) - z_i^{(r|k)})^2 + \mu E(\mathbf{x}) + const., \end{aligned}$$

where $z_i^{(r|k)} \triangleq \alpha^{(r+1|k)} p(\theta_i) - t_i^{(r+1|k)} - \rho^{(k)} \lambda_i^{(k)}$.

The $\mathbf{x}$-subproblem is given as follows:

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \frac{1}{2\rho} \sum_{i=1}^{|\Theta|} (P(\theta_i, \mathbf{x}) - z_i^{(r|k)})^2 + \mu E(\mathbf{x}) \\ \text{subject to} & \mathbf{x} \in \mathcal{X}, \end{array} \tag{9}$$

which is a constrained nonconvex problem. The majorization-minimization method can be used which is given in the following result (To simplify the discussion, we first set $\mu = 0$.).

Lemma 3. *[10] Function $\mathcal{L}(\alpha^{(r+1|k)}, \mathbf{t}^{(r+1|k)}, \mathbf{x}; \boldsymbol{\lambda}^{(k)}, \rho^{(k)})$ ($\mu = 0$) can be linearly majorized over $\mathcal{X}$ as follows:*

$$\begin{aligned} &\mathcal{L}(\alpha^{(r+1|k)}, \mathbf{t}^{(r+1|k)}, \mathbf{x}; \boldsymbol{\lambda}^{(k)}, \rho^{(k)}) \\ &\quad \leq \operatorname{Re}\left(\mathbf{x}^H \mathbf{y}_J^{(r|k)}\right) + const., \end{aligned}$$

where $\mathbf{y}_J^{(r|k)} \triangleq 4(\mathbf{M}_J^{(r|k)} \mathbf{x}^{(r|k)} - M c_e^2 \psi_1^{(k)} \mathbf{x}^{(r|k)} - \psi_2^{(r|k)} \mathbf{x}^{(r|k)})$ with $\mathbf{M}_J^{(r|k)} \triangleq \frac{1}{\rho^{(k)}} \sum_{i=1}^{|\Theta|} (P(\theta_i, \mathbf{x}) - z_i^{(r|k)}) \mathbf{A}(\theta_i)$, $\psi_1^{(k)} \geq \frac{1}{\rho^{(k)}} \lambda_{\max}(\sum_{i=1}^{|\Theta|} vec(\mathbf{A}(\theta_i)) \, vec(\mathbf{A}(\theta_i))^H)$ which is iteration-independent and can be computed in advance, and $\psi_2^{(r|k)} \geq \lambda_{\max}(\mathbf{M}_J^{(r|k)})$ which can be efficiently computed by the FFT.

A similar majorization trick can be easily applied on $E(\mathbf{x})$ (note that $E(\mathbf{x})$ is also quartic in $\mathbf{x}$) to get a majorized function as $\operatorname{Re}(\mathbf{x}^H \mathbf{y}_E^{(r|k)}) + const.$ with $\mathbf{y}_E^{(r|k)}$ defined properly. Based on Lemma 3, instead of solving Problem (9) directly, we can solve its majorization problem as follows:

$$\begin{array}{ll} \underset{\mathbf{x}}{\text{minimize}} & \operatorname{Re}\left(\mathbf{x}^H \mathbf{y}^{(r|k)}\right) \\ \text{subject to} & \mathbf{x} \in \mathcal{X}, \end{array} \tag{10}$$

where $\mathbf{y}^{(r|k)} \triangleq \mathbf{y}_J^{(r|k)} + \mathbf{y}_E^{(r|k)}$. Problem (10) takes a much simpler form than Problem (9). Efficient optimal solutions (mostly closed-form solutions) $\mathbf{x}^\star$ can be derived for different choices of $\mathcal{X}$, which are summarized in the following result.

Lemma 4. *[10] (The Optimal Solutions of Problem (10))*
[1] For $\mathcal{X} = \mathcal{X}_0$, $\mathbf{x}_m^\star = -c_e \mathbf{y}_m / \|\mathbf{y}_m\|_2$, where $\mathbf{y}_m$ denotes the elements in $\mathbf{y}$ corresponding to m-th antenna;
[2] for $\mathcal{X} = \mathcal{X}_0 \cap \mathcal{X}_1$, $\mathbf{x}^\star = c_d e^{-j\arg(\mathbf{y})}$; [2]
[3] for $\mathcal{X} = \mathcal{X}_0 \cap \mathcal{X}_2$ or $\mathcal{X}_0 \cap \mathcal{X}_1 \cap \mathcal{X}_2$, the $\mathbf{x}^\star$ can be found in [18, Alg. 2];
[4] for $\mathcal{X} = \mathcal{X}_0 \cap \mathcal{X}_3$, it is convex and can be solved efficiently;
[5] for $\mathcal{X} = \mathcal{X}_0 \cap \mathcal{X}_1 \cap \mathcal{X}_3$, the $\mathbf{x}^\star$ can be found in [19];
[6] for $\mathcal{X} = \mathcal{X}_0 \cap \mathcal{X}_2 \cap \mathcal{X}_3$ or $\mathcal{X}_0 \cap \mathcal{X}_1 \cap \mathcal{X}_2 \cap \mathcal{X}_3$, it is convex and can be solved efficiently.

In the inner-loop to solve Problem (5), the algorithm will iteratively update the primal variables until convergence.

3.6. The Overall PD-BMM Algorithm

Finally, the overall algorithm is summarized as follows.

