QUICKEST CHANGE DETECTION UNDER A NUISANCE CHANGE

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ABSTRACT

We consider the problem of quickest change detection (QCD) for a signal which may undergo both a nuisance and a critical change. Our goal is to detect the critical change without raising a false alarm over the nuisance change. An optimal sequential change detection procedure is proposed for the Bayesian formulation of our QCD problem. A sequential change detection procedure based on the generalized likelihood ratio test (GLRT) statistic is also proposed for the non-Bayesian formulation. We show that our proposed test statistics can be computed efficiently via respective recursive update schemes. We compare our proposed stopping rules with the naive 2-stage procedures, which attempt to detect the changes using separate optimal stopping procedures (i.e., the Shiryaev procedure in the Bayesian formulation, and the CuSum procedure in the non-Bayesian formulation) for the nuisance and critical changes. Simulations demonstrate that our proposed rules outperform the 2-stage procedures.

***Index Terms*—** Quickest change detection, Nuisance change, Optimal Stopping Time, Recursive update, GLRT statistic

1. INTRODUCTION

The problem of detection for a deviation in the statistical properties of a signal with the shortest possible delay is known as quickest change detection (QCD). Usually, we are given a sequence of independent and identically distributed (i.i.d) observations $\{x_t : t \in \mathbb{N}\}$ with distribution f up to an unknown change point ν and i.i.d. with distribution $g \neq f$ after. As the signal is observed sequentially, the goal is to detect this change as quickly as possible while subject to some false alarm constraints. QCD is traditionally applied to manufacturing, in areas such as quality control [1, 2] where any change in the quality of products must be quickly detected. With the decrease in cost and size of modern-day sensors, QCD methods have found applications in other areas such as fraud detection [3], cognitive radio [4], network surveillance [5–8], structural health monitoring [9], spam detection [10], bioinformatics [11], power system line outage detection [12], remote sensing [13] etc.

In the Bayesian formulation of QCD, the change-point is assumed to be a random variable possessing a prior distribution. Shiryaev [14–16] solved the QCD problem when both the pre- and post-change distributions are known and the change-point has a geometric prior distribution. The detection procedure is based on testing the posterior probability of the change currently in effect against a certain detection threshold is proved to be optimal. The procedure stops once the posterior probability exceeds the threshold. In [17, 18], the authors developed an asymptotic Bayesian theory of QCD for a class of non-i.i.d. signal models and prior distributions.

For the non-Bayesian formulation of QCD, the change-point is assumed to be unknown but deterministic. When both the pre- and post-change distribution are known, Page [19] developed the Cumulative Sum Control Chart (CuSum) for quickest change detection. Lorden [20] proved that the CuSum test has asymptotically optimal worst-case average detection delay as the false alarm rate goes to zero. Moustakides [21] later established that the CuSum test is exactly optimal under Lorden's optimality criterion. Later, Lai showed in [22] that the CuSum test is asymptotically optimum under Pollak's criterion [23], as the false alarm rate goes to zero. For the case where the post-change distribution is unknown, Lorden [20] showed that the Generalized Likelihood Ratio CuSum is asymptotically optimal for the case of finite multiple post-change distributions. Other methods were also proposed for the case when the post-change distribution is unknown to a certain degree [22, 24–27].

In many applications, the assumption that the signal is generated i.i.d. with distribution f before the change-point and i.i.d. with distribution g after the change-point over-simplifies the problem. One example is the problem of fault detection using sensor readings from an engine which can be running in two states, idle or active. In a typical fault detection scenario, the engine, which is originally in the idle state, may switch to an active state resulting in a change in the statistical property of the signal. However, this change is not of interest to us. We are only interested in the change from the idle to faulty idle state or the change from active to faulty active state. Furthermore, the readings obtained during the faulty states depends on whether the engine is in idle or fault state. We distinguish the changes the signal undergoes using the concept of a nuisance change and a critical change. We seek to design a sequential change detection procedure which ignores the nuisance change but detects the critical change as quickly as possible. We model the nuisance and critical change point using two approaches, Bayesian and non-Bayesian. In each of these approaches, we propose a stopping rule which declares that a critical change has taken place once the test-statistic exceeds a prespecified threshold. In the Bayesian formulation of the problem, the stopping rule proposed is shown to be optimal. We further derive a recursive scheme to compute our proposed test statistics efficiently. Finally, we compare the proposed stopping times with a naive 2-stage change detection procedure naive 2-stage procedures, which attempt to detect the changes using separate optimal stopping procedures (i.e., the Shiryaev procedure in the Bayesian formulation, and the CuSum procedure in the non-Bayesian formulation) for the nuisance and critical changes, on simulated signals.

