MULTI-SPECTRAL IMAGE DENOISING WITH SHARED DICTIONARIES AND LOW-RANK REPRESENTATION

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ABSTRACT

As a 3-order tensor, a multi-spectral image (MSI) has dozens of spectral bands, which can deliver more faithful representation for real scenes. However, MSIs are often corrupted by noise in the sensing process, which deteriorates the performance of higher-level classification and recognition tasks. In this paper, we propose a novel tensor dictionaries learning method for MSI denoising, where two shared dictionaries are learned from MSI groups of similar blocks in the spatial domain and the spectral domain, respectively. In addition, we enforce a low rank structure for the representations of MSI groups under the learned dictionaries, which captures the latent structure in MSIs. Our experiments demonstrate that the proposed method achieves the best performance in comparison with the state-of-the-art methods.

Index Terms— Multi-spectral image denoising, dictionary learning, low-rank tensor model

1. INTRODUCTION

Signal processing techniques for tensors have attracted growing interest of researchers in recent years. As a 3-order tensor, an MSI has dozens of spectral bands which range from infrared and ultra-violet. Compared with RGB images which only have three spectral bands, MSIs convey more information of real scenes. However, an MSI always suffers from corruption or noise in the sensing process [1]. As a low level signal processing technique, MSI denoising is the key to many high-level computer vision tasks, such as segmentation and classification whose performance highly rely on the quality of the data.

As a model based approach, dictionary learning (DL) aims to find a set of atoms from some training data, where each signal can be represented by a few of these atoms. By using a learned dictionary of some signal ensemble, noise can be effectively removed by solving a sparse signal recovery problem [2, 3]. Various dictionary learning methods have been proposed in literature [4, 5, 6, 7, 8], where they treat each signal as a vector.

Tensor dictionary learning, which keeps the multidimensional structure of tensors, has attracted growing interest of researchers to restore 3D image from noise in the past years. By using the CANDECOMP/PARAFAC (CP) decomposition, Duan et al. extend the K-SVD method for tensor data, where a set of rank-one tensors are learned to represent the tensor signals [9]. Based on the Tucker model of tensors, Zubair and Wang propose to learn multiple orthogonal dictionaries along different modes of tensors, where the core tensor have sparse non-zeros elements [10]. In [11], Ding et al. consider joint sensing matrix and sparsifying dictionary optimization for tensor compressive sensing. In [12], Qi et al. divide an MSI into small 3-order tensor blocks, and learn overcomplete dictionaries for each mode of the blocks via a two-phase block-coordinate-relaxation approach including sparse coding and dictionary updating. These methods are based on dictionary learning that exploit a sparse model, while they fails to further employ structural information for real MSI denoising. In applications of image denoising, in order to further improve image quality with redundant information, non-local similar small patches in space are clustered into groups, instead of using the whole image [13, 14]. For example, there exist many similar blocks in space of an MSI, which can be grouped into a tensor. In addition, images of different spectral bands have high correlations, and thus an MSI may have a low rank along the spectrum mode. To exploit these structural information, the low-rank model is exploited in MSI denoising. In [15], Peng et al. propose to enforce a smaller core tensor of Tucker low-rank approximation for each tensor group stacked by similar patches of the MSI. In [16], Xie et al. consider both the low-rank model and the sparsity model for each tensor that is consisted of a group of similar blocks of an MSI. Each group are processed

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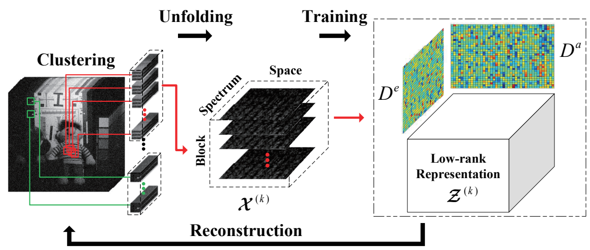

Fig. 1. The framework for MSI denoising of the proposed method.

separately without considering inter-group correlations.

