EFFICIENT BELIEF PROPAGATION DETECTION BASED ON CHANNEL HARDENING FOR MASSIVE MIMO

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ABSTRACT

For massive multiple-input multiple-output (MIMO) detection, belief propagation (BP) based on graphical models has become a popular detection algorithm since it provides a good tradeoff between performance and complexity. To further lower the complexity of BP detection, an efficient BP detection based on channel hardening (BP-CH) is proposed. In this paper, the comparison in terms of both performance and complexity between proposed BP-CH and general BP is firstly investigated exhaustively. Simulation results have shown that the proposed BP-CH achieves similar performance behavior as general BP while keeping lower computational complexity. Additionally, an folded hardware architecture for proposed BP-CH detector is designed to improve the implementation efficiency. Meanwhile, VLSI implementation results have verified the great advantage of BP-CH regarding hardware overhead, especially for scenarios with large system loading factor.

***Index Terms*—** MIMO, belief propagation (BP), channel hardening, system loading factor, cyclic shift property.

1. INTRODUCTION

Massive multiple-input multiple-output (MIMO) has emerged as an indispensable technology for the next generation wireless communication systems [1, 2]. Compared to conventional small-scale MIMO systems, massive MIMO can achieve increased data rate, enhanced link reliability, improved energy efficiency, and reduced interference. However, its enormous antenna size incurs higher computational complexity and implementation difficulties in all aspects, then hinders its widespread application in industry.

Traditional optimal detection algorithms such as maximum likelihood (ML) and sphere decoding (SD) [3] suffer from prohibitive complexity for high-dimensional MIMO and high-order modulation, because the computational complexity increases exponentially with the number of transmitting antennas (TAs). With regard to linear detection methods like minimum mean square error (MMSE) [4], they can achieve near-optimal performance but with inevitable matrix inversion operation. The computational complexity of matrix inversion operation is proportional to the third power of antennas' number, which may disable the practical use of MMSE in massive MIMO. Therefore, a feasible and efficient detection method is extremely desired to offer low-complexity and satisfying performance.

In recent years, message passing detection (MPD) algorithms have drawn much attention since they can strike a good balance between complexity and performance in massive MIMO systems [5–7]. As a representative of MPD algorithms, belief propagation (BP) has strong robustness and does not require careful selection of the initial solution. Most importantly, it avoids intractable matrix inversion operation, which makes it very attractive for high-dimensional MIMO detection. To further lower the implementation complexity of general BP detection, a novel pre-processing strategy based on channel hardening is proposed in [8]. Based on [8], the contributions of our research in this paper can be summarized as follows: (1) proposal of BP based on channel hardening (BP-CH) and a new perspective on the relationship between BP and BP-CH, (2) proposal of an folded hardware architecture for proposed BP-CH detector to improve the implementation efficiency, (3) performance comparison and VLSI implementation results to illustrate the advantage of the proposed BP-CH in terms of hardware efficiency.

The remainder of this paper is organized as follows. Section 2 introduces necessary preliminaries briefly. The proposed BP-CH detection and numerical results are presented in Section 3. Section 4 provides an efficient hardware architecture for the proposed BP-CH detector. Section 5 concludes the entire paper.

2. PRELIMINARIES

2.1. System Model

Consider an uplink MIMO system equipped with M TAs at the user side and N receiving antennas (RAs) at the BS ($M \ll N$). Define the system loading factor $\rho = N/M$ ($\rho > 1$). The complex transmitted symbol vector is denoted by $\tilde{\mathbf{x}} = [\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_M]^{\mathrm{T}} \in \mathbf{\Theta}^M$, where $\mathbf{\Theta}$ corresponds to the modulation alphabet with $\|\mathbf{\Theta}\| = Q = 2^{M_c}$. In addition, the channel gain matrix is represented by $\tilde{\mathbf{H}} = \{\tilde{h}_{ij}\}_{N\times M} \in \mathbb{C}^{N\times M}$. Hence, the received signal $\tilde{\mathbf{y}}$ is expressed by

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}\tilde{\mathbf{x}} + \tilde{\mathbf{n}}, \tag{1}$$

where $\tilde{\mathbf{n}} = [\tilde{n}_1, \tilde{n}_2, \dots, \tilde{n}_N]^T$ is denoted as the additive white *Gaussian* noise (AWGN) vector with $\tilde{n}_i \sim \mathcal{CN}(0, \sigma_n^2),\ 1 \le i \le N$.