To the best of the authors' knowledge, there are no existing works that consider the QCD problem for a signal which may undergo a change that is not of interest. Existing works in QCD which

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consider the problem where the observations are not generated i.i.d. with distribution f before and i.i.d. with distribution g after the change-point ν can be categorized into two main categories. In the first category, the papers [28–30] consider the problem where the pre-change distribution and the post-change distribution can be modelled as a hidden Markov model (HMM). The authors of [30] considers the problem where the vector parameter of a two-state HMM changes at some unknown time. The second category of papers [18, 31] consider a QCD problem with non i.i.d. observations. In [31], the authors extended the optimality of CuSum and Shiryayev-Roberts stopping rule to a class of random processes with likelihood ratios that are subjected to independence and stationary conditions.Unlike the papers mentioned above, in this paper, the signal model we consider here has more than 2-states as a HMM in the Bayesian setting. Furthermore, in the non-Bayesian setting, the signal model cannot be modelled by a HMM. Also, the likelihood ratios generated by the signal model considered in this paper are non-stationary.

The rest of this paper is organized as follows. In Section 2, we present our signal model and problem formulation. We propose stopping times and derive the recursive update scheme for the test statistics in their respective formulations is presented in Section 3 and 4. In Section 5, we present numerical simulations to illustrate the performance of our proposed stopping time. We conclude in Section 6.

2. PROBLEM FORMULATION

In this paper, we assume that the signals observed may be subjected to two types of change, a critical change at ν_c and a nuisance change at ν_n. We are interested in detecting the critical change while the nuisance change is not of interest. In our signal model, the nuisance change point also affects the distribution which generates the observations after the critical change point. This creates a dependence between the nuisance change point and the distribution after the critical change point. Formally, the signal model can be described as follows. Let f, f_n, g, g_n be distinct distributions and $X_1, X_2, ...$ be a sequence of random variables satisfying the following:

$$\text{If } \nu_c \leq \nu_n \quad \begin{cases} X_t \sim f & \text{i.i.d. for all } t < \nu_c, \\ X_t \sim g & \text{i.i.d. for all } \nu_c \leq t < \nu_n. \\ X_t \sim g_n & \text{i.i.d. for all } t \geq \nu_n. \end{cases}$$
$$\text{else} \quad \begin{cases} X_t \sim f & \text{i.i.d. for all } t < \nu_n, \\ X_t \sim f_n & \text{i.i.d. for all } \nu_n \leq t < \nu_c. \\ X_t \sim g_n & \text{i.i.d. for all } t \geq \nu_c. \end{cases} \tag{1}$$

where $\nu_n, \nu_c \geq 0$ are the nuisance and critical change-points respectively. We denote the distribution $h_{\nu_c,\nu_n,t}$ to be the distribution that generates X_t when the nuisance change point is at ν_n and the critical change point is at ν_c. The quickest change detection problem is to detect the critical change ν_c as quickly as possible by sequentially observing $X_1, X_2, ...,$ while keeping the false alarm rate low.

In the Bayesian formulation of our QCD problem, the change-points ν_n, ν_c are independent random variables modelled using a zero-modified geometric distribution [32] where:

$$\mathbb{P}(\nu_i = k) = \begin{cases} \alpha_i & \text{if } k = 0 \\ (1-\alpha_i)\rho_i(1-\rho_i)^{k-1} & \text{if } k > 0 \end{cases} \tag{2}$$

where $i \in \{c, n\}$, α_c and α_n is the probability that the critical and nuisance change (respectively) has already occurred when we start observing the sequence, ρ_c and ρ_n is the conditional probability that the critical and nuisance change point (respectively) is at time k given that the change has not taken place before time k. Our Bayesian QCD problem can be formulated as an optimization problem [15]: find a stopping time τ with respect to the filtration $\{\sigma(X_1^t)\}_{t\geq 0}$ with $X_1^t = X_1, ..., X_t$ to minimize