Motivated by the observation that different groups of similar blocks in the MSI have some degree of correlation. In this paper, we propose an MSI denoising method that learns shared dictionaries for all the groups. This method differs with the denoising methods in [15, 16] which train different dictionaries for distinct groups. The overcomplete space dictionary and spectral dictionary are designed that consider the correlation of different groups among the whole image. Furthermore, these groups have the low-rank structure under the learned dictionaries along different modes. As shown in Fig. 1, full bands blocks of an MSI can be extracted by window and then similar blocks are clustered into a 3-order tensor group with three modes corresponding to the spatial domain, the spectral domain and blocks. All grouped tensors share the same overcomplete dictionaries in both the spatial mode and the spectral mode. An effective algorithm based on the alternating direction method of multipliers(ADMM) [17] is used to solve the newly proposed dictionary learning framework for multi-spectral image denoising. Experimental results demonstrate that the new method outperforms the state-of-the-art for MSI denoising.

2. LOW-RANK TENSOR DICTIONARY LEARNING FOR MULTI-SPECTRAL IMAGE DENOISING

2.1. Notations

For convenience, the following notations are used. The order of a tensor is the number of modes. Elements of an N-order tensor $\mathcal{X} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$ are denoted by $x_{i_1 \ldots i_n \ldots i_N}$, where i_n $(1 \leq i_n \leq I_N)$ refers to the nth mode index. The mode-n vectors of an N-order tensor $\mathcal{X}$ are the I_n dimensional vectors obtained from $\mathcal{X}$ by varying index in the nth mode while keeping the indices of other modes fixed. The unfolding matrix of tensor $\mathcal{X}$ at the nth mode is $X_{(n)} \in \mathbb{R}^{I_n \times (I_1 \ldots I_{n-1} I_{n+1} \ldots I_N)}$, where the columns are mode-n vectors of $\mathcal{X}$. Conversely, the matrix can be folded to a tensor at the nth mode by arranging its columns as mode-n vectors of the tensor. The n-mode product of a tensor $\mathcal{X}$ and a matrix $U \in \mathbb{R}^{J \times I_n}$ is defined as $\mathcal{Y} = \mathcal{X} \times_n U \in \mathbb{R}^{I_1 \times \ldots \times I_{n-1} \times J \times I_{n+1} \times \ldots \times I_N}$. The unfolding matrix of tensor $\mathcal{Y}$ at the nth mode can be expressed as $Y_{(n)} = UX_{(n)}$. The inner product of two tensors $\mathcal{X}, \mathcal{Y} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$ is the sum of the products of their entries, i.e., $\langle \mathcal{X}, \mathcal{Y} \rangle = \sum_{i_1=1}^{I_1} \cdots \sum_{i_N=1}^{I_N} x_{i_1 \ldots i_N} y_{i_1 \ldots i_N}$. The Frobenius norm of a tensor is defined as $\|\mathcal{X}\|_F = (\sum_{i_1=1}^{I_1} \cdots \sum_{i_N=1}^{I_N} x_{i_1 \ldots i_N}^2)^{1/2}$.

2.2. The Proposed MSI Denoising Framework

For an MSI $\mathcal{M} \in \mathbb{R}^{L \times W \times H}$ with H bands, we extract S overlapping blocks $\mathcal{M}^i \in \mathbb{R}^{d_L \times d_W \times H}$ by using a sampling window that traverses the whole image with step lengthes p_L and p_W of the two spacial coordinates. Each block $\mathcal{M}^i$ is unfolded in spectrum domain to be a matrix $M_{(3)}^i \in \mathbb{R}^{H \times d_L d_W}$, which has a spatial mode and a spectral mode. To exploit the non-local similarity property of images, blocks of the MSI can be clustered into K groups, where the kth group forms a tensor group denoted as $\mathcal{X}^{(k)} \in \mathbb{R}^{d_L d_W \times H \times s^{(k)}}$ $(\sum_{k=1}^{K} s^{(k)} = S)$ with the 3rd mode referring to different blocks. For kth tensor group $\mathcal{X}^{(k)}$, the spatial local correlation between adjacent pixels in the 1st mode, the spectral high correlation in 2nd mode and the similarity of similar blocks in the 3rd mode imply the latent low-rank structure.

