ANALYSIS OF INFORMATION DIFFUSION WITH IRRATIONAL USERS: A GRAPHICAL EVOLUTIONARY GAME APPROACH

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ABSTRACT

Modeling and analysis of information diffusion over networks is of crucial importance to better understand the avalanche of information flow over social networks and to investigate its impact on economy and our social life. Different from prior works that study rational behavior in information diffusion, we focus on "irrational users", e.g., those who always intentionally forward fake news even when they know it contains false information. We extend the graphical evolutionary game model for information diffusion, and analyze the impact of such irrational behavior on information propagation. Our simulation results on synthetic networks are consistent with our analytical results, and they show that even a few irrational users can significantly increase the number of users who adopt the forwarding strategy.

Index Terms— information diffusion, social networks, irrational behavior, graphical evolutionary game

1. INTRODUCTION

Recently, the popularity of social networks and the advance of mobile technology enable people to share information and exchange opinions with each other anywhere and anytime. They also provide a convenient channel for rumor spreading, which may pose serious threats to our society. One example is the "salt panic" in China after the 2011 Tohoku Tsunami, where rumors circulated over internet forums, microblogs and text messages triggered "panic-shopping of salt", "long lines and mob scenes at stores" and "10-fold jump of salt price" throughout China [1, 2]. Therefore, it is of crucial importance to study how information propagates over networks, and to design effective mechanisms to prevent the spreading of such detrimental rumors.

There have been numerous works on the modeling and analysis of information diffusion over social networks. A classical work was the SIR model originating from the study of epidemics [3], and there were many follow-up works [4–6]. These works classified users into different groups: those who had not heard the news, those who had received and forwarded the news, and those who had stopped forwarding. Using a few parameters to model the transition rates between groups, their works derived the mean-field equations and analyzed the dynamics of population in different groups. To take collective behavior into consideration, the linear threshold (LT) model was proposed in [7], where a user accepted the information if the percentage of his/her neighbors who adopted the information was above a threshold. The works in [8–12] extended the LT model into different scenarios. The authors in [13] proposed a graphical evolutionary game theoretic framework to model information diffusion over networks, and analyzed the evolutionary stable states.

The work in [13] assumed that all users were rational, shared the same payoff and fitness functions, and used the same rule to update their strategies. In real social networks, we observe "irrational behavior". For example, some users intentionally spread the rumors even though they know the news is fake. Their goals are different from others, e.g., they might be paid to spread the rumors, and intuitively, their existence may significantly impact the propagation of information. In this paper, we extend the graphical evolutionary game model for information diffusion in [13], and analyze the influence of such irrational behavior on others' decisions and information propagation. We also estimate the average percentage of irrational users needed to achieve a desired propagation state. To our knowledge, this is the first work on game theoretic modeling of such irrational behavior in information diffusion. This investigation is critical to the understanding of user behavior and rumor spreading in social networks, and to the design of effective detection and defensive mechanisms.

2. GRAPHICAL EVOLUTIONARY GAME FOR INFORMATION DIFFUSION

This section reviews the graphical evolutionary game model for information diffusion in [13].

Graphical evolutionary game includes five basic elements: graph structure, players, strategies, fitness (payoff) and evo-

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$$\dot{p}_f = \frac{2\alpha(\overline{k}-3v)}{\overline{k}N(\overline{k}-2v)^2(1+q)^2} \times \frac{p_f(p_f-x_0)(1-p_f)g(p_f)}{(p_f-y_0)^2}, \quad \text{where} \quad g(p_f) = \gamma_2 p_f^2 + \gamma_1 p_f + \gamma_0 \frac{q}{\alpha},$$
$$\gamma_2 = s(\overline{k}-3v)(\delta_1 - 2\delta_2), \quad \gamma_1 = s[(\overline{k}-3v)\delta_2 - (\overline{k}-2v)\delta_1], \quad \gamma_0 = v(\overline{k}-2v)^2,$$
$$s = (\overline{k}-2v)[(v-w)(\overline{m}-1) + (v-2w)(\overline{m}+1)] - (\overline{k}-3v)(v-2w)(\overline{m}-1), \tag{2}$$
$$\overline{k} = \sum_{k=1}^{+\infty} k\lambda(k), \ \ v = \sum_{k=1}^{+\infty}\lambda(k)\frac{k}{k+1}, \ \ w = \sum_{k=1}^{+\infty}\lambda(k)\frac{k}{(k+1)^2}, \ \ \overline{m} = \sum_{k=1}^{+\infty}\frac{k^2\lambda(k)}{\overline{k}}, \ \ x_0 = \frac{\overline{k}-2v}{\overline{k}-3v}q, \ \text{and} \ \ y_0 = \frac{q}{1+q}.$$