With real value decomposition (RVD) [9], corresponding equivalent real-domain system is obtained as follows:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2}$$

where $\mathbf{y} = [y_1, y_2, \dots, y_{2N}]^T$, $\mathbf{x} = [x_1, x_2, \dots, x_{2M}]^T$ and $\mathbf{H} = \{h_{ji}\}_{1\le j\le 2N, 1\le i\le 2M}$. With RVD, a complex-domain $M \times N$ MIMO is converted into a real-domain $2M \times 2N$ MIMO.

2.2. Channel Hardening

In massive MIMO wireless communications, channel hardening refers to the phenomenon that a fading channel behaves as if it was a non-fading (deterministic) channel with the increase of the number of antennas. In other words, the communication quality is less and less affected by random fluctuations of channel gain values owing to the spatial diversity. According to [10], a hardened channel satisfies the following requirement:

$$\frac{\|\mathbf{h}_m\|^2}{\mathrm{E}\{\|\mathbf{h}_m\|^2\}} \to 1, \quad \text{as } N \to \infty, \tag{3}$$

where $\mathbf{h}_m$ stands for the m-th column vector of the channel gain matrix ($m = 1, 2, \ldots, 2M$).

Channel hardening brings in several positive implications to large-scale MIMO systems. One of the most valuable aspects is that the diagonal terms of the $\mathbf{H}^T\mathbf{H}$ matrix become increasingly stronger compared to the off-diagonal terms with the growth of antenna size. Briefly speaking, the Gram matrix $\mathbf{H}^T\mathbf{H}$ gradually approaches a diagonal matrix with the increase of antennas' number.

3. THE PROPOSED BP-CH DETECTION

In general BP detection, message passing occurs in a bi-directional network iteratively. For massive MIMO in uplink, the network will become extremely enormous if hundreds of RAs are equipped at the BS. Based on the channel hardening characteristic, an improved BP detection is proposed to lower the computational complexity by reducing the dimension of FG.

Firstly, multiplying $\mathbf{H}^T$ on both sides of Eq. (2) to get the $\mathbf{H}^T\mathbf{H}$ matrix, then Eq. (2) can be rewritten as follows:

$$\mathbf{H}^T\mathbf{y} = \mathbf{H}^T\mathbf{H}\mathbf{x} + \mathbf{H}^T\mathbf{n}. \tag{4}$$

According to Eq. (4), we have the following equation:

$$\mathbf{z} = \mathbf{R}\mathbf{x} + \mathbf{w}, \tag{5}$$

where

$$\mathbf{z} = \frac{\mathbf{H}^T\mathbf{y}}{N}, \quad \mathbf{R} = \frac{\mathbf{H}^T\mathbf{H}}{N}, \quad \mathbf{w} = \frac{\mathbf{H}^T\mathbf{n}}{N}. \tag{6}$$

Comparing Eq. (2) with Eq. (5), the original $2M \times 2N$ real MIMO system can be considered as an equivalent $2M \times 2M$ MIMO model established on Eq. (5). In the new equivalent MIMO model, $\mathbf{z}$, $\mathbf{w}$ and the matrix $\mathbf{R}$ represent the equivalent received signal, the equivalent additive noise and the equivalent channel gain matrix, respectively. Here, the covariance of $\mathbf{w}$ can be written as

$$\mathbf{C}_{\mathbf{w}} = \mathrm{E}\{\mathbf{w}\mathbf{w}^T\} = \frac{1}{N^2}\mathrm{E}\{\mathbf{H}^T\mathbf{n}\mathbf{n}^T\mathbf{H}\} = \frac{\sigma_n^2}{N^2}\mathrm{E}\{\mathbf{H}^T\mathbf{H}\}.$$

