A WEIGHTED LEAST SQUARES BEAM SHAPING TECHNIQUE
FOR SOUND FIELD CONTROL

A. Canclini [1], *D. Marković* [1], *M. Schneider* [2,3], *F. Antonacci* [1], *E. A. P. Habets* [2,3], *A. Walther* [3], *A. Sarti* [1]

[1] Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di Milano,
Piazza Leonardo da Vinci, 32, 20133 Milano, Italy
[2] International Audio Laboratories Erlangen*
[3] Fraunhofer Institute for Integrated Circuits (IIS),
Am Wolfsmantel 33, 91058 Erlangen, Germany

ABSTRACT

A weighted least squares beam shaping technique for sound field control using a loudspeaker array is proposed. Given a desired spatial response at prescribed control points, the space-time filter is designed by solving a least squares minimization problem. To reduce the computational effort, we propose to place control points only along an arc of circumference centered at the center of the array and passing through a region of interest. Furthermore, we adopt a weighted least squares approach for the design of the space-time filter, so that control points at directions towards which we admit a looser control of the sound field are less relevant in the filter design. The choice of the weights depends on the specific application and we demonstrate the feasibility of the proposed approach for sound zones scenario with one bright and one dark zone.

Index Terms— Loudspeaker array, sound field rendering, sound zones

1. INTRODUCTION

The reproduction of a soundfield through loudspeaker arrays has several applications, e.g. virtual acoustics and delivery of specific audio contents for different listeners without the use of headphones. The desired effect is typically obtained by the application of a space-time filter, which is applied to the loudspeaker signals to reproduce a target soundfield. The most general techniques aim at reproducing the effect of simple acoustic sources [1, 2, 3, 4], or to synthesize more complex wavefields [5]. Other techniques aim at controlling the directivity of the reproduced sources, by steering the maximum directivity of the array towards a desired direction [6, 7]. More advanced techniques were introduced for controlling two or more sound zones, with the aim of creating personal sound spots [8, 9, 10, 11, 12, 13].

One of the most common approaches to tackle the design of the space-time filter is based on a Least Squares (LS) formulation of the problem [5, 14, 15, 16]: a desired spatial response is given at prescribed control points, and the space-time filter best approximates the desired response in LS sense, possibly with the use of other constraints at some control points. Other approaches formulate a total LS problem for designing arbitrary spatial responses [17, 18]. In some applications, the number of transducers present in the array is constrained. In such cases, the design of the space-time filters of the transducers becomes more difficult, due to the smoothness of the spatial response. Moreover, the limited number of transducers in the array decreases the number of dimensions of the search space of the LS solution to the given problem. In such a scenario, it is worth noticing that: 1) many control points yield basically very similar information. Indeed, the information provided by additional control points does not grow linearly with their number. Instead, the amount of information grows with the spatial diversity of the control points, where varying directions are more important than varying distances; 2) not all the control points have the same relevance in the design of the space-time filter: some of them will be related to directions towards which a looser control of the sound field could be sufficient; 3) reflected sound due to the reproduction environment can impair the desired reproduction effect.

The technique proposed in this paper addresses these issues by: i) placing control points along a circle centered at the center of the array. This way, we avoid having multiple control points that are "seen" from the array under the same angle; ii) adopting a weighted LS approach in the design of the spatial filter, so that control points at directions towards which we admit a looser control of the sound field are less relevant in the filter design, while the spatial filter more accurately approximates the desired response in other directions. The choice of the weights depends on the specific application.

