WAVEFORM MODELING BY ADAPTIVE WEIGHTED HERMITE FUNCTIONS

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ABSTRACT

Modern medical science demands sophisticated signal representation methods in order to cope with the increasing amount of data. Important criteria for these methods are mainly low computational and storage costs, whereas the underlying mathematical model should still be interpretable and meaningful for the data analyst. One of the most promising models fulfilling these criteria is based on Hermite functions, however having some important limitations for specific biomedical wave shapes. We extend this model by using weighted Hermite functions and develop a gradient based constrained optimization method to adapt the system for different types of signals. In order to demonstrate the potential of our approach, we consider the problem of electrocardiogram signal compression. The experiments on the MIT/BIH arrhythmia database show a significant improvement compared to the former works using classical Hermite functions.

Index Terms— Signal modeling, weighted Hermite functions, optimization, variable projection, ECG compression.

1. INTRODUCTION

The advances in data acquisition of biomedical signals over the last decades have raised new challenges in various signal processing disciplines, such as waveform modeling, information extraction, or signal compression. Depending on the biomedical signal of interest the applications for these disciplines are manifold. Examples are modeling or denoising single-trial evoked brain responses [1, 2], waveform learning for modeling the variability in neurophysiological signals [3], or waveform modeling of the electrocardiogram (ECG), on which we will focus as a case study in the following. The ECG is widely used for tracking and evaluating the electrical activity of the heart, thereby providing remarkable diagnostic information about the cardiovascular state of health for a specific subject. The three main segments (QRS complex, P, T wave) of an ECG beat usually follow a rather characteristic shape leading to the development of various mathematical models for describing the single waves or the whole beat.

One of the most promising model was introduced by Sörnmo et al. who used Hermite functions to evaluate QRS shape features [4]. Based on their work the model was further improved and applied for ECG data compression [5, 6, 7], clustering of ECG complexes [8], detection of myocardial infarction [9], and ECG beat segmentation [10]. The latter work illustrates the suitability of adaptive Hermite functions to segment and describe the waves of ECG beats individually, thereby providing shape information in a low-dimensional space, whereas, due to the orthogonality of the system, the redundant information is eliminated and the important one is kept. This is a major advantage of adaptive Hermite functions since the extracted information allows to track (subtle) wave shape changes over time, which could be of great medical interest. Additionally this system is well suited for beat classification and compression tasks. Despite the many benefits of applying Hermite functions in ECG signal processing, there are some limitations when it comes to modeling of atypical, special wave shapes, e.g., heart beats with ventricular pacing, delta waves, or pulmonale P waves. A major demand in case of working with ECG signals – or in general with biomedical signals – is, that diagnostic features must not be distorted when applying a specific algorithm. However, this exactly happens when modeling the above examples by adaptive Hermite functions since this system has limitations in approximating very peaky or asymmetric waveforms.

Therefore, the contribution of this work is to extend the mathematical model suggested in [7] with additional free parameters which allow to adjust the Hermite weight function and consequently to model more complex wave shapes with a higher accuracy thereby keeping orthogonality of the system. These so called weighted Hermite functions are introduced in Section 2 followed by the optimization of the free parameters in Section 3, which are restricted to a feasible domain in Section 4. In Section 5 we illustrate improvements of our work by applying the model to ECG beat compression. However, this is only a case study to make our work comparable. The novel mathematical model has also the potential to be applied to other biomedical signals, as neurophysiological signals or evoked brain responses. Section 6 concludes the work.