lutionary stable state (ESS). In the information diffusion scenario, graph structure refers to the structure of the social network where the news is propagated. Here, nodes represent users and an edge connecting two nodes represents a certain relationship between the two corresponding users. Graph structure can be described by the degree distribution $\lambda(k)$, the probability of a randomly selected user having k neighbors.

Each user has two possible strategies in the game: forward (S_f) or do not forward (S_n) the information. The utility function in the evolutionary game can be defined as “fitness”. In the information diffusion scenario, users with larger fitness values have a bigger impact on their neighbors, and more users imitate their strategies. In [13], the fitness is defined as $\pi = (1-\alpha) + \alpha U$ where α is a parameter with small values. Following the works in [13–19], we consider the scenario where $\alpha \ll 1$ in this work. U is the payoff received from interactions with neighbors. When a pair of users meet and both adopt strategy S_f, each receives payoff u_{ff}; when they both adopt strategy S_n, their payoffs are both u_{nn}, and when two users with different strategies meet, they both get the payoff u_{fn}. The corresponding payoff matrix is

$$\begin{array}{cc} & \begin{array}{cc} S_f & S_n \end{array} \\ \begin{array}{c} S_f \\ S_n \end{array} & \begin{pmatrix} u_{ff} & u_{fn} \\ u_{fn} & u_{nn} \end{pmatrix} \end{array}. \tag{1}$$

We consider the simple scenario where all users in the network share the same payoff matrix and use the same fitness function.

Users may change their strategies from time to time. Following the work in [13], the strategy evolving process is divided into time units. In each time unit, a user is randomly selected as the focal user to update his/her strategy while others keep theirs unchanged. We consider the imitation (IM) update rule here, where the focal user can either imitate the strategy of one neighbor or keep his/her own unchanged. The probability that the focal user imitates a user’s strategy (including his/her own) is proportional to that user’s fitness.

The whole population evolves under the IM strategy update rule and finally reaches the evolutionary stable state (ESS). To find and analyze the ESS, let p_f and p_n be the proportions of users with strategy S_f and S_n, respectively. At the ESS, the evolution dynamics satisfy $\dot{p}_f = 0$, that is, the proportion of users with strategy S_f does not change. Let p_f^* denote the percentage of users with strategy S_f at the ESS, which quantifies the extent to which the information is propagated across the network. Detailed analysis of p_f, $\dot{p}_f = 0$ and p_f^* can be found in [13] and omitted here.

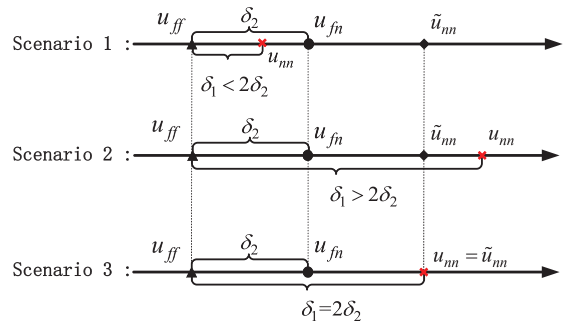


Fig. 1: Three different scenarios of the payoff values. Here $\tilde{u}_{nn} = 2u_{fn} - u_{ff}$.

