EVENT-TRIGGERED PARTICLE FILTERING VIA DIFFUSION STRATEGIES FOR DISTRIBUTED ESTIMATION IN AUTONOMOUS SYSTEMS

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ABSTRACT

The paper is motivated by recent advancements and developments in large, distributed, autonomous, and self-aware systems such as autonomous vehicles and vehicle-to-everything (V2X) technologies, where bandwidth, security, privacy, and/or power considerations limit the number of information transfers between neighbouring agents. In this regard, we propose an event-triggered distributed state estimation via diffusion strategies (ET/DPF), which is a systematic and intuitively pleasing distributed state estimation algorithm that jointly incorporates point and set-valued measurements within the particle filtering framework. In the absence of a measurement form a neighbouring node (i.e., having a set-valued measurement), each local agent/node evaluates the probability that the unknown measurement belongs to the event-triggering set based on its particles which is then used to update the corresponding particle weights. In our Monte Carlo simulations, the proposed ET/DPF outperforms its counterparts in environments with limited bandwidth or/and intermittent connectivity.

Index Terms— Autonomous and self-aware systems, Autonomous Vehicles, Event-triggered estimation, Particle filtering.

1. INTRODUCTION

Autonomous and self aware agent/sensor networks (AN/SN) play a critical role in emergence of cyber-physical systems (CPSs) [1] where communication, control, and signal processing are integrated with the physical world. Autonomous vehicles [2, 3] are one of the key players in this category as they enable deployment of advanced safety features in near future including, but not limited to, automatic collision avoidance, automatic braking, and vehicle-to-everything (V2X) technologies. Conventionally, the Kalman filter (KF) [4] is used to provide sequential state estimates in such multi-agent/sensor autonomous networks. However, the KF is a time-driven estimation methodology and requires synchronous sensor measurements in a periodic manner resulting in extensive communication overhead. Consequently, it is of great practical importance and theoretical significance to reduce the communication overhead (data transfer rate) of agents in distributed autonomous systems, which have resulted in a recent surge of interest in developing event triggered (ET) transmission, scheduling, and estimation schemes [5–12].

The ET concept emerged by the seminal work of Astrom and Bernhardsson [13] where it was shown that Lebesgue sampling is superior for state estimation purposes in some dynamical systems.

This work was partially supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada through the NSERC Discovery Grant RGPIN-2016-049988.

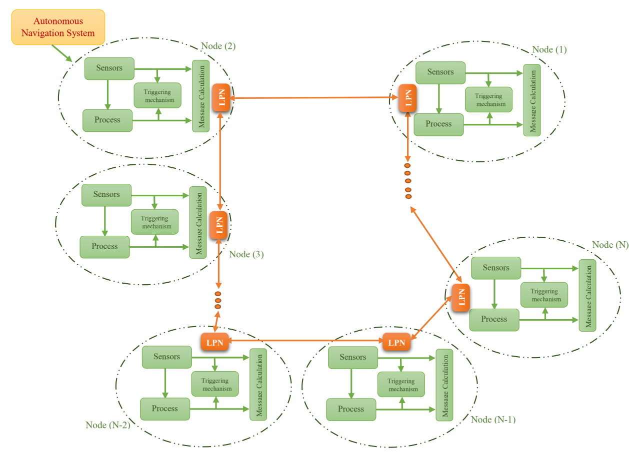


Fig. 1. Block diagram of the multi-agent event-triggered distributed state estimation framework.

References [14, 15] are among the early event-based methodologies and proposed the send-on-delta (SOD) triggering mechanism where the transmission is triggered only when the difference between the current measurement and the previously transmitted one is greater than a pre-defined threshold (delta). In such event-based estimation scenarios and in the absence of an observation (i.e., the triggering conditions are not satisfied) the estimator still has access to side information, i.e., the measurement belongs to the set characterized by the triggering mechanism. Incorporation of the side information from the event-triggering mechanism during non-event iterations results in a hybrid update strategy, i.e., state estimation with joint set-valued and point-valued measurements which is first considered in [16]. In the hybrid scenarios, due to joint incorporation of set and point valued measurements, the posterior distribution becomes non-Gaussian. Some efforts have been recently considered specially by imposing a Gaussian assumption on the posterior distribution, e.g., using single Gaussian approximation [9, 10], Gaussian sum approximation [11], and non-linear filtering scenarios [12]. However, while Gaussian-based approximation of the ET posterior has been investigated extensively, application of non-Gaussian filtering using particle filters [17, 18] is still in its infancy.