In this paper we consider the application of reproducing content in a given area (*bright zone*), while minimizing the energy in another area (*dark zone*). To demonstrate the effectiveness of a weighted LS solution, we consider a reverberant environment, and we assume only the knowledge of its geometry. For this specific scenario, in the space-time filter design we adopt a weighting strategy aimed at controlling the effect of undesired wall reflections impinging into the controlled zones, based on the geometric prediction of the main directions of arrival of wall reflections. The weighted-LS problem is formulated by assigning higher weights to these directions, along with that related to the direct paths between the array and the controlled zones. The remaining directions are inherently less important and are associated to lower weights. Within the framework of the considered application, we compare the proposed technique with the baseline (unweighted) LS approach. As a reference, we include in the analysis the iterative LS technique described in [19], which exploits more a priori information (i.e. room impulse responses) to derive the space-time filter. Simulative results prove that the proposed method attains effectiveness only slightly lower than that of [19], with a clear advantage in terms of computational costs. More-
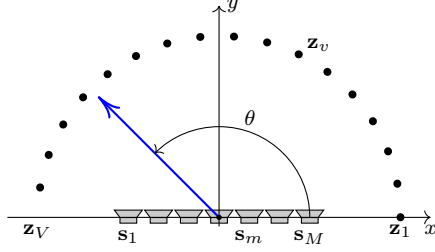

Fig. 1: Loudspeaker array.

over, the proposed technique, since it relies on limited a priori information, is suitable also for moderately dynamic environments (e.g. doors and windows opening), whereas techniques based on previous knowledge of the RIRs would require a relevant update effort.

The rest of the paper is structured as follows. The problem formulation is discussed in Section 2, while Section 3 describes the proposed weighted LS solution for reproducing a desired beam shape. In Section 4 we discuss how the proposed method can be applied to a sound zones scenario. Section 5 reports the evaluation results, while the conclusions are drawn in Section 6.

2. PROBLEM FORMULATION

An M-element loudspeaker array, with emitters placed at $\mathbf{s}_m$, $m = 1, \ldots, M$ is used for generating a desired beampattern. For the sake of simplicity, we consider a linear array, as depicted in Figure 1; nonetheless, this is not a constraint, as the derived solution can be easily adapted to other array geometries. The input signal is a single-channel discrete time signal $x(n)$, with n being the time index.

In order to generate the desired beampattern, we derive suitable space-time filters and apply them to the input signal, i.e.

$$y_m(n) = x(n) * h_m(n), \quad m = 1, \ldots, M, \qquad (1)$$

where $*$ denotes linear convolution, $h_m(n)$ is the space-time filter and $h_m(n)$ is the resulting time-domain signal to feed the mth loudspeaker. In the frequency domain, (1) can be rewritten as

$$y_m(\omega) = x(\omega)h_m(\omega), \quad m = 1, \ldots, M, \qquad (2)$$

where $y_m(\omega)$, $x(\omega)$, and $h_m(\omega)$, are discrete time Fourier transforms of $y_m(n)$, $x(n)$, and $h_m(n)$, respectively, computed at frequency ω. Given an application scenario, the goal of the beamforming algorithm is to compute the frequency-dependent filters $h_m(\omega)$, $m = 1, \ldots, M$, which, applied to the input signal and reproduced by the loudspeaker array, produce the desired sound field.

3. PROPOSED SOLUTION

We now consider the problem of designing space-time filters for sound reproduction, independently of a specific application scenario.

The LS beamformers are designed by minimizing the sum of squared residuals between the desired and produced sound fields. Let $\mathbf{f}(\omega)$ and $\mathbf{p}(\omega)$ be the $V \times 1$ vectors that contain the values of the desired and obtained spatial responses, respectively, computed at V sample control points. In particular, the desired spatial response $\mathbf{f}(\omega)$ depends on the specific application scenario (e.g., it may represent the soundfield generated by a virtual source located at a given position behind the loudspeaker array). The produced spatial response at a set of positions can be written as

$$\mathbf{p}(\omega) = \mathbf{G}(\omega)\mathbf{h}(\omega), \qquad (3)$$

where $\mathbf{h}(\omega)$ is the M-elements column vector of beamforming coefficients obtained by stacking the filters $h_m(\omega)$, and $\mathbf{G}(\omega)$ is the $V \times M$ free-field propagation matrix between the loudspeakers and control points, given by

$$[\mathbf{G}(\omega)]_{vm} = \frac{e^{-j\frac{\omega}{c}\|\mathbf{s}_m - \mathbf{z}_v\|}}{4\pi\|\mathbf{s}_m - \mathbf{z}_v\|}, \qquad (4)$$

where $\mathbf{z}_v$, $v = 1, \ldots, V$, indicate the positions of the control points, and j represents the imaginary unit ($j^2 = -1$).