With channel hardening, $\mathbf{H}^T\mathbf{H}$ can be approximately viewed as a diagonal matrix. Hence, the entries of the equivalent noise $\mathbf{w}$ still can be considered independent of each other. Basically, performing a matched filter operation on the received signal $\mathbf{y}$ can be viewed as carrying out a pre-processing measure, then the same procedures about message passing as general BP are executed. The proposed BP-CH detection is described in Algorithm 1, where $\mathbf{p}_{ij}^{(l)}$ stands for the prior probability vector delivered to the j-th RA from the i-th TA at l-th iteration. In this way, those MIMO systems with the same number of TAs have almost the same computational complexity in message passing regardless of the number of RAs.

For independent identically distributed (i.i.d.) *Rayleigh* fading channel, numerical results of proposed BP-CH and general BP are given with different antenna configurations and modulations. Here, maximum iteration number is set as 7. All messages are passed over an additive white *Gaussian* noise (AWGN). No channel coding and decoding scheme is considered here.

Algorithm 1 The Proposed BP-CH Detection

Input: $\mathbf{y} \in \mathbb{R}^{2N}, \mathbf{H} \in \mathbb{R}^{2N\times 2M}, \mathbf{s} \in \mathbb{R}^{\sqrt{Q}}, Var\{\mathbf{n}\} = \sigma_n^2$
Output: estimated symbol $\hat{\mathbf{x}}$

$\mathbf{z} = \mathbf{H}^T\mathbf{y}/N, \mathbf{R} = \mathbf{H}^T\mathbf{H}/N, \sigma_w^2 = 2\sigma_n^2/N$
$\forall i, j, \mathbf{p}_{ij}^{(0)} = \{p_{ij}^{(0)}(s_1), p_{ij}^{(0)}(s_2), \ldots, p_{ij}^{(0)}(s_K)\}$
$p_{ij}^{(0)}(s_k) = 1/K$
for $l = 1 : L$ **do**
  for $j = 1 : 2N$ **do**
    $\mu_{g_j} = \sum_{k=1}^{2M} r_{jk}\mathbf{s}^T\mathbf{p}_{kj}$
    $\sigma_{g_j}^2 = \sum_{k=1}^{2M} r_{jk}^2((\mathbf{s}\odot\mathbf{s})^T\mathbf{p}_{kj} - (\mathbf{s}^T\mathbf{p}_{kj})^2)$
    for $i = 1 : 2M$ **do**
      $\mu_{g_{ji}}^{(l)} = \mu_{g_j} - r_{ji}\mathbf{s}^T\mathbf{p}_{ij}$
      $(\sigma_{g_{ji}}^2)^{(l)} = \sigma_{g_j}^2 - r_{ji}^2((\mathbf{s}\odot\mathbf{s})^T\mathbf{p}_{ij} - (\mathbf{s}^T\mathbf{p}_{ij})^2) + \sigma_w^2$
      $\forall s, \beta_{ji}^{(l)}(s_k) = \frac{2r_{ji}(z_j - \mu_{g_{ji}}^{(l)})(s_k - s_1) - r_{ji}^2(s_k^2 - s_1^2)}{2(\sigma_{g_{ji}}^2)^{(l)}}$
    end for
  end for
  for $i = 1 : 2M$ **do**
    $\forall s, \gamma_i^{(l)}(s_k) = \sum_{k=1}^{2N}\beta_{ki}^{(l)}(s_k)$
    for $j = 1 : 2N$ **do**
      $\forall s_k, \alpha_{ij}(s_k) = \gamma_j(s_k) - \beta_{ij}(s_k)$
      $\forall s_k, p_{ij}^{(l)}(s_k) = \frac{\exp\{\alpha_{ij}(s_k)\}}{\sum_{m=1}^{K}\exp\{\alpha_{ij}(s_m)\}}$
      $\forall s_k, p_{ij}^{(l)}(s_k) = (1-\delta)p_{ij}^{(l)}(s_k) + \delta p_{ij}^{(l-1)}(s_k)$
    end for
    Symbol-based decision: $\boldsymbol{\gamma} \Rightarrow \hat{x}_i = s_k$
  end for
end for