The paper addresses this gap and proposes an event-triggered distributed state estimation via diffusion strategies (ET/DPF). In the proposed ET/DPF framework, each agent communicates its sensor measurements only to its neighbouring nodes (no long distance or broadcast communication) without inclusion of a fusion centre (FC), and only in an ET fashion (Fig. 1). Diffusion strategies [19, 20] are used to fuse the ET information in a distributed fashion as these strategies are robust to changes in the underlying network topology and outperform [21] consensus approaches for distributed estimation in autonomous AN/SN systems. The proposed ET/DPF is developed based on a systematic and intuitively pleasing mechanism to jointly incorporate point and set-valued measurements within the particle filter framework. More specifically, we capitalize on the fact that in particle filtering framework the observations's nature (being point or set-valued) will mainly affect the likelihood function which is used to update each particle's weight. In presence of an observation (point-valued measurements), the likelihood function can exactly be evaluated for each particle [22, 23]. In absence of an observation (set-valued measurement case), the proposed ET/DPF evaluates the probability that the unknown observation belongs to the event-triggering set based on its particles, which is then used to update the corresponding particle weights. Simulation experiments, as proof-of-concept, confirms the effectiveness of the of the proposed ET/DPF framework. Extensive simulations with application to navigation of autonomous vehicles is the focus of our ongoing research.

The rest of the paper is organized as follows: Section 2 formulates the problem. Section 3 presents the proposed ET/DPF followed by simulation results in Section 4. Section 5 concludes the paper.

2. PROBLEM FORMULATION AND PARTICLE FILTER

We consider the following overall state-space model to represent multi-agent estimation problem

$$\text{State Model:} \qquad \boldsymbol{x}_k = \boldsymbol{F}_k \boldsymbol{x}_{k-1} + \boldsymbol{\xi}_k \tag{1}$$

$$\text{Observation Model:} \quad \underbrace{\begin{bmatrix} z_k^{(1)} \\ \vdots \\ z_k^{(N)} \end{bmatrix}}_{\boldsymbol{z}_k} = \underbrace{\begin{bmatrix} {\boldsymbol{h}_k^{(1)}}^T \boldsymbol{x}_k \\ \vdots \\ {\boldsymbol{h}_k^{(N)}}^T \boldsymbol{x}_k \end{bmatrix}}_{\boldsymbol{H}_k \boldsymbol{x}_k} + \underbrace{\begin{bmatrix} v_k^{(1)} \\ \vdots \\ v_k^{(N)} \end{bmatrix}}_{\boldsymbol{v}_k}, \tag{2}$$

with N local agents observing a set of n_x state variables, $z_k^{(l)}$ denotes the local measurement made at node l, for ($1 \leq l \leq N$), at time instant k. Symbol T denotes transposition, and $\{\boldsymbol{\xi}(\cdot), v^{(l)}(\cdot)\}$ are, respectively, the global and local non-Gaussian uncertainties in the state and observation models. Matrices $\boldsymbol{F}_k$ and $\boldsymbol{H}_k$, respectively, represent the global state and observation dynamics.

We consider a distributed estimation architecture (Fig. 1) where each agent shares its measurements within its local neighbourhood and recursively updates the posterior distribution based on the collective set of neighbourhood measurements $z_k^{(\aleph^{(l)})} \triangleq \{z_k^{(i)} : i \in \aleph^{(l)}\}$, where $\aleph^{(l)}$ denotes the set of agents connected to agent l. In a fully connected system (i.e., each agent has a direct connection to all the other agents), $\boldsymbol{z}_k^{(\aleph^{(l)})} = \boldsymbol{z}_k$. In the autonomous and self-aware problem considered here, however, the network is not fully connected, besides, a local agent can not also afford to communicate periodically with its neighbours. This could be due to bandwidth, security, privacy, and/or power considerations. Therefore, we consider an ET communication/fusion framework [5], where after making each measurement the sensor decides on keeping or sharing its measurements with its local neighbourhood. In an ET fusion architecture, local decisions at sensor node l is governed by a binary triggering criteria denoted by $\gamma_k^{(l)}$ which is defined as follows

$$\begin{cases} \gamma_k^{(l)} = 1 : & \text{Event occurs, Sensor } l \text{ communicates.} \\ \gamma_k^{(l)} = 0 : & \text{Idle case, no communication from Sensor } l. \end{cases}$$