Algorithm 1 PD-BMM Algorithm for Minimax-TxBM

Require: $\mathbf{a}(\theta_i)$, $p(\theta_i)$ with $\theta_i \in \Theta$ and $\theta_s \in \Theta_s$; pick two deminishing sequences $\{\epsilon^{(k)} > 0\}$ and $\{\eta^{(k)} > 0\}$

Initialize $\alpha^{(0)}$, $\mathbf{t}^{(0)}$, $\mathbf{x}^{(0)}$, $\rho^{(0)}$, and $\boldsymbol{\lambda}^{(0)}$ and set $k = 0$
repeat
  $(\alpha^{(k+1)}, \mathbf{t}^{(k+1)}, \mathbf{x}^{(k+1)}) \leftarrow$ solve [Problem (5)]
  if $\|\mathbf{t}^{(k)} - \mathbf{h}^{(k)}\|_\infty < \eta^{(k)}$ **then**
    $\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} + \frac{1}{\rho^{(k)}}(\mathbf{t}^{(k)} - \mathbf{h}^{(k)})$, $\rho^{(k+1)} = \rho^{(k)}$
  else
    $\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)}$, $\rho^{(k+1)} = 0.9\rho^{(k)}$
  end if
  $k = k + 1$
until Convergence

4. NUMERICAL SIMULATIONS

We show the performance of our proposed algorithm by numerical simulations. Consider a MIMO transmission system equipped with $M = 16$ antennas and each of them will send a sequence with length $N = 12$. Without loss of generality, the total transmit power is set to $c_e^2 = 4$. The range of angles is $\Theta = (-90^\circ, 90^\circ)$ with a spacing of 2° and three interested targets are chosen as $\theta_1 = -40^\circ$, $\theta_2 = 0^\circ$, $\theta_3 = 40^\circ$. Then the desired beampattern is given as follows:

$$p(\theta) = \begin{cases} 1 & \theta \in [\theta_k - 10^\circ, \theta_k + 10^\circ], k = 1, 2, 3 \\ 0 & \text{otherwise.} \end{cases}$$

The parameter setting for our simulation is chosen as $\mu = 0$, $\epsilon^{(k)} = 0.6\epsilon^{(k-1)}$, and $\eta^{(k)} = 1/\sqrt{k}$. For comparison, we use the classical two-stage approach [20] as the benchmark. To illustrate the advantages of our proposed one-stage approach, we set the PAR constraint as 1, which actually reduces to the constant modulus constraint ($\mathcal{X} = \mathcal{X}_0 \cap \mathcal{X}_1$).

As shown in Fig. 1, our proposed algorithm can converge to a stationary solution faster than the benchmark. Although in the first stage of the two-stage approach the SOCP can obtain slightly better result (which is reasonable since the waveform constraint is relaxed), the final synthesized waveforms from the second stage lead to a worse performance.

[2] The operation $\arg(\mathbf{y})$ is applied element-wise for $\mathbf{y}$.

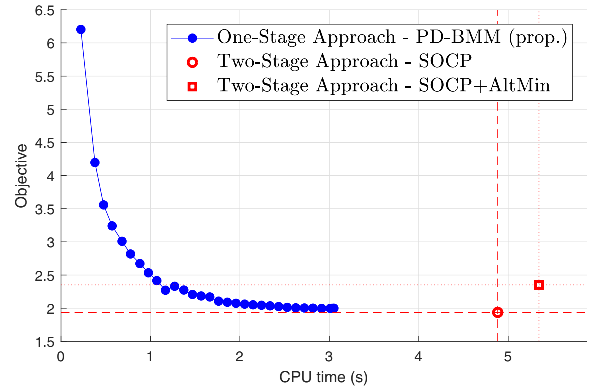

Fig. 1. Convergence comparisons for objective value.

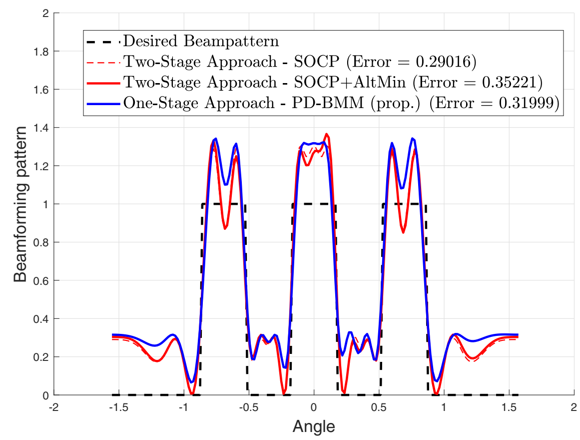

Fig. 2. Transmit beampattern design with 3 targets.

We further plot the designed beampatterns from different algorithms and the desired beampattern in Fig. 2. The matching error is defined as $\max_{i=1,\ldots,|\Theta|} |p(\theta_i) - P(\theta_i, \mathbf{x})/\alpha|$. It shows that the beampattern generated by only solving an SOCP is the best among all the methods. But it gets worse when the waveform is finally synthesized by the AltMin algorithm in the second stage. The center beam is even distorted since perfect waveform synthesis cannot be attained when a low PAR constraint is considered [7]. By comparison, our proposed one-stage approach, i.e., PD-BMM, can achieve a lower error level than the two-stage approach.

5. CONCLUSIONS

In this paper, the minimax MIMO transmit beampattern matching problem has been considered under multiple practical waveform constraints. A novel and unified one-stage approach has been proposed for efficient problem solving. Numerical simulations have shown that compared to the classical two-stage approach the proposed algorithm can obtain better results.

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