SLOW-TIME CODING FOR MUTUAL INTERFERENCE MITIGATION

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ABSTRACT

The mutual interference between similar radar systems can result in reduced radar sensitivity and increased false alarm rates. To address the interference mitigation problems in similar radar systems, we propose herein two slow-time coding schemes to modulate the pulses within a coherent processing interval (CPI). Specifically, the first coding scheme is designed through Doppler shifting and the second is devised via an optimization method. The proposed coding schemes are very easy to implement in practice and the incorporation of the coding schemes only requires slight modification of the existing systems. Our numerical examples indicate that the proposed coding schemes can reduce the interference power level in a desired area of the cross-ambiguity function significantly.

***Index Terms*—** Radar systems, mutual interference, interference mitigation, slow-time coding, code optimization.

1. INTRODUCTION

In civilian radar applications, such as the automotive radar used in vehicles [1–4] and millimeter-wave radar used in Google's hand gesture recognition systems [5, 6], the radar systems massively produced on a commercial scale tend to be quite similar, or even almost identical.

However, the increasing number of similar or identical radar systems will result in severe mutual interferences. A direct consequence of the mutual interference is the severely reduced radar sensitivity and increased false alarm rates. Thus, it is vitally important to enhance the radar performance in severe mutual interference scenarios. However, the analysis of the mutual interference problem and its associated suppression methods have not been widely discussed in the literature. In [7], the author analyzed the mutual interference between frequency-modulated continuous waveforms (FMCW) radar systems, and proposed several techniques to mitigate the interference problem, including prepossessing and finite impulse response (FIR) filtering. The authors in [8–10] investigated the mutual interference problems between automotive radar systems with different types of transmissions.

In this paper, we address the mutual interference mitigation problem in similar or identical radar systems. We propose two slow-time coding schemes to reduce the interference power level. The first coding scheme aims to shift the Doppler frequency of the interference and separate it from the target in the Doppler region. The second coding scheme aims to minimize the discrete periodic cross-ambiguity function in a desired area. We show that both methods can suppress the mutual interferences significantly. Both approaches are low cost and can be implemented in practical systems easily.

2. PROBLEM FORMULATION

Consider two identical radar systems operating within the same frequency band, as shown in Fig. 1(a). We assume that the two systems use FMCW for their transmissions (shown in Fig. 1(b)). In addition, they have the same sweep bandwidth and chirp duration (i.e., sweep time), denoted by B and T_{chirp}, respectively. Mathematically, the transmitted waveform $s(t)$ can be written as follows:

$$s(t) = \sum\nolimits_{n=-\infty}^{\infty} u(t - nT_{\text{chirp}}), \tag{1}$$

where $u(t) = \exp(j(2\pi f_{\text{c}} t + \pi K t^2))$, f_{c} is the carrier frequency, and $K = B/T_{\text{chirp}}$ is the chirp rate.

When the two radar systems are operating simultaneously, the received signal by one radar includes not only the target reflections, but also the interference signal due to the transmission from the other radar system. As a result, we can write the received signal by one radar (e.g., the radar mounted on Car 1 in Fig. 1(a)) as follows:

$$r(t) = y_{\text{T}}(t) + y_{\text{I}}(t) + w(t), \tag{2}$$

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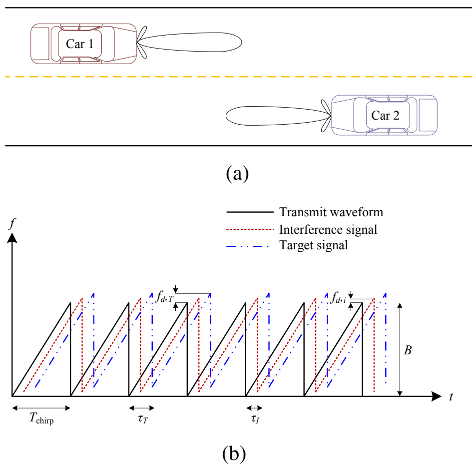

Fig. 1. Mutual interference between automotive radars. (a) Potential source of mutual interference; (b) time-frequency illustration of the transmit waveform, the target signal, and the interference.

where $y_{\rm T}(t) = \alpha_{\rm T} s(t-\tau_{\rm T}) \exp(j2\pi f_{d,\rm T} t)$ represents the target returns, $\alpha_{\rm T}$ is the target amplitude, $\tau_{\rm T}$ denotes the (two-way) target propagation delay, $f_{d,\rm T}$ is the target Doppler frequency, $y_{\rm I}(t) = \alpha_{\rm I} s(t-\tau_{\rm I}) \exp(j2\pi f_{d,\rm I} t)$ represents the interference signal, $\alpha_{\rm I}$ is the interference amplitude, $\tau_{\rm I}$ is the (one-way) delay associated with the interference, $f_{d,\rm I}$ denotes the interference Doppler frequency, and $w(t)$ stands for the internal disturbance (e.g., receiver noise).