$$\mathbb{P}(\tau < \nu_c) + C\mathbb{E}\left[(\tau - \nu_c + 1)^+\right] \tag{3}$$

where $x^+ = \max\{x, 0\}$, $\sigma(X_1^t)$ is the sigma-algebra generated by X_1^t and $C > 0$ is a constant controlling the relative importance of the false alarm $\mathbb{P}(\tau < \nu_c)$ and the expected delay $\mathbb{E}\left[(\tau - \nu_c + 1)^+\right]$.

In the non-Bayesian formulation of our QCD problem, the critical and nuisance change-points ν_c, ν_n are assumed to be unknown but deterministic. We denote $\mathbb{E}_{\nu_c,\nu_n}$ to be the expectation assuming the critical change-point is at ν_c and nuisance change-point is at ν_n. Our quickest change detection problem can be formulated as a minimax problem [20]: find a stopping time τ with respect to the filtration $\{\sigma(X_1^t)\}_{t\geq 0}$ to minimize $\text{WADD}(\tau)$ subject to $\text{ARL}(\tau) = \inf_{\nu_n \geq 1} \mathbb{E}_{\infty,\nu_n}[\tau] \geq \gamma$ for some given γ where $\text{WADD}(\tau) = \sup_{\nu_c,\nu_n\geq 1} \operatorname{ess\,sup} \mathbb{E}_{\nu_c,\nu_n}\left[(\tau - \nu_c + 1)^+ \,\middle|\, X_1^{\nu_c - 1}\right]$ is the worst case average detection delay and $\text{ARL}(\tau)$ is the average run length.

3. BAYESIAN QCD WITH NUISANCE CHANGE

In this section, we present the optimal stopping time τ_{Bayes} for our QCD problem (3) in the Bayesian formulation. We begin by presenting a proposition [16] that transforms the cost function (3) into an optimal stopping problem.

Proposition 1. *Suppose* $\mathbb{E}[\nu_c] < \infty$, $\mathbb{E}[\tau] < \infty$ *and define the sequence* $\{\pi_k\}$ *by* $\pi_k = \mathbb{P}\left(\nu_c \leq k \,\middle|\, \sigma(X_1^k)\right)$ *for* $k \geq 0$. *Then, we can write*

$$\mathbb{P}(\tau < \nu_c) + C\mathbb{E}\left[(\tau - \nu_c + 1)^+\right] = \mathbb{E}\left[1 - \pi_\tau + C\sum_{m=0}^{\tau} \pi_m\right] \tag{4}$$

Since the prior of the critical change-point ν_c, given in (2), satisfies $\mathbb{E}[\nu_c] < \infty$, we can rewrite the Bayesian QCD problem in (3) as: find a stopping time τ with respect to the filtration $\{\sigma(X_1^t)\}_{t\geq 0}$ to minimize $\mathbb{E}\left[1 - \pi_\tau + C\sum_{m=0}^{\tau} \pi_m\right]$. The optimal solution [16] that minimize this cost function is of the form $\tau_{\text{Bayes}} = \inf\{k \geq 0 \,|\pi_k \geq \pi^*\}$ where π^* is a threshold to be chosen. Furthermore, if $C \geq 1$, we have $\pi^* = 0$.

In order to have an efficient implementation of the stopping time τ_{Bayes}, we will derive a recursive update scheme for π_k. By applying Baye's rule [33], we obtain

$$\pi_k = \mathbb{P}\left(\nu_c \leq k \,\middle|\, \sigma(X_1^k)\right) = \frac{\mathbb{P}\left(\nu_c \leq k, X_1^k\right)}{\mathbb{P}\left(X_1^k\right)}$$