The tensor sparse representation (TenSR) model [12] learns the shared overcomplete dictionaries from all blocks which contain the information of the whole MSI. However, the TenSR model, which consider a sparse representation under the dictionaries, fails to consider the naturally low rank structure in a tensor group. To consider the prior for one tensor group and the correlation of all tensor groups, we consider a low-rank representation under the learned dictionaries rather than a sparse representation. This model is inspired from the observation that similar blocks in a group have similar representation in the 3rd mode and the tensor group have some degree of correlations in the spatial domain and the spectral domain. Similar idea of low-rank representation has been applied for 2D image processing in [18]. For convenience, we define τ as the dictionary redundancy, which is the ratio of the number of columns to the number of rows of the dictionary. The proposed low-rank tensor dictionary learning framework for MSI denoising is given as follows:

$$
\begin{aligned}
\min_{\substack{D^a, D^e \\ \{\mathcal{Z}^{(k)}\}}} \sum_{k=1}^{K} & \left(\left\| \mathcal{X}^{(k)} - \mathcal{Z}^{(k)} \times_1 D^a \times_2 D^e \right\|_F^2 + \lambda^{(k)} \left\| \mathcal{Z}^{(k)} \right\|_* \right) \\
\text{s.t.} \quad & \|D^a(:,r)\|_2^2 = 1 \text{ for } r = 1, \ldots, \tau_a d_L d_W \\
& \|D^e(:,r)\|_2^2 = 1 \text{ for } r = 1, \ldots, \tau_e H,
\end{aligned}
\tag{1}
$$

where $D^a \in \mathbb{R}^{d_L d_W \times \tau_a d_L d_W}$ and $D^e \in \mathbb{R}^{H \times \tau_e H}$ are dictionaries of the spatial mode and the spectral mode, respectively, τ_a and τ_e determine the redundancy of the two dictionaries, $\mathcal{Z}^{(k)} \in \mathbb{R}^{\tau_a d_L d_W \times \tau_e H \times s^{(k)}}$ is the tensor representation associated with the dictionaries, and $\lambda^{(k)} > 0$. By adding up the nuclear norm of unfolding matrix in each mode, we define the tensor nuclear norm to measure tensor multilinear rank as $\|\mathcal{X}\|_* = \sum_{n=1}^{N} \beta_n \left\|X_{(n)}\right\|_*$ and the weights in each mode satisfy $\sum_{n=1}^{N} \beta_n = 1$, which is validated in [19].

2.3. The Proposed Algorithm Based on ADMM

Solving the optimization problem in (1) is difficult, as the dictionaries and tensor representations are coupled together. Here we apply the ADMM [17, 19, 20] that splits the original problem into several subproblems and update variables iteratively. By introducing auxiliary matrices $C_n^{(k)}$ ($n = 1, 2, 3$ and $k = 1, ..., K$), the optimization problem in (1) is equivalently converted into

$$\begin{aligned}\min_{\substack{D^a,\, D^e \\ \left\{\mathcal{Z}^{(k)},\, C_n^{(k)}\right\}}} \quad & \sum_{k=1}^{K}\Big(\left\|\mathcal{X}^{(k)} - \mathcal{Z}^{(k)} \times_1 D^a \times_2 D^e\right\|_F^2 \\ & \qquad + \lambda^{(k)} \sum_{n=1}^{3} \beta_n \left\|C_n^{(k)}\right\|_* \Big) \\ \text{s.t.} \quad & \|D^a(:,r)\|_2^2 = 1 \text{ for } r = 1, \ldots, \tau_a d_L d_W \\ & \|D^e(:,r)\|_2^2 = 1 \text{ for } r = 1, \ldots, \tau_e H \\ & C_n^{(k)} = Z_{(n)}^{(k)} \text{ for } n = 1,2,3 \text{ and } k = 1, \ldots, K.\end{aligned} \tag{2}$$