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2. WEIGHTED HERMITE FUNCTIONS

Let us consider the usual weighted Lebesgue spaces $L^2_w(\Omega)$ ($\Omega \subseteq \mathbb{R}$) with a nonnegative weight function $w : \Omega \to [0, \infty)$. For $f, g \in L^2_w(\Omega)$ the inner product and the corresponding induced norm can be defined as:

$$\langle f, g \rangle_w = \int_\Omega f(t)g(t)w(t)\mathrm{d}t\,, \quad \|f\|_2 = \sqrt{\langle f, f\rangle}\,. \qquad (1)$$

Then there is associated a system of orthogonal polynomials $\{p_k \,|\, k \in \mathbb{N}\}$ that satisfy the three-term recurrence relation

$$p_{k+1}(t) = (t - \alpha_k)\, p_k(t) - \beta_k p_{k-1}(t), \quad (k \in \mathbb{N})\,, \qquad (2)$$

$$p_{-1}(t) = 0, \qquad p_0(t) = 1\,, \qquad (3)$$

where $\beta_0 = \langle 1, 1\rangle_w$, $\alpha_0 = \langle t, 1\rangle_w / \beta_0$, and

$$\alpha_k = \frac{\langle tp_k, p_k\rangle_w}{\langle p_k, p_k\rangle_w}\,, \quad \beta_k = \frac{\langle p_k, p_k\rangle_w}{\langle p_{k-1}, p_{k-1}\rangle_w} \quad (k \in \mathbb{N}^+)\,.$$

We denote the system of classical monic Hermite polynomials by $\{h_k \,|\, k \in \mathbb{N}\}$ for which $\Omega = \mathbb{R}$, $w(t) = e^{-t^2}$, $\alpha_k = 0$, $\beta_0 = \sqrt{\pi}$, and $\beta_k = k/2$ ($k \in \mathbb{N}^+$). The family of Hermite functions $\{\Phi_k \,:\, k \in \mathbb{N}\}$ can be derived as

$$\Phi_k(t) = h_k(t)e^{-t^2/2} \cdot 2^{k/2} \big/ \sqrt{\pi^{1/2} k!} \qquad (k \in \mathbb{N}),$$

which forms an orthonormal and complete system in $L^2(\mathbb{R})$ with respect to the corresponding inner product and induced norm (see e.g. [11]). These properties also hold for affine transformations of these functions, which allow to define the adaptive Hermite-Fourier series and their partial sum $S_n^{\tau,\lambda}$ as

$$S_n^{\tau,\lambda} f(t) = \sum_{k=0}^{n} \langle f, \Phi_k^{\tau,\lambda}\rangle \Phi_k^{\tau,\lambda}(t) \qquad (f \in L^2(\mathbb{R})), \qquad (4)$$

where the translation τ, and the dilation λ are free parameters that define the functions

$$\Phi_k^{\tau,\lambda}(t) := \sqrt{\lambda}\Phi_k(\lambda(t - \tau)) \qquad (t, \tau \in \mathbb{R}, \lambda > 0)\,. \qquad (5)$$

The resulting signal representation is localized in time and frequency due to the free parameters τ, λ, which can be found automatically via optimization. This is a useful property that was utilized by many authors in ECG signal processing, particularly in compression [5, 6, 7], wave shape modeling [4], machine learning [8, 9], and segmentation [10]. Besides their advantages, the adaptive Hermite functions have only two free parameters, which limit the family of wave shapes that can be described by these functions. This explains why the Hermite functions are favored in ECG signal processing, since they are very similar to the basic shapes of the main waves (QRS, T, P) in a heartbeat. However, abnormal heart functioning can imply diverse QRS, T, P waves, where the classical adaptive Hermite representation fails. We resolve this problem by extending Eq. (4) with additional free parameters, which potentially also allows better modeling of wave shapes different from the ECG, e.g., ictal and interictal spikes in EEG [12], or action potentials of cells such as neurons [13, 14].