We introduce the probabilities $\mu_{k,1}, \mu_{k,2}, \mu_{k,3}, \mu_{k,4}$ where

$$\mu_{k,1} = \mathbb{P}\left(\nu_c > k, \nu_n > k, X_1^k\right)$$
$$\mu_{k,2} = \mathbb{P}\left(\nu_c > k, \nu_n \leq k, X_1^k\right)$$
$$\mu_{k,3} = \mathbb{P}\left(\nu_c \leq k, \nu_n > k, X_1^k\right)$$
$$\mu_{k,4} = \mathbb{P}\left(\nu_c \leq k, \nu_n \leq k, X_1^k\right),$$

for $k \geq 0$ so that π_k can be written as

$$\pi_k = \frac{\mathbb{P}\left(\nu_c \leq k, X_1^k\right)}{\mathbb{P}\left(X_1^k\right)} = \frac{\mu_{k,3}+\mu_{k,4}}{\mu_{k,1}+\mu_{k,2}+\mu_{k,3}+\mu_{k,4}}.$$

In order to obtain a recursive scheme for updating π_k, we only need to derive a recursive update scheme for updating the terms $\mu_{k,i}$ for $i = 1, ..., 4$. For $\mu_{k,1}$, using properties of the signal model, we have

$$\begin{aligned}\mu_{k+1,1} &= \mathbb{P}\left(\nu_c > k+1, \nu_n > k+1, X_1^{k+1}\right) \\ &= \mathbb{P}\left(X_{k+1} \,\middle|\, \nu_c > k+1, \nu_n > k+1, X_1^k\right) \\ &\quad \times \mathbb{P}\left(\nu_c \neq k+1, \nu_n \neq k+1 \,\middle|\, \nu_c > k, \nu_n > k, X_1^k\right) \\ &\quad \times \mathbb{P}\left(\nu_c > k, \nu_n > k, X_1^k\right) \\ &= f(X_{k+1})(1-\rho_c)(1-\rho_n)\mu_{k,1}.\end{aligned}$$

For $\mu_{k,2}$, we obtain

$$\begin{aligned}\mu_{k+1,2} &= \mathbb{P}\left(\nu_c > k+1, \nu_n \leq k+1, X_1^{k+1}\right) \\ &= \mathbb{P}\left(\nu_c > k+1, \nu_n \leq k, X_1^{k+1}\right) \\ &\quad + \mathbb{P}\left(\nu_c > k+1, \nu_n = k+1, X_1^{k+1}\right) \\ &= f_n(X_{k+1})\left((1-\rho_c)\mu_{k,2} + (1-\rho_c)\rho_n\mu_{k,1}\right).\end{aligned}$$

For $\mu_{k,3}$, using similar techniques as above

$$\mu_{k+1,3} = g(X_{k+1})\left((1-\rho_n)\mu_{k,3} + (1-\rho_n)\rho_c\mu_{k,1}\right).$$

Finally for $\mu_{k,4}$, we have

$$\begin{aligned}\mu_{k+1,4} &= \mathbb{P}\left(\nu_c \leq k+1, \nu_n \leq k+1, X_1^{k+1}\right) \\ &= \mathbb{P}\left(\nu_c = k+1, \nu_n = k+1, X_1^{k+1}\right) \\ &\quad + \mathbb{P}\left(\nu_c = k+1, \nu_n \leq k, X_1^{k+1}\right) \\ &\quad + \mathbb{P}\left(\nu_c \leq k, \nu_n = k+1, X_1^{k+1}\right) \\ &\quad + \mathbb{P}\left(\nu_c \leq k, \nu_n \leq k, X_1^{k+1}\right) \\ &= g_n(X_{k+1})\left(\rho_c\rho_n\mu_{k,1} + \rho_c\mu_{k,2} + \rho_n\mu_{k,3} + \mu_{k,4}\right).\end{aligned}$$

Putting everything together, we obtain the following update scheme for π_k:

$$\begin{cases}\pi_k &= \frac{\mu_{k,3}+\mu_{k,4}}{\mu_{k,1}+\mu_{k,2}+\mu_{k,3}+\mu_{k,4}} \\ \mu_{k,1} &= f(x_k)(1-\rho_c)(1-\rho_n)\mu_{k-1,1} \\ \mu_{k,2} &= f_n(x_k)\left((1-\rho_c)\mu_{k-1,2} + (1-\rho_c)\rho_n\mu_{k-1,1}\right) \\ \mu_{k,3} &= g(x_k)\left((1-\rho_n)\mu_{k-1,3} + (1-\rho_n)\rho_c\mu_{k-1,1}\right) \\ \mu_{k,4} &= g_n(x_k)\left(\rho_c\rho_n\mu_{k-1,1} + \rho_c\mu_{k-1,2} + \rho_n\mu_{k-1,3} + \mu_{k-1,4}\right),\end{cases}$$