The augmented Lagrangian function for the above problem is given as:

$$\begin{aligned}&\mathcal{L}_\rho\Big(D^a, D^e, \{\mathcal{Z}^{(k)}\}, \{C_n^{(k)}\}, \{Y_n^{(k)}\}\Big) \\ &= \sum_{k=1}^{K}\Big(\left\|\mathcal{X}^{(k)} - \mathcal{Z}^{(k)} \times_1 D^a \times_2 D^e\right\|_F^2 + \sum_{n=1}^{3}\Big(\lambda^{(k)}\beta_n \left\|C_n^{(k)}\right\|_* \\ &\qquad + \langle C_n^{(k)} - \mathcal{Z}_{(n)}^{(k)}, Y_n^{(k)}\rangle + \frac{\rho}{2}\left\|C_n^{(k)} - \mathcal{Z}_{(n)}^{(k)}\right\|_F^2\Big)\Big),\end{aligned} \tag{3}$$

where $Y_n^{(k)}$ ($n = 1, 2, 3$ and $k = 1, ..., K$) are the Lagrange multipliers, ρ is a positive scalar, and both D^a and D^e must satisfy column normalization. Now the goal is to minimize the augmented Lagrange function (3) under the column normalization constraints of dictionaries.

With fixed dictionaries, i.e., D^a and D^e, and Lagrange multipliers $Y_n^{(k)}$, the problem in (3) can be split into K subproblems which can be conquered in parallel.

For each subproblem, we update $C_n^{(k)}$ of the kth group with other variables fixed, which requires to solve the following optimization problem

$$\min_{C_n^{(k)}} \quad \lambda^{(k)}\beta_n \left\|C_n^{(k)}\right\|_* + \frac{\rho}{2}\left\|C_n^{(k)} - T_n^{(k)}\right\|_F^2, \tag{4}$$

where $T_n^{(k)} = Z_{(n)}^{(k)} - Y_n^{(k)}/\rho$. Define the singular value decomposition $T_n^{(k)} = U\Sigma V^T$, and σ_i is the ith element of the diagonal matrix Σ. The solution of the optimization problem in (4) is $C_n^{(k)} = UD_\kappa(\Sigma)V^T$, where $D_\kappa(\Sigma) = \text{diag}[\max\{0, \sigma_i - \kappa\}]$ denotes the singular value thresholding operator [21] and $\kappa = \lambda^{(k)}\beta_n/\rho$. To update the tensor representation $\mathcal{Z}^{(k)}$ with other variables fixed, the problem of $\min_{\mathcal{Z}^{(k)}} \mathcal{L}_\rho$ becomes

$$\begin{aligned}\min_{\mathcal{Z}^{(k)}} \quad & \left\|\mathcal{X}^{(k)} - \mathcal{Z}^{(k)} \times_1 D^a \times_2 D^e\right\|_F^2 \\ & + \sum_{n=1}^{3}\Big(\langle C_n^{(k)} - \mathcal{Z}_{(n)}^{(k)}, Y_n^{(k)}\rangle + \frac{\rho}{2}\left\|C_n^{(k)} - \mathcal{Z}_{(n)}^{(k)}\right\|_F^2\Big),\end{aligned} \tag{5}$$

which also has a close-form solution. The matrix unfolded form of the solution is

$$Z_{(3)}^{(k)} = \left[2X_{(3)}^{(k)} + \sum_{n=1}^{3}\left(\rho C_{n3}^{(k)} + Y_{n3}^{(k)}\right)\right]\left[2DD^T + 3\rho I\right]^{-1}, \tag{6}$$

where $C_{n3}^{(k)}$ and $Y_{n3}^{(k)}$ are derived by first folding the matrices $C_n^{(k)}$ and $Y_n^{(k)}$ in the nth ($n = 1, 2, 3$) mode and then unfolding them at the 3rd mode, respectively, $D = D^e \otimes D^a$ ($\otimes$ denoting Kronecker product), and I is an identity matrix. The tensor representation $\mathcal{Z}^{(k)}$ can be obtained by folding the matrix solution $Z_{(3)}^k$ in the 3rd mode. For different groups, we can conduct the above steps in parallel.