When the event-triggering condition is satisfied (i.e., $\gamma_k^{(l)} = 1$), the exact value of the sensor measurement z_k is known at all its neighbouring nodes, referred to as "point-valued observation information". On the other hand, when the ET condition is violated (i.e., $\gamma_k^{(l)} = 0$), some information contained in the ET sets is known to the neighbouring nodes instead, referred to as "set-valued information". The main issue here comes from the non-Gaussianity of the a posteriori distribution due to joint incorporation of point and set-valued measurements, i.e., the posterior distribution no longer follows a Gaussian distribution. Next, we present the proposed ET/DPF implementation which systematically uses point and set-valued observation to approximate this non-Gaussian ET posterior.

3. DIFFUSIVE EVENT-TRIGGER PARTICLE FILTER

In the proposed ET/DPF, each agent implements a localized filter to compute an intermediate local estimate based on the ET measurements limited to its immediate neighbourhood. Local agents then cooperate distributively in an ET fashion to improve the accuracy of their intermediate localized state estimates. Below, we explain these steps in more details.

3.1. Local Filtering Step

In the ET/DPF, the local filter at node l computes an intermediate state estimate of the entire state vector $\boldsymbol{x}_k$ by running one localized Gaussian particle filter [23]. In computing the localized state estimates, communication is limited to the local neighbourhoods and ET measurements. Without loss of generality and for simplicity of the presentation, we consider the practical "Send-on-Delta" triggering criteria/condition [14]. In order to decide whether or not to send new measurements, sensor l computes the distance between its current measurement and the previously transmitted measurement based on the following ET schedule

$$\gamma_k^{(l)} = \begin{cases} 1, & \text{if } |z_k^{(l)} - z_{\tau_k}^{(l)}| \geq \Delta^{(l)} \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where $\tau_k^{(l)}$ denotes the time of last communication from sensor l, and $\Delta^{(l)}$ denotes its local triggering threshold. Based on the above triggering mechanism, we define the following hybrid observation

$$\tilde{z}_k^{(l)} = \begin{cases} z_k^{(l)} & \text{if } \gamma^{(l)} = 1 \\ \{z_k^{(l)} : z_k^{(l)} \in (z_{\tau_k^{(l)}}^{(l)} - \Delta^{(l)}, z_{\tau_k^{(l)}}^{(l)} + \Delta^{(l)})\} & \text{if } \gamma^{(l)} = 0 \end{cases}$$

The collective set of ET measurements is denoted by

$$\boldsymbol{y}_k^{(l)} = \{\tilde{z}_k^{(i)} : i \in \aleph^{(l)}\}, \tag{4}$$

and over time defined as $\boldsymbol{Y}_k^{(l)} = \{\boldsymbol{y}_1^{(l)}, \ldots, \boldsymbol{y}_k^{(l)}\}$. The posterior distribution $P(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)})$ based on collective set of hybrid observations is no longer Gaussian, eliminating the application of linear filters such as the KF. In such a non-Gaussian scenario, the optimal Bayesian filtering recursion for iteration k is

$$P(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)}) = \frac{P(\boldsymbol{y}_k|\boldsymbol{x}_k)P(\boldsymbol{x}_k|\boldsymbol{Y}_{k-1}^{(l)})}{P(\boldsymbol{y}_k|\boldsymbol{Y}_{k-1}^{(l)})}, \tag{5}$$

$$P(\boldsymbol{x}_k|\boldsymbol{Y}_{k-1}^{(l)}) = \int P(\boldsymbol{x}_{k-1}|\boldsymbol{Y}_{k-1}^{(l)}) f(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}) d\boldsymbol{x}_{k-1}. \tag{6}$$

