COMBATING JAMMING IN WIRELESS NETWORKS: A BAYESIAN GAME WITH JAMMER'S CHANNEL UNCERTAINTY

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ABSTRACT

Due to the shared and open-access nature of the wireless medium, wireless networks are vulnerable to jamming attacks. In this paper we study the problem of resource assigning in a single carrier communication system, where a user is communicating with a destination in the presence of a jammer. The jammer's channel to the destination is assumed flat fading, and its gain is known in probabilistic terms. In particular, the jammer's channel gain could take any value out of a finite set, with an a priori known probability. We model the problem in a Bayesian jamming game framework with utility the user throughput. We prove the existence and uniqueness of Nash and Stackelberg equilibria, and derive the equilibrium strategies in closed form. Our theoretical results, also supported by simulations, suggest that the Nash strategy is more sensitive to varying a priori probabilities, as compared to the Stackelberg strategy.

Index Terms— Bayesian game, Jamming, Nash equilibrium, Stackelberg equilibrium

1. INTRODUCTION

Due to the shared and open-access nature of the wireless medium, wireless networks are vulnerable to malicious attacks, such as jamming or spoofing. Due to the severity of the potential consequences of such threats, wireless security has continued to receive considerable attention by the research community. A comprehensive survey of security threats and detection techniques in cognitive radio networks can be found in [1]. Since jamming problems involve multiple agents (users and adversaries), each having their own objective, non-cooperative game theory, and the powerful concepts of equilibrium strategies are natural tools to study such problems [2]. One solution to the non-cooperative game is the Nash equilibrium, according to which, none of the agents has a motivation to diverse from equilibrium strategy due to loss in gain if all the other agents follow their own equilibrium strategies. Examples of designing Nash equilibrium in anti-jamming problems include [3–9]. Another solution

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is the *Stackelberg equilibrium*, where one of the agents is the leader and others are the followers, which impels a hierarchical behavior among the agents. Examples of designing Stackelberg equilibrium in anti-jamming problems include [10–13].

A key characteristic of wireless access networks is that the agents (users and adversaries) might not have complete information regarding the other agent's identities, traffic dynamic and channel characteristics and agent's location [14]. Uncertainty about the jammer's location was examined in [15], within an FDMA scheme and applying a matrix game. As the fading channel gains depend on the distance between the jammer and the receiver, uncertainty on fading channel gains can also be interpreted as uncertainty about the jammer's location. Such uncertainty on fading channel gains was also considered in [16–18]. In [16], non-hostile interference caused by selfish user communication was considered in a single carrier network. In [17], hostile interference caused by a malicious user in low SINR OFDM communications was considered. In [18], the jamming problem was investigated in a single carrier transmission scheme, assuming a flat fading channel and using the SINR as the user utility. In particular, such SINR utility can approximately linearize the throughput expression if the SINR is low enough. Also, a hierarchical iteration between a user and a jammer (i.e., a Stackelberg equilibrium framework) was employed.

The most related paper to our contribution is [18], but here we consider throughput as the user's utility instead of SINR employed as user's utility in [18]. To the best knowledge of the authors, this is the first work where, under uncertainty on the jammer's channel and using throughput as user utility, the equilibrium strategy is found in closed form for the Nash equilibrium framework (i.e., without assuming a hierarchy on the rivals' behavior) as well as for the Stackelberg equilibrium framework (i.e., impelling hierarchy on the rivals' behavior by considering the user as a leader and the jammer as the follower).

The organization of this paper is as follows: in Sec-



Fig. 1. The user and the jammer.

tion 2, basic model is formulated within the Nash equilibrium scenario. Uniqueness of the equilibrium is proven as well as equilibrium strategies are derived in closed form. In Section 3, the model is modified and solved for the Stackelberg scenario. Finally, in Section 4, conclusions and discussions are offered.