Typically, FMCW radar systems collect the received signal from N consecutive pulses within a coherent processing interval (CPI) for target detection and parameter estimation. To this end, the received signal is usually conjugately mixed with the transmitted signal to produce a low-frequency beat signal (i.e., dechirping). As a result, the dechirped version of $r(t)$ for the n^{th} pulse is given by

$$\begin{aligned} r_{\rm dc}^n(t) =& \alpha_T \exp(j2\pi(f_{B,\rm T} t + n f_{d,\rm T} T_{\rm chirp})) \\ &+ \alpha_I \exp(j2\pi(f_{B,\rm I} t + n f_{d,\rm I} T_{\rm chirp})) + w^n(t), \end{aligned} \tag{3}$$

where $f_{B,\rm T} = K\tau_{\rm T} + f_{d,\rm T}$ and $f_{B,\rm I} = K\tau_{\rm I} + f_{d,\rm I}$ denote the beat frequencies corresponding to the target and the interference signal, respectively, and to lighten the notations, we absorb the constant phase terms into $\alpha_{\rm T}$ and $\alpha_{\rm I}$, and use $w^n(t)$ to denote the dechirped noise.

Denote the digital samples associated with $r_{\rm dc}^n(t)$ as

$$\begin{aligned} r(m,n) =& \alpha_T \exp(j2\pi(\hat{f}_{B,\rm T} m + \hat{f}_{d,\rm T} n)) \\ &+ \alpha_I \exp(j2\pi(\hat{f}_{B,\rm I} m + \hat{f}_{d,\rm I} n)) + w(m,n), \end{aligned} \tag{4}$$

where we use m to denote the fast-time index, $\hat{f}_{B,\rm T} = f_{B,\rm T} T_s$ and $\hat{f}_{B,\rm I} = f_{B,\rm I} T_s$ denote the normalized beat frequencies, $\hat{f}_{d,\rm T} = f_{d,\rm T} T_{\rm chirp}$ and $\hat{f}_{d,\rm I} = f_{d,\rm I} T_{\rm chirp}$ denote the normalized Doppler frequencies, $T_s = 1/f_s$, f_s stands for the sampling frequency, and $w(m,n)$ denotes the noise. Assume that M samples are collected for each period. Then we can obtain the range-Doppler image by applying the 2-D fast Fourier transform (FFT) to $r(m,n), m = 1,2,\cdots,M, n = 1,2,\cdots,N$:

$$\begin{aligned} {\rm RD}(k,p) =& \alpha_T D_M(\hat{f}_{B,\rm T} - k/M) D_N(\hat{f}_{B,\rm T} - k/M) \\ &+ \alpha_I D_M(\hat{f}_{B,\rm I} - k/M) D_N(\hat{f}_{B,\rm I} - k/M) \\ &+ W(k,p), \end{aligned} \tag{5}$$

where $D_n(x) = \sin(n\pi x)/\sin(\pi x)$ denotes the Dirichlet function, and $W(k,p)$ represents the 2-D FFT of noise.

We can observe from (5) that the interference signal will form a sharp peak in the range-Doppler image, even when the two systems are not exactly identical. In particular, though the interference might be attributable to the transmission from the antenna sidelobe of one radar and reception by the antenna sidelobe of the other, the potential interference level can still be significantly higher than the target reflections, due to the one-way propagation characteristic of the interference and the direct (without reflection) blast from one's transmission to the other's reception.