In order to compute the non-Gaussian posterior distribution given by Eq. (5) jointly based on point and set-valued measurements, each localized filter approximates the filtering distribution $P(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)})$ using a set of particles $\{\mathbb{X}_{k_i}^{(l)}\}_{i=1}^{n_p}$ derived from a proposal distribution $q(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)})$, and computes their associated weights $W_{k_i}^{(l)}$. The ET/DPF implements the filtering recursions by propagating the particles $\mathbb{X}_{k_i}^{(l)}$ and associated weights $W_{k_i}^{(l)}$, $(1 \leq i \leq n_p)$, as

$$\mathbb{X}_{k_i}^{(l)} \sim q(\mathbb{X}_{k_i}^{(l)}|\mathbb{X}_{k-1_i}^{(l)}, \boldsymbol{Y}_k^{(l)}) \tag{7}$$

$$W_{k_i}^{(l)} \propto W_{k-1_i}^{(l)} \frac{P(\boldsymbol{y}_k^{(l)}|\mathbb{X}_{k_i}^{(l)})P(\mathbb{X}_{k_i}^{(l)}|\mathbb{X}_{k-1_i}^{(l)})}{q(\mathbb{X}_{k_i}^{(l)}|\mathbb{X}_{k-1_i}^{(l)}, \boldsymbol{Y}_k^{(l)})}. \tag{8}$$

Consequently, the local filter at Agent l computes a particle-based approximation of the local ET conditional posterior $p(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)})$ as $p(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)}) = \sum_{i=1}^{n_p} W_{k_i}^{(l)} \delta\big(\boldsymbol{x}_k - \mathbb{X}_{k_i}^{(l)}\big)$. The local intermediate state estimate denoted by $\boldsymbol{\psi}_k^{(l)}$ at iteration (k) is defined as the expected value of the posterior distribution, i.e.,

$$\boldsymbol{\psi}_k^{(l)} = \mathbb{E}\{\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)}\} = \int \boldsymbol{x}_k p(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)}) \boldsymbol{d}\boldsymbol{x}_k \approx \sum_{i=1}^{n_p} W_{k_i}^{(l)} \mathbb{X}_{k_i}^{(l)}. \tag{9}$$

Node l fuses its local intermediate state estimate $\boldsymbol{\psi}_k^{(l)}$ with those of its neighbouring nodes using diffusive strategies to form its updated local state estimate, denoted by $\hat{\boldsymbol{x}}_k^{(l)}$. Assume all local filters are at steady-state at the end of iteration $(k-1)$, i.e., node l, has computed $\hat{\boldsymbol{x}}_k^{(l)}$ and its corresponding error covariance $\boldsymbol{P}_k^{(l)}$. At iteration k, the local filtering step is then completed at each node l, $(1 \leq l \leq N)$ based on the following sub-steps:

Sub-Step L1. Observation Collection: Node l collects observations made in its neighbourhood to form $\boldsymbol{y}_k^{(l)}$, i.e., the collection of ET measurements available in the local neighbourhood $\aleph^{(l)}$ of node l.

Sub-Step L2. Local State Estimation: Node l computes the local state estimate $\boldsymbol{\psi}_k^{(l)}$ by generating n_p particles from the transitional density $p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$ and computes the mean $\bar{\boldsymbol{\mu}}_k^{(l)}$ and covariance $\bar{\boldsymbol{\Sigma}}_k^{(l)}$ of its predictive particles as $\bar{\boldsymbol{\mu}}_k^{(l)} = 1/n_p \sum_{i=1}^{n_p} \mathbb{X}_{k_i}^{(l)}$ and $\bar{\boldsymbol{\Sigma}}_k^{(l)} = 1/n_p \sum_{i=1}^{n_p} \big(\bar{\boldsymbol{\mu}}_k^{(l)} - \mathbb{X}_{k_i}^{(l)}\big)\big(\bar{\boldsymbol{\mu}}_k^{(l)} - \mathbb{X}_{k_i}^{(l)}\big)^T$. Node l then updates the corresponding weights of its predictive particles as follows

$$\tilde{W}_{k_i}^{(l)} = \frac{p\big(\boldsymbol{y}_k^{(l)}|\mathbb{X}_{k_i}^{(l)}\big) \overbrace{\mathcal{N}[\mathbb{X}_{k_i}^{(l)}; \bar{\boldsymbol{\mu}}_k^{(l)}, \bar{\boldsymbol{\Sigma}}_k^{(l)}]}^{p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})}}{\pi\big(\mathbb{X}_{k_i}^{(l)}|\boldsymbol{Y}_k^{(l)}\big)}, \tag{10}$$