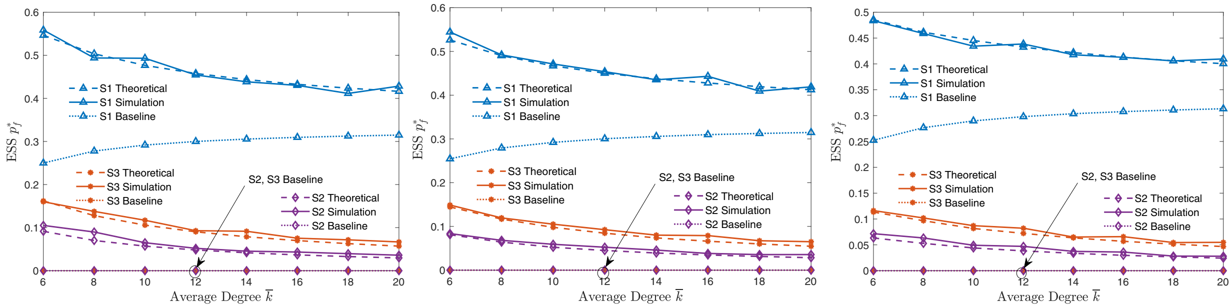
3. INFORMATION DIFFUSION WITH IRRATIONAL USERS

In this section, we consider the scenario where there exist “irrational” users whose behavior is different from those in Section 2, and who always forward the information. That is, they always use strategy S_f and never change. The analysis of the scenario where they always take strategy S_n and never forward the information is similar and omitted.

We consider an N-user undirected and connected network where the proportion of irrational users is q. We assume that q is relatively small and these irrational users are randomly distributed in the network. Compared to the model in [13], with irrational users, the biggest difference is when these irrational users are selected as the focal user, they will never change their strategies. Unaware of the existence of such irrational users, rational users assume that all users are rational, and use the same rule as in [13] to update their strategies. Thus, there always exist users who forward the information, and intuitively, the information will potentially reach more people and have a bigger impact.

Define $\delta_1 = u_{nn} - u_{ff}$ and $\delta_2 = u_{fn} - u_{ff}$. Following the same analysis as in [13], we obtain the evolution dynamics of p_f as in (2). In (2), $\overline{k}$ is the average degree of the network, and $\{v, w, \overline{m}, s\}$ are variables determined by the graph structure. Detailed derivation can be found in [20].

In this work, we consider the scenario where u_{ff} is smaller than u_{fn} and u_{nn}. That is, the information is of little



(a) Regular networks (b) ER random networks (c) BA scale free networks

Fig. 2: ESS (p_f^*) in the three scenarios with $q = 0.0025$.

value to most users, and without irrational users' forwarding, it will only reach a small group of people and die out very soon. We are interested in whether a few irrational users can help change the propagation of such information and how much more people it can reach. For the other two scenarios where $u_{ff} > u_{fn} > u_{nn}$ and $u_{fn} > u_{ff} > u_{nn}$ in [13], most people are interested in the information, and it will propagate across the entire network with and without irrational users. It is less interesting to our study and the analysis is omitted here. Therefore, $\delta_1, \delta_2 > 0$, as shown in Fig. 1.

From [13], p_f should satisfy $\dot{p}_f = 0$ at the ESS. Also, q of the entire population are irrational users who always use strategy S_f, so p_f^* should be in the range $[q, 1]$. In addition, the second order derivative of p_f should be negative at the ESS to satisfy the stability requirement. Same as in [13], we assume that $\overline{k} > 4$ and $\alpha, q \ll 1$. From (2), the roots of $\dot{p}_f = 0$ are: $p_f = 0$, $p_f = 1$, $p_f = x_0 \approx q$, and the roots of $g(p_f) = 0$. Note that the pole y_0 in (2) will not influence the ESS since $y_0 < q$. Also p_f^* cannot be 0 since $p_f^* \geq q > 0$. In addition, we can show that $p_f = x_0 \approx q$ is an unstable state and thus excluded from our analysis.

Therefore, the ESS candidates are 1 and the positive root(s) of $g(p_f) = 0$. From (2), the roots of $g(p_f) = 0$ are

$$x_{\pm} = \frac{-\gamma_1 \pm \sqrt{\gamma_1^2 - 4\gamma_2\gamma_0\frac{q}{\alpha}}}{2\gamma_2}. \tag{3}$$

Note that $\gamma_0 > 0$, and we can show that $s(\overline{k} - 3v) > 0$. Thus, depending on $\delta_1 - 2\delta_2$, $\gamma_2 = s(\overline{k} - 3v)(\delta_1 - 2\delta_2)$ may take different values, and there are three possible scenarios.