Let p_1, p_2 be polynomials of degree ℓ, m such that the rational function $u = p_1/p_2$ is nonnegative on Ω. Now we consider the modified real valued weight function $v = u \cdot w$, and the set of polynomials $\{q_k \,|\, k \in \mathbb{N}\}$ which are orthogonal with respect to $\langle \cdot, \cdot \rangle_v$. If the parameters of the weight function v are given, e.g., zeros/poles or coefficients, then the new system of orthogonal polynomials is defined. Here, we confine our investigations to the modifications of the classical Hermite weight function $w(t) = e^{-t^2}$, ($t \in \Omega = \mathbb{R}$) by using only quadratic factors (qf), quadratic divisors (qd), symmetric quadratic divisors (sqd) and their sums:

$$u_1(t) = (t - x)^2\,, \qquad u_2(t) = 1/((t - y)^2 + z^2)\,, \qquad (6)$$

$$u_3(t) = 1/(t^2 + s^2)\,, \quad u_4(t) = u_1(t) + u_2(t) + u_3(t)\,,$$

where $x, y \in \mathbb{R}$, $z, s \in \mathbb{R}^+$ provides the nonnegativity of $v_i = u_i \cdot w$. Let us denote the vector of free parameters by $\boldsymbol{\eta} = (x, y, z, s)^T$. Then, the corresponding family of modified Hermite functions $\{\Psi_{i,k}(\cdot; \boldsymbol{\eta}) \,:\, k \in \mathbb{N}\}$ for a proper index i is as follows:

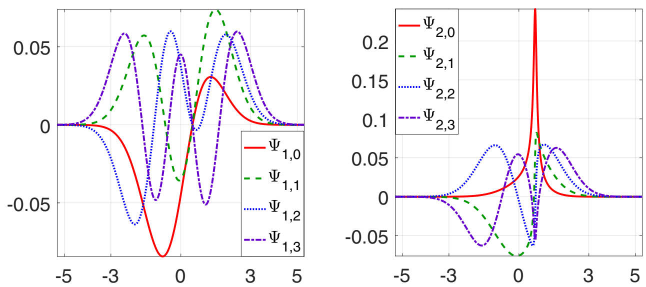
$$\Psi_{i,k}(t; \boldsymbol{\eta}) = q_k(t) / \|q_k\|_2 \cdot \sqrt{v_i(t; \boldsymbol{\eta})} \qquad (k \in \mathbb{N}). \qquad (7)$$

Fig. 1 shows the first four elements of the new systems for $i = 1, 2$, and Fig. 2(a) demonstrates the influence of the parameters, i.e., x, y modifies the center of the waves, while z, s is for squeezing and stretching. In analogy to Eq. (5), the set of parameters can further be extended by translation and dilation.

We highlight that the new recurrence coefficients $\widehat{\alpha}_k := \widehat{\alpha}_k(\boldsymbol{\eta})$, $\widehat{\beta}_k := \widehat{\beta}_k(\boldsymbol{\eta})$ depend on the vector of free parameters $\boldsymbol{\eta}$. There are various ways to determine $\widehat{\alpha}_k$, $\widehat{\beta}_k$ from the old ones in Eq. (2). Here, we are applying the work of Gautschi [15, 16], which utilizes the generalized Christoffel theorems to calculate the recurrence coefficients with respect to v_1, v_2, v_3, then the recurrence coefficients corresponding to their sum v_4 is computed by [17].

3. OPTIMIZATION METHOD

In case of analog signals, Ω is a continuous interval, which is restricted to a proper countable set for discrete-time series. Here, for the sake of simplicity, we consider Ω to be equal to the uniform discretization of a finite time interval, and we apply the composite trapezodial rule to calculate the integral in Eq. (1) using N samples. Let us consider now a discrete-time signal $\mathbf{f} \in \mathbb{R}^N$, the vectors $\boldsymbol{\Psi}_{i,k}(\boldsymbol{\eta}) \in \mathbb{R}^N$, which are the modified Hermite functions in Eq. (7) uniformly sampled along the time axis t, and the corresponding matrix $\mathbb{R}^{N\times(n+1)} \ni \boldsymbol{\Psi}_i(\boldsymbol{\eta}) := (\boldsymbol{\Psi}_{i,0}(\boldsymbol{\eta}), \ldots, \boldsymbol{\Psi}_{i,n}(\boldsymbol{\eta}))$. Furthermore, we will use the Moore–Penrose pseudoinverse $\boldsymbol{\Psi}_i(\boldsymbol{\eta})^+$