2. NASH EQUILIBRIUM

Let us consider a single carrier transmission scheme with two agents, namely, a user and an adversary. The adversary is a jammer, who intends to degrade the user's communication by generating interference. We study the resource allocation problem in the framework of game theory. The resource for the user is its transmission power P, with $P \geq 0$, and for the adversary is its jamming power J, with $J \ge 0$. Let C_P and C_J be the user transmission cost per unit transmission power and the jamming cost per unit jamming power, respectively. Let σ^2 and h be the background noise variance and gain of the fading source-destination channel, respectively. All channels are considered to be flat fading. The jammer is assumed to have complete knowledge of h. The user does not know the jammer's exact location. The user only knows that the jammer can be at distance d_i from the receiver with a priori known probability q_i , for $i = 1, \ldots, n$, and $\sum_{i=1}^{n} q_i = 1$. Such knowledge could have been obtained based on past observations of jammer behavior. Equivalently, the user knows that the gain of the jammer's channel to the destination is $g_i = G/d_i^2$ (with G > 0 some constant) with probability q_i [3] (see Fig. 1). In the following, by *jammer type-i* we refer to a jammer employing strategy J_i and having fading channel gain q_i .

Let $J = (J_1, \ldots, J_n)$. The payoff for the user is taken as the difference between expected throughput and the transmission cost, i.e.,

$$v_U(P, \mathbf{J}) \triangleq \sum_{i=1}^n q_i \ln\left(1 + \frac{hP}{\sigma^2 + g_i J_i}\right) - C_P P.$$

Further, let us define the cost function of the type-i jammer to be the sum of the user's throughput and the jam-

ming cost, i.e.,

$$v_{J,i}(P,J_i) \triangleq \ln\left(1 + \frac{hP}{\sigma^2 + g_i J_i}\right) + C_{J,i} J_i \text{ for } i = 1, \dots, n.$$

The user wants to use power P that maximizes its payoff, $v_U(P, \mathbf{J})$, while each jammer type wants to minimize its cost, $v_{J,i}(P, J_i)$. Thus, we look for Nash equilibrium (NE) strategies [2], which due to incomplete information can also be interpreted as Bayesian equilibrium. In particular, we look for (P_*, \mathbf{J}_*) , such that for any (P, \mathbf{J}) the following inequalities hold:

Let us denote by Γ_{NG} the above described Nash game (NG). By (1), (P, \mathbf{J}) is a NE if and only if P is the best response strategy to $\mathbf{J} = (J_1, \ldots, J_n)$, while J_i , for each i, is the best response strategy to P, i.e., they are solutions of the following best response equations:

$$P = BR_U(\boldsymbol{J}) \triangleq \operatorname*{arg\,max}_{P \ge 0} v_U(P, \boldsymbol{J}), \tag{2}$$

$$J_i = \operatorname{BR}_{J,i}(P) \triangleq \underset{J_i \ge 0}{\operatorname{arg\,min}} v_{J,i}(P, J_i), \text{ for } i = 1, \dots, n.$$
(3)

The following theorem provides the equilibrium strategies in closed form.

Theorem 1 The game Γ_{NG} has a unique NE (P, \mathbf{J}) . In particular,

(a) If

$$1/\sigma^2 \le C_P/h \tag{4}$$

then

$$P = 0 \text{ and } \boldsymbol{J} = \boldsymbol{0}. \tag{5}$$

(b) If

$$1/\sigma^2 - C_J/\underline{g} \le C_P/h < 1/\sigma^2 \tag{6}$$

with $g \triangleq \min g_i$ then

$$P = 1/C_P - \sigma^2/h \text{ and } \boldsymbol{J} = 0$$
(7)

(c) If

$$C_P/h < 1/\sigma^2 - C_J/\underline{g} \tag{8}$$

then

$$J_{i} = \begin{cases} \frac{1}{g_{i}} \left(\sqrt{\left(\frac{hP}{2}\right)^{2} + \frac{g_{i}}{C_{J}}hP} - \frac{hP}{2} - \sigma^{2} \right), & i \in \mathcal{I}(P), \\ 0, & i \notin \mathcal{I}(P), \end{cases}$$
(9)

where

$$\mathcal{I}(P) = \left\{ i \in \{1, \dots, n\} : g_i \ge C_J \sigma^2 (\sigma^2 / (hP) + 1) \right\}$$
(10)

and P is the unique positive root of equation

$$\mathcal{F}_U(P) = C_P,\tag{11}$$

with

$$\mathcal{F}_{U}(P) \triangleq \sum_{i \in \mathcal{I}(P)} q_{i} \frac{h}{hP/2 + \sqrt{(hP/2)^{2} + g_{i}hP/C_{J}}} + \sum_{i \notin \mathcal{I}(P)} q_{i} \frac{h}{\sigma^{2} + hP}.$$
(12)