Typically, the control points sample the entire room area in front of the array or just the region of interest. In the first case, a high computational effort is required. On the other hand, in the second case there is no control over the whole beampattern, except for directions that reach the region of interest. In both cases, however, many control points yield basically very similar information. In fact, to obtain higher spatial diversity, varying directions of control points is more important than varying their distances. Therefore, we propose to place the control points only on the arc of circumference passing through a region of interest, i.e.

$$\mathbf{z}_v = \rho[\cos(\theta_v), \sin(\theta_v)]^T, \quad v = 1, \ldots, V, \qquad (5)$$

where ρ is the radius of the circumference, V is the number of points, and θ_v are the corresponding sample directions, as shown in Figure 1. In particular, the region of interest and the radius ρ are determined by a specific application scenario. Furthermore, by controlling the radius ρ the beamformer can work both in near and far field.

The LS optimization problem is given by

$$\text{minimize } \|\mathbf{p}(\omega) - \mathbf{f}(\omega)\|_2^2, \qquad (6)$$

which yields the normal equations

$$\left(\mathbf{G}^H(\omega)\mathbf{G}(\omega)\right)\mathbf{h}(\omega) = \mathbf{G}^H(\omega)\mathbf{f}(\omega). \qquad (7)$$

In the following, for reasons of conciseness in the notation, we omit the dependency on the frequency.

Because not all directions θ_v corresponding to control points on the arc of circumference contribute equally to the sound field inside the region(s) of interest, the weighted ARC-LS beamformer introduces a weight matrix $\mathbf{W}$ to the LS minimization process. The modified normal equations become

$$\left(\mathbf{G}^H\mathbf{W}\mathbf{G}\right)\mathbf{h} = \mathbf{G}^H\mathbf{W}\mathbf{f}, \qquad (8)$$

where $\mathbf{W} = \text{diag}(\mathbf{w})$, $\mathbf{w} = [w_1, \ldots, w_V]$, and w_v is the weight given to a particular direction θ_v (at given frequency ω).

The solution of (8) could be ill-conditioned. In order to alleviate this problem, a regularization is in order. Let $\mathbf{G}_w = \text{diag}(\sqrt{\mathbf{w}})\mathbf{G}$ and $\mathbf{f}_w = \text{diag}(\sqrt{\mathbf{w}})\mathbf{f}$. The pseudo-inverse matrix $\mathbf{G}_w^\dagger$ is found through Tikhonov regularization. We denote the conditioning number of $\mathbf{G}_w$ with $\Gamma = \frac{\lambda_{\max}}{\lambda_{\min}}$, where $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum singular values of $\mathbf{G}_w$, respectively. Let κ denote the desired conditioning number of the pseudo-inverse after regularization. The following regularization rules are based on the theoretical results reported in [20]. Two cases arise:

$$\begin{array}{ll} \mathbf{G}_w^\dagger = (\mathbf{G}_w^H\mathbf{G}_w)^{-1}\mathbf{G}_w^H & \text{if } \Gamma < \kappa, \\ \mathbf{G}_w^\dagger = (\mathbf{G}_w^H\mathbf{G}_w + \sigma^2\mathbf{I})^{-1}\mathbf{G}_w^H & \text{if } \Gamma \geq \kappa, \end{array} \qquad (9)$$

where the Tikhonov parameter σ^2 is determined by

$$\sigma^2 = \frac{\lambda_{\max}^2 - \lambda_{\min}^2\kappa^2}{\kappa^2 - 1}. \qquad (10)$$