(a) $x = 0.5$ (b) $y = 0.7,\ z = 0.06$

Fig. 1. Examples of the modified weighted Hermite functions.

of $\boldsymbol{\Psi}_i(\boldsymbol{\eta})$, and define $\mathbf{P}_{\Psi_i}(\boldsymbol{\eta}) := \boldsymbol{\Psi}_i(\boldsymbol{\eta})\boldsymbol{\Psi}_i(\boldsymbol{\eta})^+$, which is the orthogonal projector on the linear space spanned by the columns of $\boldsymbol{\Psi}_i(\boldsymbol{\eta})$. The projector to the orthogonal complement is $\mathbf{P}^{\perp}_{\Psi_i}(\boldsymbol{\eta}) := \mathbf{I} - \mathbf{P}_{\Psi_i}(\boldsymbol{\eta})$. Then, the best parameters of the new weight function v_i $(i = 1, \ldots, 4)$ can be found by solving the following nonlinear optimization problem:

$$\min_{\boldsymbol{\eta}\in\Gamma} r_{i,2}(\boldsymbol{\eta}) = \min_{\boldsymbol{\eta}\in\Gamma} \|\mathbf{f} - \mathbf{P}_{\Psi_i}(\boldsymbol{\eta})\mathbf{f}\|_2^2 = \min_{\boldsymbol{\eta}\in\Gamma} \left\|\mathbf{P}^{\perp}_{\Psi_i}(\boldsymbol{\eta})\mathbf{f}\right\|_2^2 , \tag{8}$$

where Γ denotes the feasible domain that we define later. For a certain i, $r_{i,2}$ is called variable projection (VP) functional [18]. For the sake of simplicity, we will omit the vector of free parameters $\boldsymbol{\eta}$ and the index i from the notations. Then, according to the work of Golub and Pereyra [18], the jth coordinate of the gradient of this functional can be calculated explicitly as follows:

$$\frac{1}{2}\nabla r_2^{(j)} = \left(-\left(\mathbf{P}^{\perp}_{\Psi}\mathbf{D}_j\boldsymbol{\Psi}^+ + \left(\mathbf{P}^{\perp}_{\Psi}\mathbf{D}_j\boldsymbol{\Psi}^+\right)^T\right)\mathbf{f}\right)^T \mathbf{P}^{\perp}_{\Psi}\mathbf{f}\,, \tag{9}$$

where the matrix $\mathbf{D}_j$ denotes the partial derivatives of the modified Hermite functions with respect to the free parameters, i.e., $\mathbf{D}_j := \partial\boldsymbol{\Psi}(\boldsymbol{\eta})/\partial\eta_j$.

Although Eq. (9) defines the gradient explicitly, it is still difficult to compute due to the recalculation of the partial derivatives $\mathbf{D}_j$ and the pseudoinverse $\boldsymbol{\Psi}^+$ at each optimization step. Note that the new recurrence coefficients $\widehat{\alpha}_k$, $\widehat{\beta}_k$ of q_k in Eq. (7) should be also recalculated for every parameter setup $\boldsymbol{\eta}$ by using a time-consuming iterative method [15, 16]. In order to avoid that we restrict the optimization to the first element $\Psi_{i,0}$ of the modified Hermite system. Since $q_0 \equiv 1$ for any weight function v_i, the new recurrence coefficients are not required to define $\Psi_{i,0}(t;\boldsymbol{\eta}) = \sqrt{v_i(t;\boldsymbol{\eta})}$. Therefore, choosing $\boldsymbol{\Psi} := \boldsymbol{\Psi}_{i,0} \in \mathbb{R}^{N\times 1}$ in Eq. (9), the pseudoinverse reduces to $\boldsymbol{\Psi}^+ = \boldsymbol{\Psi}^T$, and the partial derivatives simplify to $\mathbf{D}_j = \partial\sqrt{\mathbf{v}_i(\boldsymbol{\eta})}/\partial\eta_j$, where the vector $\mathbf{v}_i$ denotes the uniformly sampled weight function. From now on, we will refer to the resulting optimization as the reduced problem. By using this approach, one can design an orthonormal function system, in which the first element is very similar to the original signal $\mathbf{f}$. This heuristic is sometimes satisfactory in practical applications as well. For instance, Sielużycki et al. [1] applied Gabor functions for sparse representation of evoked potentials, where the first element of the representation usually matched the main characteristic of the signals.