- **Scenario 1 (S1): $\boldsymbol{\delta_1 < 2\delta_2}$ and $\boldsymbol{\gamma_2 < 0}$**

In this scenario, $g(p_f) = 0$ has only one positive root x_-. We can show that $x_- > q$. If $x_- \in (q, 1)$, $\ddot{p}_f$ at x_- and 1 are negative and positive, respectively, which indicates that $p_f^* = x_-$. When $x_- \geq 1$, the only candidate for ESS is 1, and we can show that $\ddot{p}_f < 0$ at 1. Summarizing the above analysis, the ESS in Scenario 1 is $p_f^* = min(x_-, 1)$.

As shown in Fig. 1, u_{nn} is comparable to u_{fn} and u_{ff} in this scenario, and forwarding may potentially bring some rewards to the users. Thus, some users may still consider taking strategy S_f.

- **Scenario 2 (S2): $\boldsymbol{\delta_1 > 2\delta_2}$ and $\boldsymbol{\gamma_2 > 0}$**

In this scenario, we can show that $\gamma_1 < 0$, and the analysis depends on $\Delta = (\gamma_1^2 - 4\gamma_2\gamma_0 q/\alpha)$.

- When $\Delta < 0$, $g(p_f) = 0$ has no real roots. The only candidate for ESS is 1.
- When $\Delta = 0$, we can show that the only root of $g(p_f) = 0$ is not stable. The only candidate for ESS is 1.
- When $\Delta > 0$, both x_- and x_+ are positive. We can show that $g(q)g(1) < 0$, $x_- \in (q, 1)$ and $x_+ > 1$. Consequently, both 1 and x_- are candidates for ESS.

By checking $\ddot{p}_f$ at each candidate, the ESS in Scenario 2 is:

$$p_f^* = \begin{cases} x_-, & \Delta > 0, \\ 1, & \Delta \leq 0. \end{cases} \tag{4}$$

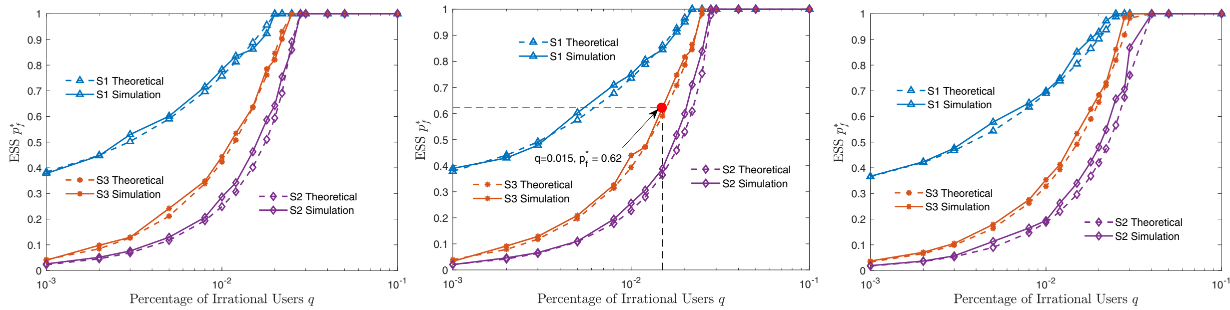
In this scenario, not forwarding has a much larger payoff than forwarding, so compared to Scenario 1, fewer people will take strategy S_f.

- **Scenario 3 (S3): $\boldsymbol{\delta_1 = 2\delta_2}$ and $\boldsymbol{\gamma_2 = 0}$**

This is the boundary between Scenario 1 and Scenario 2, and at this point, $g(p_f)$ is a linear function whose root is $x_3 = -q\gamma_0/(\alpha\gamma_1)$. We can show that $x_3 > q$. Similar to the analysis in Scenario 1, we can show that the ESS in Scenario 3 is $p_f^* = min(x_3, 1)$.