Moreover, \mathcal{F}_U decreases from h/σ^2 to 0 as P increases from 0 to ∞ . Thus, the unique positive root of (11) can be found via the bisection method.

Finally, the best response strategy $BR_{J,i}(P)$ of the type-*i* jammer is given by (9).

Thus, (4) yields non-active user's strategy P = 0. The set $\mathcal{I}(P)$ defines the types of jammers such that $J_i > 0$, i.e., active jammer types.

The following corollary establishes the condition for the set $\mathcal{I}(P)$ to be empty, i.e., the condition which guarantees the absence of hostile interference.

Corollary 1 (a) The set of active jammer types, $\mathcal{I}(P)$, is empty for any $P \ge 0$ if and only if:

$$\underline{g} \le C_J \sigma^2. \tag{13}$$

(b) If (13) holds, then J = 0 and $P = \max\{1/C_P - \sigma^2/h, 0\}$.

3. STACKELBERG EQUILIBRIUM

In this section we consider a hierarchical relation between the user and the jammer, namely, the user is the leader and the jammer is the follower. Such scenario is formulated as a two-level optimization problem, with the user (top-level) to the jammer (low-level). The problem can be solved by backward induction, and the solution is referred to as the Stackelberg equilibrium (SE), while the game is called Stackelberg game (SG) [2]. In the first step of this game, for a fixed P, determined by the user, the jammer of each type tries to minimize its cost function. Thus, per Theorem 1, the type-*i* jammer intends to apply strategy BR_{J,i}(P), which is given in closed form by (9). In the second step of the two-level game, the user selects the optimal P to get a maximal payoff, i.e., to solve the following optimization problem:

$$\max_{P \ge 0} \Psi(P), \text{ with } \Psi(P) \triangleq v_U(P, (\mathrm{BR}_{J,1}(P), \dots, \mathrm{BR}_{J,n}(P))).$$
(14)

Let us denote by Γ_{SG} the above described SG. By (9), we can present (14) in closed form as follows:

$$\Psi(P) = \sum_{i \in \mathcal{I}(P)} q_i \psi_i(P) + \psi_0(P) \sum_{i \notin \mathcal{I}(P)} q_i - C_P P, \quad (15)$$

where
$$\psi_i(P) \triangleq \ln\left(1 + \frac{2}{\sqrt{1 + 4g_i/(C_J h P)} - 1}\right)$$
 and $\psi_0(P) \triangleq \ln\left(1 + hP/\sigma^2\right)$.

Theorem 2 If the set of active jammer types is empty (i.e., (13) holds), or the user is non-active (i.e., (4) holds), then Γ_{SG} has a unique SE, which coincides with NE given by Corollary 1(b) and (5) correspondingly.

Let us now consider the case in which the set of active jammer types is non-empty and user is active, i.e.,

$$g > C_J \sigma^2$$
 and $h > C_P \sigma^2$. (16)

By (16), the set $\mathcal{I}(P)$ defined in (10) can be presented in the following equivalent form:

$$\mathcal{I}(P) = \{i \in \{1, \dots, n\} : P \ge P_i\},\$$
where $P_i \triangleq \sigma^4 / (h(g_i/C_J - \sigma^2))$. Based on (16), it holds
that $P_i > 0$ for any *i*. Without loss of generality we can
assume that $g_1 > g_2 > \ldots > g_n$. Thus, $P_1 < P_2 < \ldots < P_n$. Let $P_0 = 0$ and $P_{n+1} = \infty$. For each *P* there is an
integer $t = t(P) \in \{0, \dots, n\}$ such that $P_t \le P < P_{t+1}$.
Then, (15) can be presented as follows:

$$\Psi(P) = \Psi_t(P) \triangleq \sum_{i=1}^t q_i \psi_i(P) + \psi_0(P) \sum_{i=t+1}^n q_i - C_P P.$$
(17)