4. CONSTRAINTS

In this section, we restrict the free parameters $x, y \in \mathbb{R}$, $z, s \in \mathbb{R}^+$ of the weight function in Eq. (6) to a feasible domain Γ in order to ease the optimization. Recall that the modified weight function v_i is equal to $u_i \cdot w$, where w is a Gaussian function $e^{-\frac{(t-\mu)^2}{2\sigma^2}}$ centered at $\mu = 0$ and with $\sigma = 1/\sqrt{2}$. Since $w(t)$ decreases much faster than $u_i(t)$ as $t \to \pm\infty$, it does not make sense positioning these waves far from the center of the Gaussian. In order to define what "far" means here, we will apply the well-known three-sigma rule, which states that around 68%, 95%, 99% of the overall integral of w lies within the intervals $[-\ell\sigma, \ell\sigma]$ for $\ell = 1, 2, 3$, respectively. Therefore, we restrict the values of the parameters x, y to the interval $I_x = I_y := [-3\sigma, 3\sigma]$. Choosing x, y outside of this interval does not significantly change the weight function and so the system, since the corresponding quadratic factors and divisors are smoothed out by the tails of the Gaussian.

The parameters z, s control the value of the maximum and the width of the main lobe of the qd and the sqd factors in Eq. (6). The lower these parameters, the more peaky the functions u_2, u_3. High values are not preferred, since the resulting qd and sqd factors are so smooth they do not significantly change the original weight function. Hence, we derive our constraints as follows:

$$0.8 \cdot \int_{-\infty}^{\infty} u_2(t)\mathrm{d}t \le \int_{-3\sigma}^{3\sigma} u_2(t)\mathrm{d}t, \tag{10}$$

which means that main lobe of u_2 cannot be too wide, i.e., 80% of its overall integral should lie in the interval $[-3\sigma, 3\sigma]$. For predefined parameters y, z, one can easily show that the definite integral on the right hand side of Eq. (10) is equal to

$$\frac{1}{z} \cdot \left(\arctan\left(\frac{3\sigma - y}{z}\right) - \arctan\left(\frac{-3\sigma - y}{z}\right)\right). \tag{11}$$

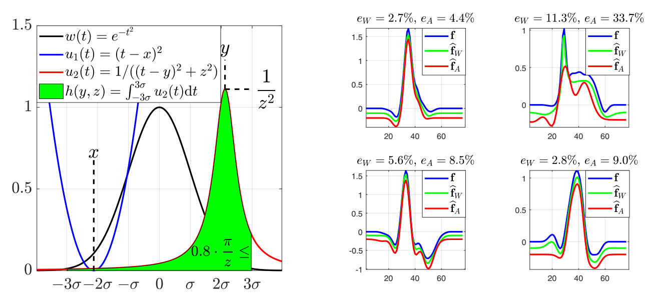
Taking the limit for $\sigma \to \infty$ gives π/z, which is the improper integral of u_2 over $\mathbb{R}$. Substituting these results back to Eq. (10) and simplifying by z, we can write the constraint in the form

$$0.8 \cdot \pi \le \arctan\left(\frac{3\sigma - y}{z}\right) + \arctan\left(\frac{3\sigma + y}{z}\right) := h(y, z). \tag{12}$$