Fig. 1 shows the performance comparison of proposed BP-CH and general BP detection with quadrature phase shift keying (QPSK) modulation. Here, the 3 configurations are: $N = 32, 64, 128$ with a fixed $M = 8$, associated with the three kinds of loading factors ($\rho = 4, 8, 16$), respectively. According to Fig. 1, it is observed that the performance gap between proposed BP-CH and general BP is getting smaller and smaller with the increase of loading factor ρ. For 8×32 MIMO ($\rho = 4$), at a BER of 10^{-3}, the performance degradation of proposed BP-CH detection is about 2 dB compared to general BP, which is far from satisfaction. With respect to the scenarios with large system loading factor ($\rho = 8, 16$), proposed BP-CH suffers from only about 0.4 dB loss at a BER of 10^{-3}.

In Fig. 2, the performance comparison of proposed BP-CH and general BP detection is presented when 16-Quadrature Amplitude Modulation (16-QAM) is employed. Similar to the simulation results under QPSK modulation, the BER performance curve of proposed BP-CH becomes closer to that of general BP as the growing of the number of RAs when the number of TAs is fixed. For 8×128 MIMO ($\rho = 16$), proposed BP-CH detection achieves almost the same performance gain as general BP.

Actually, the above results can be explained by the theory of channel hardening. The core concept of BP-CH detection is to establish a simple symbol model by compressing the channel information. In the other word, the loss of channel information will lead to the detection performance degradation to some extent. With the increase of antennas' number, the channel hardening characteristic becomes more and more obvious. As the less channel information is ignored, the performance of BP-CH detection becomes more satisfied.

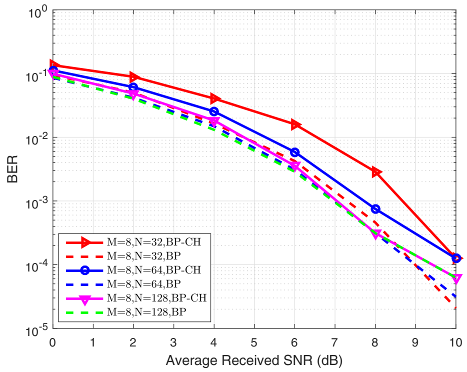

Fig. 1. Performance comparison of BP-CH and BP (QPSK).

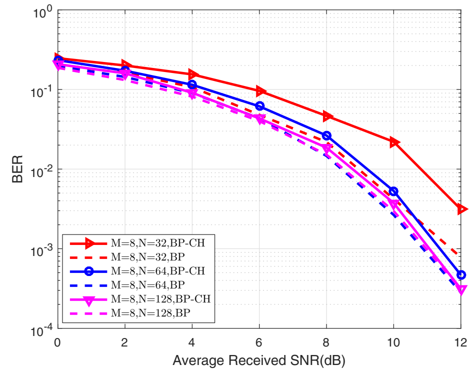

Fig. 2. Performance comparison of BP-CH and BP (16-QAM).

In summary, the proposed BP-CH detection is especially suitable for large loading factor. For those scenarios, the performance gap between proposed BP-CH and general BP is tiny enough to be neglected while keeping lower implementation complexity.

4. EFFICIENT ARCHITECTURE FOR PROPOSED BP-CH

In this section, a folded hardware architecture for proposed BP-CH detector is proposed firstly. The efficiency of the hardware architecture is improved greatly based on the reduced FG.