Based on (17), $\Psi(P)$ is continuous for P > 0 and differentiable everywhere except on the finite set $\mathcal{P} = \{P_1, \ldots, P_n\}$. Moreover,

$$\frac{d\Psi}{dP}(P) = \frac{d\Psi_t}{dP}(P) = \xi_t(P) - C_P \text{ for } P \in (P_t, P_{t+1}), \quad (18)$$

where

$$\xi_t(P) \triangleq \sum_{i=1}^t q_i \sqrt{\frac{C_J h}{(C_J h P + 4g_i)P}} + \frac{h}{\sigma^2 + hP} \sum_{i=t+1}^n q_i.$$
(19)

Let us formulate the following two auxiliary lemmas.

Lemma 1 (a)
$$\frac{d\psi_0}{dP} = \frac{h}{\sigma^2 + hP}$$
 and $\frac{d^2\psi_0}{dP^2} = -\frac{h^2}{(\sigma^2 + hP)^2}$,
(b) $\frac{d\psi_i}{dP} = \sqrt{\frac{C_Jh}{(C_JhP + 4g_i)P}}$ and $\frac{d^2\psi_i}{dP^2} = -\sqrt{\frac{C_Jh(C_JhP + 2g_i)^2}{(C_JhP + 4g_i)^3P^3}}$,
(c) $\xi_t(P_t) < \xi_{t-1}(P_t)$.

Lemma 2 There is the unique t_* such that one of the following two relations holds:

$$\xi_{t_*}(P_{t_*}) \le C_P \le \xi_{t_*-1}(P_{t_*}),\tag{20}$$

$$\xi_{t_*}(P_{t_*+1}) < C_P < \xi_{t_*}(P_{t_*}). \tag{21}$$

In the following theorem we prove the uniqueness of SE and also find SE is in closed form.

Theorem 3 Let us assume that (16) holds. The Γ_{SG} has a unique SE $P, J_i = BR_{J,i}(P), i = 1, ..., n$, where (i) if (20) holds then $P = P_{t_*}$,

(ii) if (21) holds then P is the unique root in (P_{t_*}, P_{t_*+1}) of the equation $\xi_{t_*}(P) = C_P$. Moreover, because ξ_{t_*} is a decreasing function, such P can be found via the bisection method.

4. DISCUSSION OF THE RESULTS

We consider a set of channel states consisting of $q_2 =$ $1 - q_1$ (n = 2), and $\sigma^2 = 1$, $C_P = 0.2$, $C_J = 0.2$, h = 1, $(g_1, g_2) = (10, 1)$. Fig. 2 illustrates the dependence of user and jammer strategies on a priori probability q_1 . Note that the absence of a hierarchical structure for making decision in the Nash game can be interpreted as the Nash game having higher power consumption as compared to Stackelberg game. Fig. 2 illustrates that Nash equilibrium strategies assign higher power to transmit signals than Stackelberg equilibrium strategies. Thus, on one hand, the higher competition in Nash game as compared to Stackelberg game makes the user and the jammer transmit in higher power, on the other hand, the lower competition of the Stackelberg game as compared to Nash game could make the strategies less sensitive to a priori information on jammer's location varying network parameters. This phenomena is reflected by flat segments in rival's strategies occurring when case (i) of Theorem 3 holds. In this case the value of strategy P_t . does not depend on $\{q_i\}$, while the interval of such nonsensitivity given by (19) and (20), depends on $\{q_i\}$.