An upper bound for z, s can also be given by estimating the right hand side of Eq. (12). Note that $y \in I_y = [-3\sigma, 3\sigma]$, thus $h(y, z) \le 2 \cdot \arctan(6\sigma/z)$, which is still less than or equal to 0.8π, provided that $6\sigma/\tan(0.4\pi) \leq z$. Therefore, one can consider the interval $I_z := (0, 6\sigma/\tan(0.4\pi)]$. The same applies for the sqd factor by substituting z by s and setting $y = 0$, i.e., we get $s \in I_s := (0, 3\sigma/\tan(0.4\pi)]$. Then the feasible domain can be defined as follows:

$$\Gamma := \{\boldsymbol{\eta} \in \mathbb{I} \,|\, 0.8\pi \leq h(\eta_2, \eta_3),\, 0.8\pi \leq h(0, \eta_4)\}\,, \quad (13)$$

where $\sigma = 1/\sqrt{2}$, and $\mathbb{I} = I_x \times I_y \times I_z \times I_s$. In Fig. 2(b), we give some demonstrative examples by fitting real QRS complexes. Here, $\widehat{\mathbf{f}}_{\mathbf{A}}$ denotes the samples of the approximation $S_6^{\tau,\lambda}$ in Eq. (4), where we optimized only for τ and λ. Similarly, we got $\widehat{\mathbf{f}}_{\mathbf{W}}$ by replacing $\Phi_k^{\tau,\lambda}$ in Eq. (4) with the modified Hermite functions $\Psi_k^{\tau,\lambda}$, which were optimized for the parameters of the weight function as well. For better visualization, we applied a small vertical shift to the approximations. According to Eq. (14), we calculated the corresponding reconstruction errors e_A, e_W, which shows that the results are much better for the proposed method.



(a) Constraints (b) Approximations

Fig. 2. Demonstrating the modified Hermite system.

5. EXPERIMENTS

In order to make our work comparable we carried out ECG data compression since it provides a good benchmark regarding the distortion of the ECG signals. For that reason we used the MIT/BIH arrhythmia database [19], which contains 48 half-hour recordings, digitized to 11 bit. As suggested by [20] we took the first lead of 24 pre-selected recordings resulting in a total length of 12 hours ECG raw data. According to [21], the recordings were sliced into single ECG beats by cutting the raw data 130 samples before the provided QRS annotations, whereas beats shorter than 130 samples were not considered. These beats were further subdivided into P, QRS, and T by assuming the QRS duration to be 100 ms. The single segments were then approximated using weighted Hermite functions, whereas the free parameters for adjusting the weight functions were optimized once per recording for the average of all beats. This provides an individual function system for a specific recording, which thus allows to model more complex waveforms. Subsequently, the dilation and the translation were optimized for every beat as described in [10] thereby representing each wave by 4 basis functions and compensating for a possible baseline fluctuation by a linear interpolation between the start and the end of the segment. We want to emphasize that the preoptimized values for the weight function can be interpreted as subject-specific parameters (describing the general morphology for a subject) while the translation and the dilation coefficients are responsible for describing the morphological changes induced, e.g., by respiration. All parameters needed were then compressed and decompressed according to the work of [7]. For evaluation we calculated the compression ratio (CR) per recording and the average normalized percent root mean square difference (PRDN) for M ECG beats/QRS complexes

Table 1. Experimental results of 12 hours long real ECG data.