Finally, the solution to the LS problem is given by

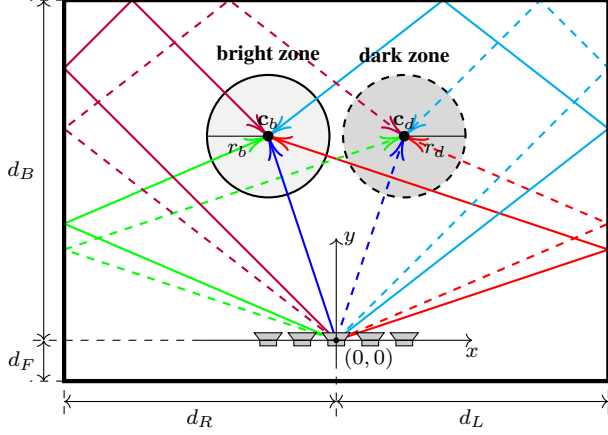$$\mathbf{h}(\omega) = \mathbf{G}_w^\dagger(\omega)\mathbf{f}_w(\omega). \qquad (11)$$

Fig. 2: Geometric setup of the application scenario.

4. CASE STUDY: SOUND ZONES

As an application we are considering a sound zones scenario in which we have one bright and one dark zone. As depicted in Figure 2, both these zones are considered to be circular, centered at $\mathbf{c}_b = [x_b, y_b]$ with radius r_b, and at $\mathbf{c}_d = [x_d, y_d]$ with radius r_d, respectively. The loudspeaker array is placed at distances d_L, d_R, d_B, and d_F from the left, right, back, and front walls, respectively.

The direction of the desired beam is computed as $\theta_0 = \arctan2(y_b, x_b)$, where $\arctan2$ is the four-quadrant inverse tangent. The width of the desired beam is computed in such a way that it covers the whole bright zone, i.e. $\phi_0 = 2\tan(r_b/\|\mathbf{c}_b\|)$. Given θ_0 and ϕ_0, the desired spatial response $\mathbf{f}(\omega)$ is computed at the control points on the arc of circumference passing by the center of the bright zone, $\rho = \|\mathbf{c}_b\|$, as

$$[\mathbf{f}(\omega)]_v = \begin{cases} \frac{ae^{-j\frac{\omega}{c}\|\mathbf{s}_m - \mathbf{z}_v\|}}{4\pi\|\mathbf{s}_m - \mathbf{z}_v\|}, & \text{if } |\theta_v - \theta_0| \leq \frac{\phi_0}{2} \\ 0, & \text{otherwise} \end{cases}, \quad (12)$$

where a is Tukey window [21] with 0.5 ratio of cosine-tapered section length to the entire length, pointing toward the desired direction θ_0. This window function is used to obtain a smooth beampattern.

For the weighted LS problem we distinguish between important and less-important emission directions. In particular, the important directions are the ones that contribute to the sound field inside both the bright zone and the dark zone, either directly or after first or second reflection as shown in Figure 2. These paths are generated by the loudspeaker array, with origin at $\mathbf{l}_0 = [0, 0]$, and its wall-reflected replicas (image sources) with origins at $\mathbf{l}_1 = [2d_L, 0]$, $\mathbf{l}_2 = [2d_L, 2d_B]$, $\mathbf{l}_3 = [-2d_R, 0]$, and $\mathbf{l}_4 = [-2d_R, 2d_B]$. The path directions are given by

$$\theta_{z,i} = \begin{cases} \arctan2(y_z, x_z), & \text{if } i = 0 \\ \arctan(\frac{[\|\mathbf{l}_i - \mathbf{c}_z\|]_2}{[\|\mathbf{l}_i - \mathbf{c}_z\|]_1}), & \text{if } i = 1, 2 \\ \pi - \arctan(\frac{[\|\mathbf{l}_i - \mathbf{c}_z\|]_2}{[\|\mathbf{l}_i - \mathbf{c}_z\|]_1}), & \text{if } i = 3, 4 \end{cases}, \quad (13)$$

where z indicates the zone, which can be either bright or dark, i.e. $z \in \{b, d\}$. Widths are computed similarly as for the desired beam, i.e. $\phi_{z,i} = 2\tan(r_z/\|\mathbf{l}_i - \mathbf{c}_z\|)$ for $z \in \{b, d\}$ and $i \in \{0, \ldots, 4\}$.