where $\mu_{0,1} = (1-\alpha_n)(1-\alpha_c)$, $\mu_{0,2} = \alpha_n(1-\alpha_c)$, $\mu_{0,3} = (1-\alpha_n)\alpha_c$ and $\mu_{0,4} = \alpha_n\alpha_c$.

4. NON-BAYESIAN QCD WITH NUISANCE CHANGE

In this section, we present the stopping time τ_{GLRT} based on the generalised likelihood ratio test (GLRT) for our QCD problem in the non-Bayesian formulation. To motivate the design of our stopping time, we consider the CuSum stopping time τ_{CuSum} and the CuSum test statistic $S_{\text{CuSum}}(t)$:

$$\begin{aligned}\tau_{\text{CuSum}} &= \inf\{t \geq 0 \mid S_{\text{CuSum}}(t) \geq \beta\} \\ S_{\text{CuSum}}(t) &= \max_{1\leq k\leq t+1} \sum_{i=k}^{t} \log\frac{g(x_i)}{f(x_i)} \qquad (5) \\ &= \log\frac{\max_{1\leq k\leq t+1}\prod_{i=1}^{k-1} f(x_i)\prod_{i=k}^{t} g(x_i)}{\prod_{i=1}^{t} f(x_i)}.\end{aligned}$$

The CuSum test statistic $S_{\text{CuSum}}(t)$ can be interpreted as a GLRT between the hypothesis that the change is currently in effect against the hypothesis that no change has taken place. Applying this to our QCD problem, we propose the following GLRT stopping time:

$$\begin{aligned}\tau_{\text{GLRT}} &= \inf\{t \geq 0 \mid S_{\text{GLRT}}(t) \geq \beta\} \\ S_{\text{GLRT}}(t) &= \log\frac{\max_{1\leq\lambda_c,\lambda_n\leq t+1}\prod_{i=1}^{t} h_{\lambda_c,\lambda_n,i}(x_i)}{\max_{1\leq\lambda_n\leq t+1}\prod_{i=1}^{t} h_{\infty,\lambda_n,i}(x_i)} \qquad (6)\end{aligned}$$

where we stop the process once the test statistic $S_{\text{GLRT}}(t)$ exceeds a threshold β. In order to efficiently update $S_{\text{GLRT}}(t)$, we derive a recursive update scheme for both the numerator and denominator within the logarithm. Firstly, we let

$$a_t = \prod_{i=1}^{t} h_{t+1,t+1,i}(x_i), \quad b_t = \max_{1\leq\lambda_c\leq t}\prod_{i=1}^{t} h_{\lambda_c,t+1,i}(x_i)$$

$$c_t = \max_{1\leq\lambda_n\leq t}\prod_{i=1}^{t} h_{t+1,\lambda_n,i}(x_i), \quad d_t = \max_{1\leq\lambda_c,\lambda_n\leq t}\prod_{i=1}^{t} h_{\lambda_c,\lambda_n,i}(x_i).$$

Splitting the domain of the maximization into 4 regions, we obtain the following relation $\max_{1\leq\lambda_c,\lambda_n\leq t+1}\prod_{i=1}^{t} h_{\lambda_c,\lambda_n,i}(x_i) = \max\{a_t, b_t, c_t, d_t\}$. Furthermore, each of the term a_t, b_t, c_t, d_t can be update recursively with a formula shown at the end of this section. Similarly, by defining p_t, q_t to be $p_t = \prod_{i=1}^{t} h_{\infty,t+1,i}(x_i)$ and $q_t = \max_{1\leq\lambda_n\leq t}\prod_{i=1}^{t} h_{\infty,\lambda_n,i}(x_i)$. It can be seen that the denominator $\max_{1\leq\lambda_n\leq t+1}\prod_{i=1}^{t} h_{\infty,\lambda_n,i}(x_i) = \max\{p_t, q_t\}$ and we have the following update rules $p_{t+1} = f(x_{t+1})p_t$ and $q_t = f_n(x_{t+1})\max\{p_t, q_t\}$. Putting everything together, we have the following recursive update scheme for $S_{\text{GLRT}}(t)$:

$$\begin{cases}S_{\text{GLRT}}(t) &= \log\frac{\max\{a_t,b_t,c_t,d_t\}}{\max\{p_t,q_t\}} \\ a_t &= f(x_t)a_{t-1} \\ b_t &= g(x_t)\max\{a_{t-1}, b_{t-1}\} \\ c_t &= f_n(x_t)\max\{a_{t-1}, c_{t-1}\} \\ d_t &= g_n(x_t)\max\{a_{t-1}, b_{t-1}, c_{t-1}, d_{t-1}\} \\ p_t &= f(x_t)p_{t-1} \\ q_t &= f_n(x_t)\max\{p_{t-1}, q_{t-1}\}\end{cases}$$

We note that the $S_{\text{GLRT}}(t)$ test statistics reduces the CuSum test statistic when $f_n = f$ and $g_n = g$.

5. NUMERICAL RESULTS

In this section, we compare the performance of our proposed stopping time, under both frameworks, with the naive 2-step stopping time which first attempts to detect for the change from f to f_n or g, and proceeds to detect for a change from f_n to g_n if a nuisance change is detected in the first stage. We denote $\mathcal{N}(\mu, \sigma^2)$ as the normal distribution with mean μ and variance σ^2. In our simulations, our signal is generated using the following distributions and parameters $f = \mathcal{N}(0,1)$, $f_n = \mathcal{N}(0,2)$, $g = \mathcal{N}(0.5,1)$, $g_n = \mathcal{N}(0.5,2)$,

$\alpha_n = \alpha_c = 0.1$ and $\rho_c = \rho_n = 0.01$. Here, the critical change is a change in mean from 0 to 0.5 and the nuisance change is a change in variance from 1 to 2.

5.1. Bayesian QCD with Nuisance Change

In the Bayesian formulation of our QCD problem, a naive solution to the QCD with nuisance change problem is a 2-stage procedure. The naive stopping time $\tau_{\text{2-stage}}$ is constructed from 3 stopping times $\tau_{f\to f_n}$,$\tau_{f\to g}$ and $\tau_{f_n\to g_n}$. These are the optimal stopping times constructed using the Shiryaev's procedure [16] to detect for a change in distribution from f to f_n, from f to g and from f_n to g_n respectively. In the first stage, we apply both the stopping times $\tau_{f\to g}$ and $\tau_{f\to f_n}$ to the observations. If $\tau_{f\to g}$ stops the process before $\tau_{f\to f_n}$ then we declare that a critical change has occurred and $\tau_{\text{2-stage}} = \tau_{f\to g}$. Otherwise, we apply $\tau_{f_n\to g_n}$ to the rest of the observations and $\tau_{\text{2-stage}} = \tau_{f_n\to g_n}$. We denote the threshold for declaring critical change as π_c^* and threshold for declaring a nuisance change as π_n^*.

In Fig. 1, we present the comparison of the trade-off between false alarm probability and detection delay for the optimal stopping time τ_{Bayes} and the naive stopping time $\tau_{\text{2-stage}}$ for different nuisance change threshold α_n^*. The results shows that our proposed stopping τ_{Bayes} outperforms the naive stopping time $\tau_{\text{2-stage}}$ as expected.

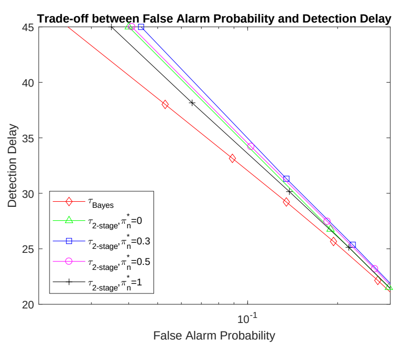


Fig. 1. Comparison of trade-off performance for the optimal stopping time τ_{Bayes} and naive stopping time $\tau_{\text{2-stage}}$ with different nuisance change threshold π_n^*.