4.1. The Characteristics of the Equivalent Channel Matrix

As mentioned above, the complex channel gain matrix is represented by $\tilde{\mathbf{H}} = \{\tilde{h}_{ij}\}_{N\times M}$. Assume that $\tilde{h}_{ij} = a_{ij} + jb_{ij}$, where a_{ij} and b_{ij} are the real and imaginary parts of $\tilde{h}_{ij}$, respectively. With RVD, the real-valued channel matrix $\mathbf{H}$ becomes

$$\mathbf{H} = \begin{bmatrix} a_{11} & -b_{11} & \cdots & a_{1\times 2M} & -b_{1\times 2M} \\ b_{11} & a_{11} & \cdots & b_{1\times 2M} & a_{1\times 2M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{2N\times 1} & -b_{2N\times 1} & \cdots & a_{2N\times 2M} & -b_{2N\times 2M} \\ b_{2N\times 1} & a_{2N\times 1} & \cdots & b_{2N\times 2M} & a_{2N\times 2M} \end{bmatrix}. \tag{7}$$

Obviously, $\mathbf{H}$ is such a $2N \times 2M$ matrix satisfying:

$$\mathbf{h}_{2m-1}^T \cdot \mathbf{h}_{2m} = 0, m = 1, 2, \ldots, M, \tag{8}$$

where $\mathbf{h}_{2m-1}$ and $\mathbf{h}_{2m}$ stands for the odd and even columns of $\mathbf{H}$, respectively. From Eq. (8), it is seen that the odd and even columns of the matrix $\mathbf{H}$ are orthogonal to each other.

Regarding the fact that $\mathbf{R} = \mathbf{H}^T\mathbf{H}$ is a symmetric matrix (i.e. $\mathbf{R} = \mathbf{R}^T$), the new equivalent channel matrix $\mathbf{R}$ can be expressed as following:

$$\begin{aligned} \mathbf{R} &= \begin{bmatrix} \mathbf{h}_1^T \\ \mathbf{h}_2^T \\ \vdots \\ \mathbf{h}_{2M}^T \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \ldots & \mathbf{h}_{2M} \end{bmatrix} \\ &= \begin{bmatrix} r_{11} & 0 & & & & & \\ 0 & r_{11} & & & & & \\ & & r_{22} & 0 & & & \\ & & 0 & r_{22} & & & \\ & & & & \ddots & & \\ & & & & & r_{MM} & 0 \\ & & & & & 0 & r_{MM} \end{bmatrix}. \end{aligned} \tag{9}$$

In reality, the equivalent channel matrix $\mathbf{R}$ is not a strict diagonal matrix with all the off-diagonal elements to be zero. Owing to the channel hardening characteristics of massive MIMO systems, the diagonal terms of the matrix $\mathbf{R}$ become increasingly stronger compared to the off-diagonal terms with the growth of antenna size. There, when system loading factor is relatively large, the matrix $\mathbf{R}$ can be approximated as a diagonal matrix.

4.2. Reduced FG

In order to reduce the FG, we consider to divide the complete FG into M subgraphs. Each subgraph includes $2M$ symbol nodes, the $(2m-1)$-th and the $2m$-th $(m = 1, 2, \ldots, M)$ observation nodes. In this paper, the compared to the symbol precessing unit (SPU) arrays is folded to lower the hardware implementation complexity, because the observation precessing unit (OPU) is more difficult to implement SPU. The folded architecture of BP-CH detector is based on the reduced FG shown in Fig. 3.

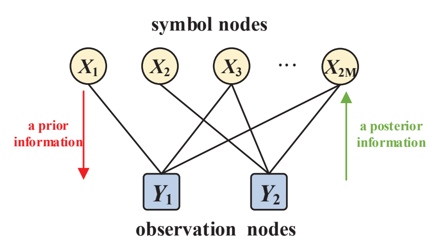

Fig. 3. Reduced FG in folded BP-CH detection.

4.3. Hardware Architecture Design

Based on the reduced FG, the overall framework of the folded hardware architecture for the proposed BP-CH detector is presented in Fig. 4, which consists of $2M$ SPUs, two OPUs , a pre-treatment module (PM), a data buffer (BUF) and a decision module (DM).

The specific procedures of folded BP-CH detector is as follows:

- The PM is responsible to calculate the equivalent channel matrix $\mathbf{R}$ and equivalent output signal $\mathbf{z}$.
- The $\mathbf{z}$ is delivered to the OPU array, and the matrix $\mathbf{R}$ is sent to the BUF to waiting. Each time, two rows of OPUs is passed to the OPU array for information calculation.