	Work of [5]			Work of [7]			Proposed work			
Rec.	$\overline{\text{PRDN}}$	$\frac{\text{QRS}}{\text{PRDN}}$	CR	$\overline{\text{PRDN}}$	$\frac{\text{QRS}}{\text{PRDN}}$	CR	$\overline{\text{PRDN}}$	$\frac{\text{QRS}}{\text{PRDN}}$	CR	System
mean	**18.18**	**20.28**	**19.75**	**15.40**	**11.40**	**18.83**	**14.88**	**9.86**	**18.82**	**-**
Selected recordings (for illustration)										
100	17.09	16.57	19.47	13.09	12.09	18.52	9.78	7.74	18.52	qf
102	33.57	36.34	20.22	33.69	24.91	19.23	31.27	16.05	19.23	qd
104	31.94	39.49	19.96	34.10	37.57	18.98	29.78	20.38	18.98	qd
232	32.40	20.28	24.39	26.00	14.55	23.22	24.18	9.59	23.21	qd+qf

$$\overline{\text{PRDN}} = 100 \cdot \frac{1}{M} \sum_{m=1}^{M} \frac{||\mathbf{f_m} - \hat{\mathbf{f}}_\mathbf{m}||_2}{||\mathbf{f_m} - \overline{\mathbf{f}}_\mathbf{m}||_2}\,, \quad (14)$$

where $\mathbf{f_m}$ and $\hat{\mathbf{f}}_\mathbf{m}$ represent the raw beats/segments and their approximations, respectively. Table 1 illustrates the comparison between the work optimizing only for the dilation [5], the dilation and the translation [7], and our work. Here, the bold row represents the main result, illustrating the average improvement over 12 hours ECG recordings. One can observe that in the mean the $\overline{\text{PRDN}}$ is better for the weighted Hermite functions while the compression ratio stays almost the same. Additionally, we want to highlight the strong improvements for specific recordings with more complex waveforms (100, 102, 104, 232), which strengthens advances of our work.

6. CONCLUSIONS

The proposed mathematical model is a generalization of former wave shape models, which use classical Hermite functions. In order to adapt the new weighted Hermite system to various types of signals, we developed a gradient based optimization. Then, we speeded up the computation, by introducing a reduced variation of the optimization problem, which was restricted to a feasible domain. As a case study, we considered ECG compression, however, our approach is of general nature that can be utilized in many applications such as information extraction, signal classification, detection, etc. The MATLAB implementation of the proposed method is available at the website [22].

7. REFERENCES

[1] C. Sielużycki, R. König, A. Matysiak, R. Kús, D. Ircha, and P. J. Durka, "Single-trial evoked brain responses modeled by multivariate matching pursuit," *IEEE Transactions on Biomedical Engineering*, vol. 56, no. 1, pp. 74–82, 2009.

[2] M. Ahmadi and R. Q. Quiroga, "Automatic denoising of single-trial evoked potentials," *NeuroImage*, vol. 66, pp. 672 – 680, 2013.

[3] S. Hitziger, M. Clerc and, S. Saillet, C. Bénar, and T. Papadopoulo, "Adaptive waveform learning: A framework for modeling variability in neurophysiological signals," *IEEE Transactions on Signal Processing*, vol. 65, no. 16, pp. 4324–4338, Aug 2017.

[4] L. Sörnmo, P. L. Börjesson, M. E. Nygårds, and O. Pahlm, "A method for evaluation of QRS shape features using a mathematical model for the ECG," *IEEE Transactions on Biomedical Engineering*, vol. 28, pp. 713–717, 1981.

[5] R. Jané, S. Olmos, P. Laguna, and P. Caminal, "Adaptive Hermite models for ECG data compression: performance and evaluation with automatic wave detection," in *Proceedings of Computers in Cardiology Conference*, Sep 1993, pp. 389–392.

[6] A. Sandryhaila, S. Saba, M. Püschel, and J. Kovacevic, "Efficient compression of QRS complexes using Hermite expansion," *IEEE Transactions on Signal Processing*, vol. 60, no. 2, pp. 947–955, 2012.