From the above analysis, the ESS p_f^* is a non-decreasing function of q. This is in agreement with the intuition that when there are more irrational users who always forward, more users will be influenced and adopt the strategy S_f. From (2), the network structure will also affect the ESS, though the analysis is much more complicated. We will show its influence on the ESS via simulation results in the next section.

We also investigate from the information source's perspective, and ask the following question: "If the information is less interesting to users and is unlikely to spread across the network without irrational users' help, what is the minimum



(a) Regular networks (b) ER random networks (c) BA scale free networks

Fig. 3: ESS (p_f^*) in the three scenarios with $\overline{k}$=10.

number of irrational users required to achieve a certain desired ESS?"

Given the desired ESS $\underline{p}_f^*$, the payoff matrix, and the network degree distribution $\lambda(k)$, we can use (2) to calculate the parameters γ_2, γ_1 and γ_0. From the above analysis, on average, we required at least

$$\underline{q} = -\alpha \frac{\gamma_1 \underline{p}_f^* + \gamma_2 \underline{p}_f^{*2}}{\gamma_0} \tag{5}$$

of the users to be irrational to achieve the desired ESS.

4. SIMULATION RESULTS

This section shows our simulation results on synthetic networks. We consider three types of synthetic networks: regular networks, Erdös Rényi (ER) random networks and Barabási-Albert (BA) scale free networks. The network sizes are all set to 2000, and 200 users (including irrational users) are randomly selected to initially use strategy S_f, and the rest use strategy S_n. α is set to 0.01. For each type of networks, 10 graphs are randomly generated, and 30 simulation runs are conducted for each graph. In each simulation run, the IM update rule is repeated until the network reaches the stable state. We consider three different payoff sets corresponding to the three scenarios in Section 3:

- **S1**: $u_{ff} = 0.3$, $u_{fn} = 0.5$, $u_{nn} = 0.4$;
- **S2**: $u_{ff} = 0.3$, $u_{fn} = 0.5$, $u_{nn} = 0.9$; and
- **S3**: $u_{ff} = 0.3$, $u_{fn} = 0.5$, $u_{nn} = 0.7$.

First we let $q = 0.0025$, i.e., there are only 5 irrational users, and change the average degree $\overline{k}$ to study the influence of network structure on the ESS. We also use the ESS without irrational users in [13] as the baseline. The result is shown in Fig. 2. We can see that the analytical results match well with the simulation results on all networks. We can also see that p_f^* decreases as the average degree $\overline{k}$ increases. This is because when $\overline{k}$ increases, each rational user has more neighbors who will influence his/her decision, and the impact of irrational users is reduced. Also, with our simulation setup, in Scenario 1, even 5 irrational users could cause at least 10% more users to adopt the forwarding strategy for all three types of networks. In Scenario 2 and Scenario 3, the information is unlikely to spread without irrational users' help. Now with only 5 irrational users, the information is able to spread out and there are about 3% to 15% users who will finally take the forwarding strategy at the ESS.

Fig. 3 shows the simulation results when the average degree $\overline{k}$ is fixed as 10 and when q changes from 0.001 to 0.1. Same as in Fig. 2, our simulation results match well with our analytical results, and more irrational users can help increase p_f^* by a larger amount. Furthermore, from Fig. 3, if we consider ER random networks with $\overline{k} = 10$ and the payoff values in S3, and set the desired ESS $\underline{p}_f^* = 60\%$, both simulation results and our analytical results in (5) show that on average, q should be no smaller than 1.5%. We observe the same trend for other parameter settings and other types of networks.

In addition, from both Fig. 2 and Fig. 3, we observe that among the three scenarios, Scenario 1 has a much larger p_f^* than Scenario 2, and that of Scenario 3 is in between. This is consistent with our analysis in Section 3.

5. CONCLUSION

In this work, we study information propagation over social networks, focus on irrational users who always forward the information, and analyze their impact on the information diffusion process. Our theoretical analysis and simulation results show that a few irrational users can significantly increase the number of users who adopt the forwarding strategy. We also show that more irrational users and a smaller average network degree will cause a wider spread of the information. In addition, we investigate from the information source's perspective and derive the minimum number of irrational users required to achieve the desired propagation state.

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