To distinguish between important and less-important emission directions, we assign the following weights $w_v(\omega)$ to the sample
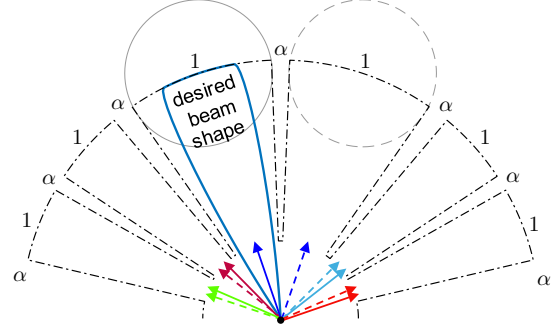

Fig. 3: Desired beam shape and weights used by the W-ARC-LS algorithm. The values of weights w_v are shown on the arc.

directions θ_v, $v = 1, \ldots, V$,

$$w_v(\omega) = \begin{cases} 1, & \text{if } |\theta_v - \theta_{z,i}| \leq \frac{\phi_{z,i}}{2}, \ z \in \{b,d\}, \ i \in \{0,\ldots,4\} \\ \alpha(\omega), & \text{otherwise} \end{cases}, \quad (14)$$

where $0 \leq \alpha(\omega) \leq 1$ controls the weight given to the less-important directions in the LS problem (8). Figure 3 shows an example of the desired beam shape given by the Tukey window (blue line), the important paths $\theta_{z,i}$ (arrows) and assigned weights w_v (dashed line).

The case $\alpha(\omega) = 1$ is equivalent to the non-weighted LS. On the other hand, $\alpha(\omega) = 0$ is the case in which we do not have any control over the less-important directions. However, because these directions could still impact the sound field in the two areas after higher order reflections or in a case of not perfect knowledge of the environment geometry, a certain control is still desired. That is why we propose to compute the parameter $\alpha(\omega)$ in such a way that the sidelobe rejection, computed as the ratio between the first and second highest peaks of the free-field beampattern $\mathbf{G}(\omega)\mathbf{h}(\omega)$, does not decrease more than a prescribed threshold with respect to the case $\alpha(\omega) = 1$. In particular, at each frequency the algorithm starts with $\alpha(\omega) = 1$ and iteratively decreases the value of the parameter, using the update rule $\alpha_{new}(\omega) = 0.95\alpha_{old}(\omega)$, as long as the change in sidelobe rejection does not reach -3 dB. To avoid infinite loops, the algorithm stops if the parameter $\alpha(\omega)$ reaches a minimum value $\alpha_{min}(\omega) = 3.5 \times 10^{-5}$ as well.

5. EVALUATION

In this section we evaluate the effectiveness of the proposed method. After introducing the evaluation setup and the objective metrics, we present the results of the conducted simulations.

Setup To evaluate the proposed algorithm, we simulated the acoustic scenario in Figure 2, considering a rectangular room of size $3.5 \times 3\,\text{m}^2$. The loudspeaker array is composed of $M = 13$ point sources, and its total length is 72 cm. The position of the array is determined by the distances $d_F = 0\,\text{m}$, $d_R = d_L = 1.75\,\text{m}$ and $d_B = 3\,\text{m}$. The bright and dark zones have radii $r_b = r_d = 0.25\,\text{m}$ and are centered at $\mathbf{c}_b = [-0.3\,\text{m}, 1.5\,\text{m}]^T$ and $\mathbf{c}_d = [0.3\,\text{m}, 1.5\,\text{m}]^T$, respectively. The wall hosting the loudspeaker array was considered as totally absorbing, while the remaining three walls were modeled as rough concrete walls, whose absorbing coefficients are tabulated in [22]. To simulate acoustic propagation, we used the fast beam tracing techniques in [5], by modeling early reflections up to the 8th order. In particular, we calculated the point-to-point transfer func-

tions to all the loudspeakers to $Q_b = Q_d = 276$ evaluation points uniformly distributed in the bright and dark zones, respectively. This led to the computation of the vectors $\hat{\mathbf{p}}_b(\omega)$ and $\hat{\mathbf{p}}_d(\omega)$, whose elements correspond to the wavefield predicted at the evaluation points in the two zones. The parameters of the proposed algorithm are set to $\kappa = 25$ and $V = 400$.