Now we consider the updating of D^a and D^e. As the dictionaries are shared among groups, all blocks of the multi-spectral image are used in the learning process, i.e., $\min_{\{D^a, D^e\}} \mathcal{L}_\rho$, which can be rewritten as

$$\begin{aligned}\min_{D^a, D^e} \; & \|\mathcal{X} - \mathcal{Z} \times_1 D^a \times_2 D^e\|_F^2 \\ \text{s.t.} \; & \|D^a(:,r)\|_2^2 = 1 \text{ for } r = 1, \ldots, \tau_a d_L d_W \\ & \|D^e(:,r)\|_2^2 = 1 \text{ for } r = 1, \ldots, \tau_e H,\end{aligned} \tag{7}$$

where $\mathcal{X} \in \mathbb{R}^{d_L d_W \times H \times S}$ is obtained by concatenating all K tensor groups $[\mathcal{X}^{(1)}, ..., \mathcal{X}^{(K)}]$ and $\mathcal{Z} \in \mathbb{R}^{\tau_a d_L d_W \times \tau_e H \times S}$ is acquired in the same way. Then we update each one of D^a and D^e with the other one fixed in (7). For updating D^a, the optimization problem in (7) becomes

$$\begin{aligned}\min_{D^a} \; & \left\|X_{(1)} - D^a A_{(1)}\right\|_F^2 \\ \text{s.t.} \; & \|D^a(:,r)\|_2^2 = 1 \text{ for } r = 1, \ldots, \tau_a d_L d_W\end{aligned} \tag{8}$$

where $X_{(1)} \in \mathbb{R}^{d_L d_W \times HS}$ and $A_{(1)} \in \mathbb{R}^{\tau_{spa} d_L d_W \times HS}$ are matrices by unfolding $\mathcal{X}$ and $\mathcal{A} = \mathcal{Z} \times_2 D^e$ respectively. (8) is a quadratically constrained quadratic programming problem and can be solved using a Lagrange dual [22]. The update of D^e can be conducted in the same manner.

At last, we update the Lagrange multipliers by

$$Y_n^{(k)} := Y_n^{(k)} + \rho\big(C_n^{(k)} - Z_{(n)}^{(k)}\big), \tag{9}$$

where we update $\rho := \mu\rho$ ($\mu > 1$) in different iterations to accelerate convergence. The iterative algorithm stops until some halting condition is satisfied, and the final denoised MSI is constructed from $\mathcal{Z}^{(k)} \times_1 D^a \times_2 D^e$ ($k = 1, \ldots, K$).

3. EXPERIMENTS

In order to verify the effectiveness of the proposed method for MSI denoising, we carry out simulated experiments on CAVE dataset [23], where each MSI has the size of 512×512 in space and includes 31 bands from $400nm$ to $700nm$ at $10nm$ steps. We compare the proposed method with various existing approaches including denoising methods for 2D images (K-SVD [5] and BM3D [13]) and denoising methods for 3D images (BM4D [14], Tdl [15], and KBRreg [16]). For K-SVD and BM3D, an MSI is processed band by band as multiple 2D images.

3.1. Experimental Settings

In the experiments, we normalize all MSIs in the CAVE dataset to the interval $[0\ 1]$. Gaussian noise is added with mean of 0 and variance of $\nu = 0.1$ on the whole MSI. Groups of an MSI are clustered by using the k-means++ [24], and each cluster forms a tensor group. The two dictionaries are initialized by extracting mode-n (n =1,2) vectors randomly of $\mathcal{X}^{(k)}$ with the redundancy ratio of $\tau_a = \tau_e = 1.5$. The spatial window size is set as $d_L = d_W = 7$ with the step $p_L = p_W = 4$. For the parameters in the proposed algorithm, we empirically set $\rho = 0.1$, $\mu = 1.3$, and $\beta = [\beta_1, \beta_2, \beta_3] = [0.1, 0.45, 0.45]$ where the small weight of β_1 on space means the weak local correlation. To ease the difficulty in the process of parameter tuning, we simply set $\lambda^{(k)} = 20\nu\sqrt{s^{(k)}}$, where $\lambda^{(k)}$ and ν has a positive correlation.