and normalize them as $W_{k_i}^{(l)} = \tilde{W}_{k_i}^{(l)} / \sum_{i=1}^{n_p} \tilde{W}_{k_i}^{(l)}$. In Eq. (10), $\mathcal{N}[\cdot]$ denotes the Gaussian distribution with mean and covariance specified within its parenthesis. Further, agent l updates its local intermediate state estimate and its corresponding covariance as $\boldsymbol{\psi}_k^{(l)} = \sum_{i=1}^{n_p} W_{k_i}^{(l)} \mathbb{X}_{k_i}^{(l)}$ and $\boldsymbol{P}_k^{(l)} = \sum_{i=1}^{n_p} W_{k_i}^{(l)} \big(\boldsymbol{\psi}_k^{(l)} - \mathbb{X}_{k_i}^{(l)}\big)\big(\boldsymbol{\psi}_k^{(l)} - \mathbb{X}_{k_i}^{(l)}\big)^T$. Consequently, the localized filtering density at node l is approximated with a single Gaussian as $p(\boldsymbol{x}_k|\boldsymbol{Y}_k^{(l)}) = \mathcal{N}\big(\boldsymbol{x}_k; \boldsymbol{\psi}_k^{(l)}, \boldsymbol{P}_k^{(l)}\big)$.

The final step to implement localized filters within the ET/DPF framework is to evaluate the ET likelihood, $p\big(\boldsymbol{y}_k^{(l)}|\mathbb{X}_{k_i}^{(l)}\big)$. For this purpose, we make the common assumption that measurements are uncorrelated, i.e.,

$$p(\boldsymbol{y}_k^{(l)}|\boldsymbol{x}_k) = \prod_{j \in \aleph^{(l)}} p(y_k^{(j)}|\boldsymbol{x}_k). \tag{11}$$

Therefore, agent l computes the likelihood function for each of its neighbouring nodes based on one of the following two approaches.

(i) Update based on Set-valued Measurements ($\gamma_k^{(j)} = 0$): In the absence of the sensor measurement from agent $j \in \aleph^{(l)}$, and based on the triggering mechanism defined in Eq. (3), the estimator at node l has the following side information

$$z_k^{(j)} \in (z_{\tau_k}^{(j)} - \Delta^{(j)}, z_{\tau_k}^{(j)} + \Delta^{(j)}), \tag{12}$$

where $z_{\tau_k}^{(j)}$ is the previously communicated observation from node j. In this case, the likelihood function can be specified as follows

$$P\big(y_k^{(j)}|\boldsymbol{x}_k, \gamma_k^{(j)} = 0\big) = P\big(z_{\tau_k}^{(j)} - \Delta^{(j)} \leq z_k^{(j)} \leq z_{\tau_k}^{(j)} + \Delta^{(j)}\big), \tag{13}$$

which by substituting from Eq. (2), we have

$$\begin{aligned} &P\big(y_k^{(j)}|\boldsymbol{x}_k, \gamma_k^{(j)} = 0\big) \\ &= P\big(z_{\tau_k}^{(j)} - \Delta^{(j)} \leq \boldsymbol{h}_k^{(j)^T} \boldsymbol{x}_k + v_k^{(j)} \leq z_{\tau_k}^{(j)} + \Delta^{(j)}\big) \\ &= P\Big(\Big[z_{\tau_k}^{(j)} - \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T} \boldsymbol{x}_k\Big] \leq v_k^{(j)} \leq \Big[z_{\tau_k}^{(j)} + \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T} \boldsymbol{x}_k\Big]\Big). \end{aligned} \tag{14}$$

Note that in the third line of Eq. (14), we kept the noise in the middle and moved other terms to the sides in order to be able to compute the likelihood function based on the probability distribution of the noise. As the observation noise $v_k^{(j)}$ has a zero-mean Gaussian distribution with variance $R_k^{(j)}$, the likelihood function reduces to

$$\begin{aligned} &P(y_k^{(j)}|\boldsymbol{x}_k, \gamma_k^{(j)} = 0) \\ &= \frac{1}{\sqrt{2\pi R_k^{(j)}}} \int_{z_\tau^{(j)} - \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T} \boldsymbol{x}_k}^{z_\tau^{(j)} + \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T} \boldsymbol{x}_k} \exp\left\{\frac{-t^2}{2R_k^{(j)}}\right\} dt \\ &= \underbrace{\Phi\left(\frac{z_\tau^{(j)} + \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T} \boldsymbol{x}_k}{\sqrt{R_k^{(j)}}}\right) - \Phi\left(\frac{z_\tau^{(j)} - \Delta^{(j)} - \boldsymbol{h}_k^{(j)^T} \boldsymbol{x}_k}{\sqrt{R_k^{(j)}}}\right)}_{\boldsymbol{h}^{(j)^T}(\boldsymbol{x}_k)}, \end{aligned} \tag{15}$$

where $\Phi(\cdot)$ is the cumulative Gaussian distribution function with zero mean and variance 1. This completes the computation of the likelihood function in idle scenarios (no transmission).