Metrics To assess the validity of the proposed approach, we considered the following objective metrics:

- *acoustic contrast*, defined as the ratio between the acoustic energy in the bright and dark zones, i.e. $\mathrm{AC}(\omega) = \frac{\|\hat{\mathbf{p}}_b(\omega)\|^2}{\|\hat{\mathbf{p}}_d(\omega)\|^2}$;

- *normalized mean squared error* of reproduction, computed as $\mathrm{NMSE}(\omega) = \frac{\|\hat{\mathbf{p}}(\omega) - \mathbf{p}(\omega)\|^2}{\|\mathbf{p}(\omega)\|^2}$, where $\hat{\mathbf{p}}(\omega) = [\hat{\mathbf{p}}_b^T(\omega), \hat{\mathbf{p}}_d^T(\omega)]^T$ and $\mathbf{p}(\omega) = [\mathbf{p}_b^T(\omega), \mathbf{p}_d^T(\omega)]^T$ is the target wavefield. Specifically, $\mathbf{p}_b(\omega)$ corresponds to free-field radiation of a point source located at the array center, while $\mathbf{p}_d(\omega) = [0 \cdots 0]^T$;

- *sidelobe rejection*, computed as the ratio between the first and second highest peaks of the free-field beampattern;

- *white noise gain* (WNG), as defined in [23].

Results The result of the evaluation are presented in Figure 4, that shows the acoustic contrast (Fig. 4a), the NMSE (Fig. 4b), the sidelobe rejection (Fig. 4c) and the white noise gain (Fig. 4d). The blue curves depict the results relative to the proposed approach (W-ARC-LS). As a matter of comparison, we included two additional algorithms in the evaluation: ARC-LS (red curves) refers to a non-weighted version of the proposed method, i.e. where the weights are forced to be unitary; I-LS (dashed gray curves) refers to the iterative LS method recently proposed in [19]. I-LS filters were computed using simulated room impulse responses at a total of 48 control points uniformly distributed on the borders of bright and dark zones. Adjacent control points are thus separated by a distance of about $3\,\mathrm{cm}$, similarly to that of control points used by (W-)ARC-LS. This sampling scheme corresponds to a spatial Nyquist frequency around $5\,\mathrm{Hz}$, which is above the spatial aliasing frequency limit of $2.8\,\mathrm{kHz}$ imposed by the considered array. As we considered exact knowledge of the impulse responses, I-LS can be considered as a benchmark for the analyzed application. Furthermore, in Fig. 4b we included the NMSE achieved by a single loudspeaker placed at the center of the array (continuous black curves), which provides an upper bound of the performance scores. This curve was omitted in the other figures, as a single speaker, by definition, achieves: $0\,\mathrm{dB}$ acoustic contrast; $0\,\mathrm{dB}$ side lobe rejection; and unitary white noise gain. We first observe that W-ARC-LS generally achieves higher acoustic contrast with respect to the non-weighted counterpart ARC-LS. This comes at the expense of lower sidelobe rejection, whose worsening is however kept limited to $3\,\mathrm{dB}$ as specified in the filter design stage. As interesting side effect, we can also observe that the higher acoustic contrast positively impacts on the NMSE at frequency below $2\,\mathrm{kHz}$. The WNG remains practically unchanged, meaning that the weighting does not reflect onto the power requested by loudspeakers. Nevertheless, the WNG values maintain reasonable in the whole considered frequency range. Notice that WNG may be increased by decreasing the regularization parameter κ; nevertheless a deeper analysis of WNG is out of the scope of this work.