5.2. Non-Bayesian QCD with Nuisance Change

We can use a naive 2-stage procedure for the problem of QCD with nuisance change in the non-Bayesian formulation. The naive stopping time $\omega_{\text{2-stage}}$ is also constructed from 3 stopping times $\omega_{f\to f_n}$,$\omega_{f\to g}$ and $\omega_{f_n\to g_n}$. In the non-Bayesian formulation, these stopping times are the optimal CuSum based stopping times described in (5). In the first stage, we apply both the stopping times $\omega_{f\to g}$ and $\omega_{f\to f_n}$ to the observations. If $\omega_{f\to g}$ stops the process before $\omega_{f\to f_n}$, we declare that a critical change has occurred and $\omega_{\text{2-stage}} = \omega_{f\to g}$. Otherwise, we apply $\omega_{f_n\to g_n}$ to the rest of the observation after the stopping time $\omega_{f\to f_n}$ and set $\omega_{\text{2-stage}} = \omega_{f_n\to g_n}$. We denote the threshold for declaring critical change as β_c and threshold for declaring a nuisance change as β_n.

We present the comparison of the trade-off between the average run length and average detection delay for the stopping times τ_{GLRT} and $\omega_{\text{2-stage}}$ for different nuisance change threshold β_n in Fig. 2. Results from the simulation suggest our proposed stopping time τ_{GLRT} out-performs the naive stopping $\omega_{\text{2-stage}}$.

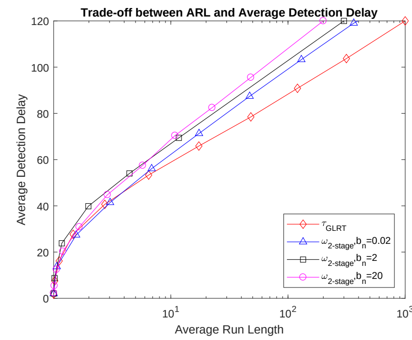
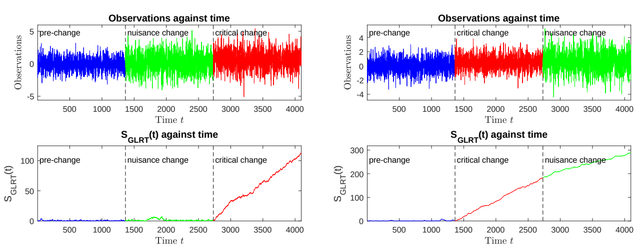


Fig. 2. Comparison of trade-off performance for the proposed stopping time τ_{GLRT} and naive stopping time $\omega_{\text{2-stage}}$.

In Fig. 3, we present the graphs of the test statistic $S_{GLRT}(t)$ together with the observations $x(t)$. The graphs in blue represent the signal before the critical and nuisance change, in red represents the signal after the critical change and in green represents the signal after the nuisance change. In Fig. 3a, the signal experiences a nuisance change followed by a critical change. We observe that $S_{\text{GLRT}}(t)$ remains low after the nuisance change and starts to rise after the critical change. On the other hand, in Fig. 3b, the signal experiences a critical change followed by a nuisance change. We observe that $S_{\text{GLRT}}(t)$ rises steadily after the critical change and continues to rise after the nuisance change. It can be seen that our test statistics $S_{\text{GLRT}}(t)$ is able to quickly react to a critical change while ignoring the nuisance change.



(a) (b)

Fig. 3. Examples of $S_{GLRT}(t)$ values with different choices of ν_c, ν_n. (a)$\nu_c = 2730, \nu_n = 1365$ (b) $\nu_c = 1365, \nu_n = 2730$.

6. CONCLUSION AND FUTURE WORK

We have studied both the Bayesian and non-Bayesian QCD problem where the signal may be subjected to a nuisance change. For the Bayesian QCD problem, we derived a recursive update scheme for the optimal stopping time. For the non-Bayesian QCD problem, we design a stopping time based on the GLRT statistic. We also derived a recursive update scheme for the proposed test statistic. The numerical simulations we ran suggests that the GLRT based stopping time outperforms the naive 2-stage CuSum stopping time. In future work, we will study more general signal models to include signals nuisance changes which are transient and also signal models with multiple post-change hypotheses.

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