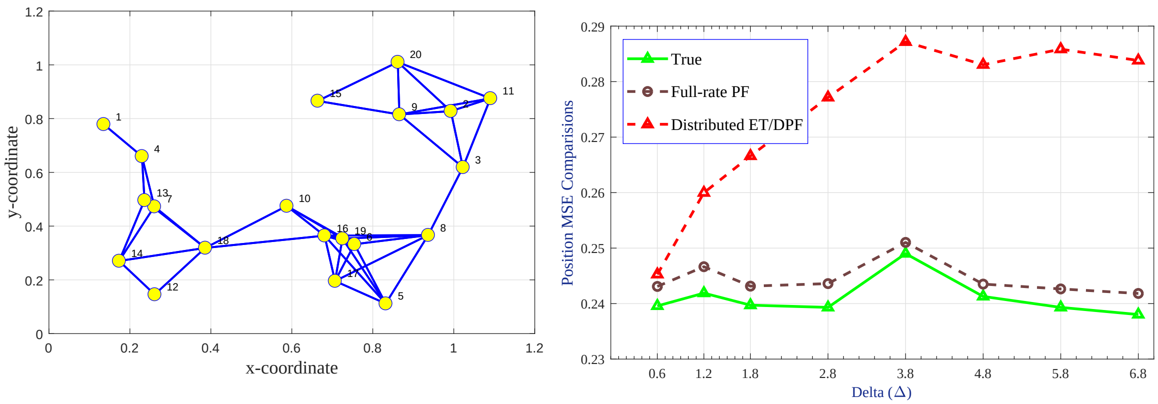
(ii) Update based on Point Measurements ($\gamma_k^{(j)} = 1$): In this case, the estimator receives the sensor measurement $z_k^{(j)}$, therefore, the hybrid likelihood function $P(y_k^{(j)}|\boldsymbol{x}_k)$ reduces to the sensor likelihood function $P(z_k^{(j)}|\boldsymbol{x}_k)$ [24]. Consequently, the hybrid likelihood function is given by

$$P(y_k^{(j)}|\boldsymbol{x}_k, \gamma_k^{(j)} = 1) = \frac{1}{\sqrt{2\pi R_k^{(j)}}} \exp\left\{\frac{-(z_k^{(j)} - \boldsymbol{h}_k^{(j)^T} \boldsymbol{x}_k)^2}{2R_k^{(j)}}\right\} \tag{16}$$

This complete the presentation of the localized filters of the ET/DPF. Next, we present our diffusive fusion where each node updates its local state estimates by collaborating with its neighbouring nodes.

3.2. Diffusion Step

The second step is based on local collaboration, where node l, $(1 \leq l \leq N)$, fuses its local intermediate estimate $\boldsymbol{\psi}_k^{(l)}$ with that of its



(a) (b)

Fig. 2. (a) Sensor placements. (b) Agent networks and connections. (c) Position MSE comparison over different values of $\Delta^{(l)}$.

neighbouring nodes as follows

$$\hat{\boldsymbol{x}}_k^{(l)} = \sum_{j \in \aleph^{(l)}} \underbrace{\gamma_k^{(j)} \times \alpha_k^{(j,l)}}_{\beta_k^{(j,l)}} \times \boldsymbol{\psi}_k^{(j)}, \tag{17}$$

such that if we collect the nonnegative weights $\beta_k^{(j,i)}$ into a $N \times N$ matrix $\boldsymbol{A}_k$, the weights $\beta_k^{(j,l)}$ satisfy the following properties: (i) $\beta_k^{(j,l)} \geq 0$; (ii) $\boldsymbol{A}_k^T \mathbf{1} = \mathbf{1}$, and; (iii) $\beta_k^{(j,l)} = 0$ if $j \notin \aleph^{(l)}$ or $\gamma_k^{(j)} = 0$. Term $\mathbf{1}$ is a vector of size N with all entries equal to one. These conditions imply that the weights on the links arriving at a single node add up to one, which is equivalent to saying that the matrix is left-stochastic. Moreover, if two nodes are not connected or an event is not triggered, then their corresponding entry is zero. The ET diffusive matrix $\boldsymbol{A}_k$ can be designed using covariance intersection [25] or updated adaptively as explained in [26]. A simple approach for choosing the ET diffusion matrix is to assign a weigh to each node according to the cardinality of its neighbourhood by considering the triggering variables. Through diffusive fusion, the filter implemented at node l, $(1 \leq l \leq N)$, forms a Gaussian approximation of the posterior distribution as