Moreover, it is worth comparing the performance of the proposed W-ARC-LS method with the results achieved by the reference technique I-LS, whose filters were designed in order to maintain a balanced trade-off between acoustic contrast and NMSE. This design strategy determined slightly better NMSE scores for I-LS, especially at higher frequencies. This, however, comes at the expense
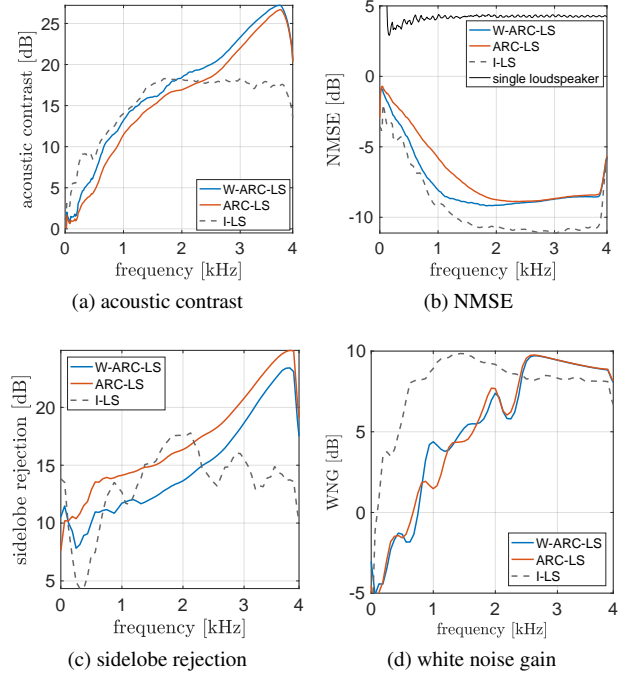

(a) acoustic contrast (b) NMSE

(c) sidelobe rejection (d) white noise gain

Fig. 4: Performance scores for the tested algorithms.

of a worser acoustic contrast in this frequency region, which clips to $17\,\mathrm{dB}$ above $2\,\mathrm{kHz}$. The sidelobe rejection is comparable for all methods. I-LS presents higher WNG values below $2.3\,\mathrm{kHz}$, however this trend is reversed above that frequency.

We can therefore conclude that the proposed technique is effective in mitigating the effect of undesired reflections reaching the controlled zones, despite the fact that it is merely based on the knowledge of the room geometry. Nonetheless, a deeper knowledge of the acoustic properties may lead to slightly better results as in the I-LS case, at the expense of measuring several impulse responses.

6. CONCLUSIONS

We proposed a weighted LS technique for controlling the spatial response of a loudspeaker array. The method was evaluated considering a sound zones application in a reverberant enclosure, whose geometry is assumed to be known, in which a content is delivered to a bright zone while minimizing the acoustic energy in a dark zone. Simulation results revealed the effectiveness of the method, which achieves reproduction error (NMSE) and acoustic contrast generally comparable to those of the reference I-LS technique. It is important to observe that I-LS relies on the knowledge of impulse responses on the border of the bright and dark zones, which take a large effort to be measured and cannot be assumed to be stationary in real environments. The room geometry, on the other hand, can be easily measured in many cases and does not change under normal conditions. Moreover, depending on the application, the positions of bright and dark zones might be altered. In that case, the room impulse responses would have to be measured again, while the proposed approach can simply consider the newly given zone positions while still relying on the unaltered room geometry. Future work is going extend the acoustic propagation model by including the radiation pattern of the loudspeakers, in order to explicitly account for arrays properties and the mutual influence of the emitters.

7. REFERENCES

[1] Ole Kirkeby and Philip A. Nelson, "Reproduction of plane wave sound fields," *The Journal of the Acoustical Society of America*, vol. 94, no. 5, pp. 2992–3000, 1993.

[2] D. B. Ward and T. D. Abhayapala, "Reproduction of a plane-wave sound field using an array of loudspeakers," *IEEE Transactions on Speech and Audio Processing*, vol. 9, no. 6, pp. 697–707, Sep 2001.

[3] Jens Ahrens, Rudolph Rabenstein, and Sascha Spors, "The theory of wave field synthesis revisited," in *Audio Engineering Society Convention 124*, May 2008.