Table 1. Implementation results for proposed BP-CH and general BP.

Detector	8×32			8×64		
	BP	**BP-CH**	**Folded BP-CH**	**BP**	**BP-CH**	**Folded BP-CH**
LUTs	$17,996(4\%)$	$12,998(2\%)$	$4,336(1\%)$	$24,837(5\%)$	$12,998(2\%)$	$4,336(1\%)$
FFs	$5,362(3\%)$	$3,717(1\%)$	$3,717(1\%)$	$7,090(4\%)$	$3,717(1\%)$	$3,717(1\%)$
DSP48s	$144(4\%)$	$144(4\%)$	$144(4\%)$	$144(4\%)$	$144(4\%)$	$144(4\%)$
Latency(T_s)	82	114	360	146	146	392
f_{max} **(MHZ)**	69	71	71	69	71	71
Throughput(Mbps)	13.5	9.96	3.2	7.6	7.78	2.9

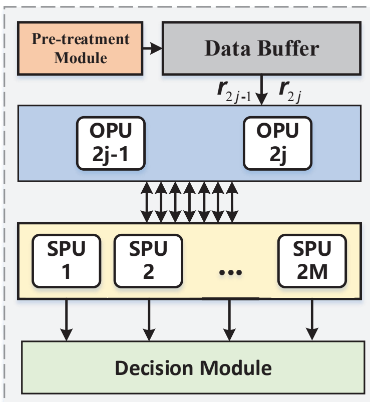

Fig. 4. The overall framework of the efficient hardware architecture.

- There are only two OPUs to finish the message updating of $2M$ OPUs in sequence. OPU 1 an OPU 2 receive the all prior information from SPUs and first two columns $(\mathbf{r}_1, \mathbf{r}_2)$ from BUF to update the posterior information.
- The BUF outputs the second two columns $(\mathbf{r}_3, \mathbf{r}_4)$ for message passing in next round.
- Until all columns of $\mathbf{R}$ is transferred to OPU array, a complete iteration is finished. When the pre-set iteration number is reached, all prior messages are delivered to the DM to determined the estimated transmitted signal.

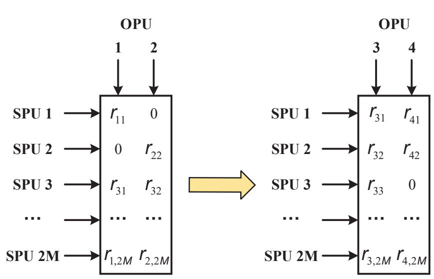

Fig. 5. Connections between SPUs and OPUs based on reduced FG.

In Fig. 4, it is seen the implementation complexity of folded BP-CH detector has reduced by only employing two OPUs. Fig. 5 shows the connections between SPUs and OPUs.

4.4. VLSI Implementation Results

We investigate the FPGA implementation results for general BP, proposed BP-CH, shift BP-CH based on Xilinx Virtex-7 XC7VX690T FPGA and compare their hardware costs in Table 1. It is shown that the hardware consumption of BP-CH detection is reduced greatly in various aspects compared to general BP detection. For instance, the LUTs cost of BP-CH detection has been reduced by about 27% in comparison with general BP detection for 8×32 MIMO system. If the folded hardware architecture is employed, the reduction of hardware resources will be more noteworthy. On the other hand, the proposed BP-CH detector has higher clock frequency than the traditional BP owing to its shorter critical path. However, the reduction in hardware resources of folded BP-CH detection is at the expense of the processing delay and data throughput. Generally speaking, for the situation with moderate throughput requirements, the proposed BP-CH can be considered as an excellent method to signal detection for massive MIMO.

5. CONCLUSION

In this paper, BP-CH detection based on channel hardening is proposed to reduce the implementation complexity for massive MIMO. To better understand the proposed BP-CH, a new perspective on the relationship between BP and BP-CH is elaborated on. Additionally, an folded hardware architecture for BP-CH is firstly proposed. Both numerical simulation and FPGA implementation of proposed BP-CH detector have verified its application feasibility for practical massive MIMO systems.

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