3. CODING SCHEMES FOR MUTUAL INTERFERENCE MITIGATION

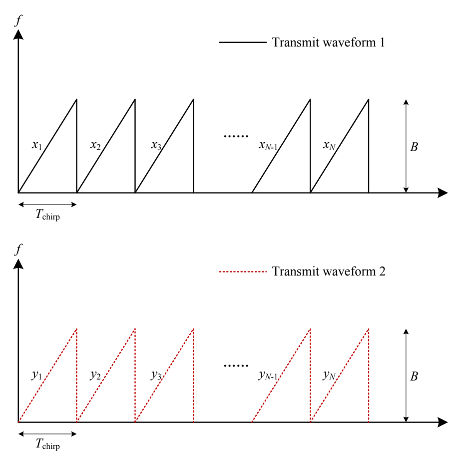

Fig. 2. The proposed coding scheme.

In this section, we propose novel coding schemes to reduce the mutual interferences between two identical (or similar) radar systems. As shown in Fig. 2, the proposed scheme utilize (periodic) slow-time coding for the N consecutive pulses. We denote the associated coding sequences by $\mathbf{x} = [x_1, x_2, \cdots, x_N]^T$ and $\mathbf{y} = [y_1, y_2, \cdots, y_N]^T$, respectively. That is to say, in the n^{th} pulse of a CPI, the first radar system transmits $x_n u(t)$ and the second radar system transmits $y_n u(t)$. Moreover, to keep constant transmit power over the N pulses, we constrain the code sequences to be unimodular, i.e., $|x_n| = |y_n| = 1, n = 1,2,\cdots,N$.

With the coding scheme, the digital samples of the n^{th} dechirped signal is denoted by

$$\begin{aligned} r^c(m,n) = &\alpha_{\mathrm{T}} \exp(j2\pi(\hat{f}_{B,\mathrm{T}}m + \hat{f}_{d,\mathrm{T}}n)) \\ &+ \alpha_{\mathrm{I}} x_n^* y_{(n+l)\mathrm{mod}N} \exp(j2\pi(\hat{f}_{B,\mathrm{I}}m + \hat{f}_{d,\mathrm{I}}n)) \\ &+ w(m,n). \end{aligned} \tag{6}$$

Therefore, the range-Doppler image formed with the proposed scheme is given by

$$\begin{aligned} \mathrm{RD}^c(k,p) = &\alpha_{\mathrm{T}} D_M(\hat{f}_{B,\mathrm{T}} - k/M) D_N(\hat{f}_{B,\mathrm{T}} - k/M) \\ &+ \alpha_{\mathrm{I}} D_M(\hat{f}_{B,\mathrm{I}} - k/M) r_{xy}^l(\hat{f}_{d,\mathrm{I}} - p/N) \\ &+ W(k,p), \end{aligned} \tag{7}$$

where $r_{xy}^l(f) = \sum_{n=1}^N x_n^* y_{(n+l)\mathrm{mod}N} \exp(j2\pi nf)$, which is the periodic cross-ambiguity function of $\mathbf{x}$ and $\mathbf{y}$.

To suppress the interference power in the range-Doppler image, we aim at designing $\mathbf{x}$ and $\mathbf{y}$ to minimize $r_{xy}^l(f)$ within a range of interest for f. To this end, we propose two methods to design $\mathbf{x}$ and $\mathbf{y}$ in the following subsections.

3.1. Doppler-shifting Scheme

First, we propose a simple heuristic coding scheme to mitigate the interference. The central idea of this scheme is to split the target reflections and the interference signal in the Doppler region. Specifically, the coding vectors $\mathbf{x}$ and $\mathbf{y}$ are given by

$$\mathbf{x} = [1, 1, \cdots, 1]^T, \tag{8}$$

$$\mathbf{y} = \begin{cases} [1, -1, \cdots, -1, 1]^T, \text{if } N \text{ is odd}, \\ [1, -1, \cdots, 1, -1]^T, \text{if } N \text{ is even}. \end{cases} \tag{9}$$

Note that for $l = -N+1, \cdots, N-1$,

$$(\mathbf{y})_{(n+l)\mathrm{mod}N} = \begin{cases} \mathbf{y}, & \text{if } l \text{ is even}, \\ -\mathbf{y}, & \text{if } l \text{ is odd}. \end{cases}$$

It is easy to verify that, with (8) and (9), $r_{xy}(f) = D_N(f + 1/2)$. Thus, with the proposed coding scheme, the Doppler frequency of the interference signal is shifted to a higher frequency area. As a result, it is possible to separate the target reflections and interference in the Doppler dimension if $|f| < 1/4$.