[4] J. Ahrens, *Analytic Methods of Sound Field Synthesis*, Tlabs Series in Telecommunication Services. Springerverlag Berlin Heidelberg, 2012.

[5] D. Marković, A. Canclini, F. Antonacci, A. Sarti, and S. Tubaro, "Visibility-based beam tracing for soundfield rendering," in *Proc. IEEE Int. Workshop on Multimedia Signal Process. (MMSP)*, 2010.

[6] E. Corteel, "Synthesis of directional sources using wave field synthesis, possibilities, and limitations," *EURASIP Journal on Advances in Signal Processing*, vol. 2007, no. 1, pp. 090509, 2007.

[7] E. Mabande and W. Kellermann, "Towards superdirective beamforming with loudspeaker arrays," in *Proc. Int. Congress on Acoustics*, 2007.

[8] Mark Poletti, "An investigation of 2-d multizone surround sound systems," in *Audio Engineering Society Convention 125*, Oct 2008.

[9] Y. J. Wu and T. D. Abhayapala, "Spatial multizone soundfield reproduction: Theory and design," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 19, no. 6, pp. 1711–1720, Aug 2011.

[10] T. Okamoto, "Analytical methods of generating multiple sound zones for open and baffled circular loudspeaker arrays," in *2015 IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA)*, Oct 2015, pp. 1–5.

[11] Y. J. Wu and T. D. Abhayapala, "Multizone 2d soundfield reproduction via spatial band stop filters," in *2009 IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, Oct 2009, pp. 309–312.

[12] T. Betlehem, W. Zhang, M. A. Poletti, and T. D. Abhayapala, "Personal sound zones: Delivering interface-free audio to multiple listeners," *IEEE Signal Processing Magazine*, vol. 32, no. 2, pp. 81–91, March 2015.

[13] T. Betlehem and C. Withers, "Sound field reproduction with energy constraint on loudspeaker weights," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 20, no. 8, pp. 2388–2392, Oct 2012.

[14] G. N. Lilis, D. Angelosante, and G. B. Giannakis, "Sound field reproduction using the lasso," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 18, no. 8, pp. 1902–1912, Nov 2010.

[15] Ferdinando Olivieri, Filippo Maria Fazi, Mincheol Shin, and Philip Nelson, "Pressure-matching beamforming method for loudspeaker arrays with frequency dependent selection of control points," in *Audio Engineering Society Convention 138*. Audio Engineering Society, 2015.

[16] F. Antonacci, A. Calatroni, A. Canclini, A. Galbiati, A. Sarti, and S. Tubaro, "Soundfield rendering with loudspeaker arrays through multiple beam shaping," in *2009 IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, Oct 2009, pp. 313–316.

[17] Simon Doclo and Marc Moonen, "Design of far-field and near-field broadband beamformers using eigenfilters," *Signal Process.*, vol. 83, no. 12, pp. 2641–2673, Dec. 2003.

[18] S. Doclo and M. Moonen, "Design of broadband beamformers robust against gain and phase errors in the microphone array characteristics," *IEEE Transactions on Signal Processing*, vol. 51, no. 10, pp. 2511–2526, Oct 2003.

[19] M. Schneider and E. A. P. Habets, "An iterative least-squares design method for filters with constrained magnitude response in sound reproduction," in *Proc. of the 43. Deutsche Jahrestagung für Akustik (DAGA)*, 2017.

[20] Andrew E Yagle, "Regularized matrix computations," *matrix*, vol. 500, pp. 10.

[21] P. Bloomfield, *Fourier Analysis of Time Series: An Introduction*, Wiley Series in Probability and Statistics. Wiley, 2004.

[22] M. Vorländer, *Auralization: Fundamentals of Acoustics, Modelling, Simulation, Algorithms and Acoustic Virtual Reality*, RWTHedition. Springer Berlin Heidelberg, 2007.

[23] Henry Cox, Robertm Zeskind, and Markm Owen, "Robust adaptive beamforming," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 35, no. 10, pp. 1365–1376, 1987.