[7] T. Dózsa and P. Kovács, "ECG signal compression using adaptive Hermite functions," *Advances in Intelligent Systems and Computing*, vol. 399, pp. 245–254, 2015.

[8] M. Lagerholm, C. Peterson, G. Braccini, L. Edenbrandth, and L. Sörnmo, "Clustering ECG complexes using Hermite functions and self-organizing maps," *IEEE Transactions on Biomedical Engineering*, vol. 47, no. 7, pp. 838–717, 2000.

[9] H. Haraldsson, L. Edenbrandt, and M. Ohlsson, "Detecting acute myocardial infarction in the 12-lead ECG using Hermite expansions and neural networks," *Artificial Intelligence in Medicine*, vol. 32, pp. 127–136, 2004.

[10] P. Kovács, C. Böck, J. Meier, and M. Huemer, "ECG segmentation using adaptive Hermite functions," in *Proceedings of the 51st Asilomar Conference on Signals, Systems, and Computers*, 2017, pp. 1476–1480.

[11] G. Szegő, *Orthogonal polynomials*, AMS Colloquium Publications, New York, USA, 3rd edition, 1967.

[12] P. J. Karoly, D. R. Freestone, R. Boston, D. B. Grayden, D. Himes, K. Leyde, U. Seneviratne, S. Berkovic, T. O'Brien, and M. J. Cook, "Interictal spikes and epileptic seizures: their relationship and underlying rhythmicity," *Brain*, vol. 139, no. 4, pp. 1066–1078, 2016.

[13] R. Q. Quiroga and S. Panzeri, "Extracting information from neuronal populations: information theory and decoding approaches," *Nature Reviews Neuroscience*, vol. 10, pp. 173 – 185, 2009.

[14] E. M. Izhikevich, "Simple model of spiking neurons," *IEEE Transactions on Neural Networks*, vol. 14, no. 6, pp. 1569 – 1572, 2003.

[15] W. Gautschi, "An algorithmic implementation of the generalized Christoffel theorem," in *Numerical integration*, G. Hämmerlin, Ed., vol. 57 of *Internat. Ser. Numer. Math.*, pp. 89–106. Birkhäuser, Basel, Switzerland, 1982.

[16] W. Gautschi, *Orthogonal Polynomials, Computation and Approximation*, Numerical Mathematics and Scientific Computation. Oxford University Press, Oxford, UK, 2004.

[17] B. Fischer and G. H. Golub, "On generating polynomials which are orthogonal over several intervals," *Mathematics of Computation*, vol. 56, pp. 711–730, 1991.

[18] G. H. Golub and V. Pereyra, "The differentiation of pseudo-inverses and nonlinear least squares problems whose variables separate," *SIAM Journal on Numerical Analysis (SINUM)*, vol. 10, no. 2, pp. 413–432, 1973.

[19] A. L. Goldberger, L. A. N. Amaral, L. Glass, J. M. Hausdorff, P. Ch. Ivanov, R. G. Mark, J. E. Mietus, G. B. Moody, C. K. Peng, and H. E. Stanley, "PhysioBank, PhysioToolkit, and PhysioNet: Components of a new research resource for complex physiologic signals," *Circulation*, vol. 101, no. 23, pp. 215–220, 2000.

[20] C. M. Fira and L. Goras, "An ECG signals compression method and its validation using NNs," *IEEE Transactions on Biomedical Engineering*, vol. 55, no. 4, pp. 1319–1326, 2008.

[21] H. H. Chou, Y. J. Chen, Y. C. Shiau, and T. S. Kuo, "An effective and efficient compression algorithm for ECG signals with irregular periods," *IEEE Transactions on Biomedical Engineering*, vol. 53, no. 6, pp. 1198–1205, 2006.

[22] P. Kovács, C. Böck, T. Dózsa, J. Meier, and M. Huemer, "Waveform modeling by adaptive weighted Hermite functions," 2018, [Online], Available: http://codeocean.com.