$$p(\boldsymbol{x}_k | z_k) = \mathcal{N}\big(\boldsymbol{x}_k; \hat{\boldsymbol{x}}_k^{(l)}, \boldsymbol{P}_k^{(l)}\big). \tag{18}$$

Note that, $\boldsymbol{P}_k^{(l)}$ in Eq. (18) is not a representative of the covariance of the diffusive state estimate $\hat{\boldsymbol{x}}_k^{(l)}$ as the diffusion update is not performed on the covariance matrices.

4. SIMULATIONS

In this section, simulation experiments are developed, as proof-of-concept, to evaluate the performance of the proposed ET/DPF. Extensive simulations with application to navigation of autonomous vehicles is the focus of our ongoing research. Following the recent literature on ET estimation [7], a tracking problem is considered where observations from an agent network of $N = 20$ nodes is used to sequentially estimate the state of the target denoted by $\boldsymbol{x}_k$ consisting of its position and speed. Sensors are distributed randomly in a square region and each sensor communicates with its neighbours within a connectivity radius of $\sqrt{2\log(N)/N}$ units. Target's dynamic is given by $\boldsymbol{x}_k = \begin{bmatrix} 0.8 & 1 \\ 0 & 0.95 \end{bmatrix} \boldsymbol{x}_{k-1} + \boldsymbol{w}_k$, where $\boldsymbol{w}_k \sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{Q} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\right)$. Each sensor periodically measures the position and speed of the target based on the following observation model $z_k^{(l)} = \begin{bmatrix} 0.7 & 0.6 \end{bmatrix} \boldsymbol{x}_k + v_k^{(l)}$. In this experiment, the observation noise variance is $\sigma^2_{v^{(l)}} = 0.01$. The following results are computed over Monte-Carlo (MC) simulations of 100 runs. The object's position and speed used in each simulation run changes randomly to provide a fair experimental benchmark. Furthermore, the following three estimators are implemented and compared for accuracy: (i) The full-rate diffusion-based KF where each sensor communicates its measurements to its neighbouring nodes every iteration; (ii) Event-based diffusive KF, where SOD triggering is used, and; (iii) The proposed ET/DPF algorithm, where the triggering decisions at the sensor level are made based on SOD mechanism and the fusion is performed distributively using diffusive strategies by jointly incorporating set-valued and point-valued measurements.

A realization of the sensor placement is shown in Fig. 2(a). Fig. 2(b) illustrates the estimated position mean-square errors (MSE) obtained from different implemented filters without inclusion of the KF-based curve for better clarity. In this experiment, the value of $\Delta^{(l)}$, for $(1 \leq l \leq N = 20)$, varies up to 6.8, which in turn results in varying values of the communication rate (we considered the same value for all the agents). It is observed that the proposed ET/DPF closely follows the ground truth (i.e., the difference is within 0.05 in position MSE sense). Besides, it is also observed that the proposed ET/DPF algorithm provides acceptable results in very low communication rates (high values of $\Delta^{(l)}$) and closely follows its full-rate counterparts in high communication rates. Finally, when the communication rate increases (i.e., small values for $\Delta^{(l)}$), the proposed event-based methodology approaches its full-rate counterpart.

5. CONCLUSION

In this paper, we proposed an event-triggered particle filter (ET/DPF) framework for distributed state estimation in autonomous agent-sensor systems. Each sensor uses practical send-on-delta (SOD) event triggering mechanism resulting in availability of side information at its neighbouring nodes in the absence of an observation. Utilization of this side information results in estimation with joint set-valued and point-valued measurements which consequently translates in to a non-Gaussian state estimation problem. The proposed ET/DPF is a systematic and intuitively pleasing non-Gaussian estimation framework within the particle filter framework and uses diffusion strategies for distributed implementations. The simulation results depicts that the proposed ET/DPF outperforms its counterparts specifically in low communication rates.

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