3.2. Experimental Results

All the 32 MSIs are used in the experiment for evaluating the performance of different methods, and we provide the averaged performance in the results. In the comparison, we employ four different performance indicators including peak signal-to-noise ratio (PSNR), structural similarity (SSIM), spectral angle mapper (SAM) [25] and dimensionless global relative error of synthesis (ERGAS) [26]. Recovered MSIs with higher PSNR and SSIM or lower SAM and ERGAS are usually considered as images with good quality. PSNR and SSIM are two conventional spatial-based indexes, while ERGAS and SAM are spectral-based evaluation indexes. The averaged denoising results are given in Table 1. It can be observed that the proposed method outperforms all the competing methods for all the four different quality indicators.

Index	Method					
	K-SVD	BM3D	BM4D	Tdl	KBRreg	**Ours**
PSNR	30.238	37.219	39.763	39.479	40.722	**41.158**
SSIM	0.603	0.918	0.941	0.946	0.944	**0.967**
SAM	0.552	0.220	0.230	0.178	0.259	**0.115**
ERGAS	173.914	78.682	58.678	60.373	53.335	**50.522**

Table 1. Average performance.

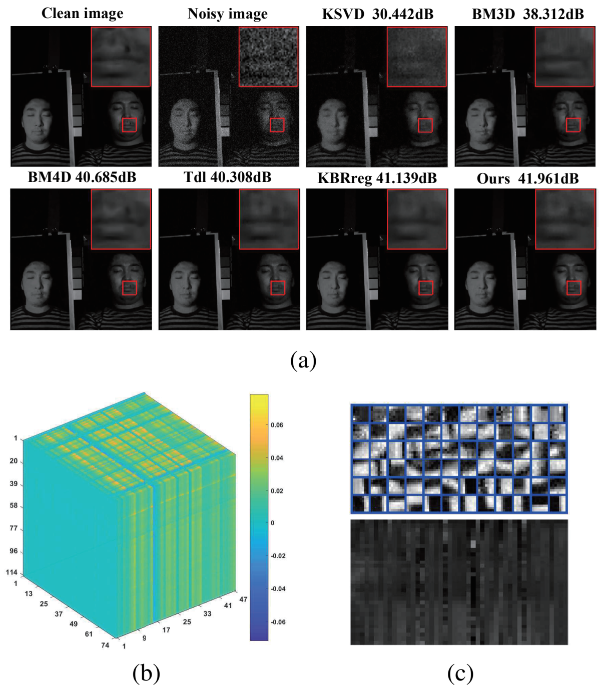

Fig. 2. Experimental results of the “photo and face” MSI in the CAVE dataset. (a) The denoised image of the 25th band and corresponding PSNR; (b) An illustration of a recovered $\mathcal{Z}^{(k)}$ which has a low-rank structure; (c) The learned space dictionary D^a(top) and the learned spectral dictionary D^e(bottom).

For visualizing the proposed method, we display the denoising results on a CAVE MSI in Fig. 2(a), where we zoom in part of the image for better comparison. In Fig. 2(b), we show a recovered tensor representation $\mathcal{Z}^{(k)}$, which exhibits a low-rank structure as we expected. The learned dictionaries are given in Fig. 2(c). Atoms of the learned spectral dictionary D^e correspond to various spectral feature of the scene. Atoms in the spatial dictionary D^a represent the spatial features of the MSI. To enhance visualization, we reorganize each atom (column) into a patch of the size 7×7.

4. CONCLUSIONS

This paper presents an effective tensor dictionary learning method for denoising high dimensional MSIs. The proposed method exploits the low-rank structure in different groups of similar blocks in the MSI and also exploits shared dictionaries among different groups, which makes the proposed method distinct to existing methods. Experimental results show the superior performance of the proposed method for denoising MSIs.

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