Appendix

Proof of Theorem 1: Since $v_U(P, \mathbf{J})$ is concave on P, and $v_{J,i}(P, J_i)$ is convex on J_i , the Nash's theorem [2]. implies existence of at least one equilibrium. To prove uniqueness as well as to design equilibrium strategies in closed form we have to solve directly the best response equations (2). Since $v_U(P, \mathbf{J})$ is concave in P, and $v_{J,i}(P, J_i)$ is convex in J_i , P and \mathbf{J} are the solution of the response equations (2) and (3) if and only if the following conditions hold:

$$\sum_{i=1}^{n} q_i \frac{h}{\sigma^2 + hP + g_i J_i} \begin{cases} = C_P, & P > 0, \\ \le C_P, & P = 0, \end{cases}$$
(22)

$$\frac{hg_i P}{(\sigma^2 + g_i J_i)(\sigma^2 + hP + g_i J_i)} \begin{cases} = C_J, & J_i > 0, \\ \le C_J, & J_i = 0. \end{cases}$$
(23)

By (23), if P = 0 then $J_i = 0$. Then, by (22), $(0, \mathbf{0})$ is an equilibrium if and only if (4) holds. Thus, we can assume that P > 0. Let us consider separately only two possible cases: (I) $\mathbf{J} = \mathbf{0}$ and (II) $\mathbf{J} \neq \mathbf{0}$.

(I) Let J = 0. On substituting J = 0 into (22) we get (7). Moreover, since P > 0, via (7), we get that

$$C_P/h < 1/\sigma^2. \tag{24}$$

On substituting (7) into (23) we get that the following relation should hold for any i:

$$g_i C_P \left(1/C_P - \sigma^2/h \right) / \sigma^2 \le C_J.$$
⁽²⁵⁾



Fig. 2. The user's strategy (left) and jammer's strategy (right) as functions on q_1 , where "(i)" and "(ii)" on the graph correspond to the cases of Theorem 3.

The last inequality is equivalent to $1/\sigma^2 - C_J/g_i \leq C_P/h$. Since the last inequality has to hold for any i, (24) yields (6), and then (b) follows.

(II) Let $J \neq 0$. Note that

$$hg_i P/((\sigma^2 + g_i J_i)(\sigma^2 + hP + g_i J_i)) = C_J$$
 (26)

is equivalent to the following quadratic equation

$$x^{2} + hPx - g_{i}hP/C_{J} = 0 (27)$$

with $x = \sigma^2 + g_i J_i$. This equation has the unique positive root

$$x = -hP/2 + \sqrt{(hP/2)^2 + g_i hP/C_J}.$$
 (28)

Since $J_i \ge 0$, then, $x \ge \sigma^2$. Substituting (28) into $x \ge \sigma^2$ implies that *i* has to belong to $\mathcal{I}(P)$ given by (10). This and (22) implies (9) and that *P* is the root of the equation (11), and the result follows.

Proof of Theorem 2: By Corollary 1(a), we have that if (13) holds then $\Psi(P) = \ln(1 + hP/\sigma^2) - C_P P$. This is utility of CDMA transmission in absent of the jammer [19]. This straightforward implies the result.

Proof of Lemma 1: (a) and (b) follow by straightforward calculation. By (16) and (19), $\xi_t(P_t) - \xi_{t-1}(P_t) = -\frac{h(g_t - C_J \sigma^2)^2}{\sigma^2(2g_t - C_J \sigma^2)} < 0$, and the result follows.

Proof of Lemma 2: The result follows from (19) and Lemma 1.

Proof of Theorem 3: $\Psi(P)$ is differentiable on P apart from finite set of points \mathcal{P} . By Lemma 1 and (18) we have that there exists \tilde{P} such that

$$\lim_{\tau\uparrow P} \frac{d\Psi}{dP}(\tau) \text{ and } \lim_{\tau\downarrow P} \frac{d\Psi}{dP}(\tau) \begin{cases} > C_P, \quad P < \tilde{P}, \\ < C_P, \quad P > \tilde{P}. \end{cases}$$
(29)

Here, upper limit and lower limit coincide for each $P \notin \mathcal{P}$. Thus, by (29), $\Psi(P)$ achieves its maximum at the unique point, and this point is $P = \tilde{P}$. By Lemma 1 and Lemma 2, $\tilde{P} = P_*$, and the result follows.

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