3.2. Optimized Coding Scheme

In this subsection, we seek to optimize $\mathbf{x}$ and $\mathbf{y}$ such that the corresponding $|r_{xy}(f)|$ has small values in a desired area. Given that the two radar systems usually have unsynchronized transmissions, the desired area should include all possible delays. Thus, we consider the following optimization problem:

$$\begin{aligned} \min_{\mathbf{x},\mathbf{y}} &\sum\nolimits_{l=-N+1}^{N-1} \sum\nolimits_{p=-P}^{P} |r_{lp}|^2 \\ \text{s.t. } &|x_n| = 1, |y_n| = 1, n = 1, 2, \cdots, N, \end{aligned} \tag{10}$$

where $r_{lp} = \sum_{n=1}^N x_n^* y_{(n+l)\,\mathrm{mod}\,N} e^{-j2\pi np/N_f}$ denotes the discrete periodic cross-ambiguity function of $\mathbf{x}$ and $\mathbf{y}$ (for a related problem of synthesizing aperiodic cross-ambiguity functions and periodic ambiguity functions, we refer to [11, 12].), $N < N_f, 0 < P < N_f$, N_f denotes the overall number of discrete (Doppler) frequencies, and the value of P is closely related to the maximum Doppler frequency of interest.

Note that $r_{lp} = \mathbf{x}^H \mathrm{Diag}(\mathbf{f}_p) \mathbf{C}_l \mathbf{y}$, where $\mathbf{C}_l = \mathbf{C}_{-l}^T = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{N-l} \\ \mathbf{I}_l & \mathbf{0} \end{bmatrix}$ is a circular shift matrix, $\mathrm{Diag}(\mathbf{f}_p)$ is a diagonal matrix with the diagonal elements $\mathbf{f}_p$, and the n^{th} element of $\mathbf{f}_p$ is given by $e^{-j2\pi np/N_f}$. Thus, we can reformulate the optimization problem in (10) as follows:

$$\begin{aligned} \min_{\mathbf{x},\mathbf{y}} &\sum_{l=-N+1}^{N-1} \sum_{p=-P}^{P} |\mathbf{x}^H \mathrm{Diag}(\mathbf{f}_p) \mathbf{C}_l \mathbf{y}|^2 \\ \text{s.t. } &|x_n| = 1, |y_n| = 1, n = 1, 2, \cdots, N, \end{aligned} \tag{11}$$

It is easy to verify that the optimization problem in (11) is non-convex and difficult to solve. Herein, we propose to tackle the problem in a cyclic manner. Specifically, in the i^{th} iteration of the cyclic optimization, we first optimize $\mathbf{x}$ for fixed $\mathbf{y}^{(i-1)}$ and then optimize $\mathbf{y}$ for fixed $\mathbf{x}^{(i)}$. Next we present the solution to the two subproblems involved in each iteration. For notational simplicity, we omit the superscripts of $\mathbf{y}^{(i-1)}$ and $\mathbf{x}^{(i)}$ if this leads to no confusion.

• **Optimization of $\mathbf{x}$ for fixed $\mathbf{y}$:**

The associated optimization problem can be recast as

$$\min_{\mathbf{x}} \mathbf{x}^H \mathbf{B}_y \mathbf{x}, \text{ s.t. } |x_n| = 1, n = 1, 2, \cdots, N, \tag{12}$$

where

$$\mathbf{B}_y = \sum_{l=-N+1}^{N-1} \sum_{p=-P}^{P} \mathrm{Diag}(\mathbf{f}_p) \mathbf{C}_l \mathbf{y}\mathbf{y}^H \mathbf{C}_l^H \mathrm{Diag}^H(\mathbf{f}_p). \tag{13}$$

The problem in (12) is called a (non-convex) unimodular quadratic programming (UQP) problem. In particular, it can be tackled with the power-method-like iterations proposed in [13, 14] (see also in [15, 16] for the application of power-method-like iterations in radar code optimization.). Specifically, let μ_y be a positive constant to ensure $\mathbf{D}_y = \mu_y \mathbf{I}_N - \mathbf{B}_y \succ \mathbf{0}$ (i.e., positive definite). It is easy to verify that the problem in (12) can be equivalently written as:

$$\max_{\mathbf{x}} \mathbf{x}^H \mathbf{D}_y \mathbf{x}, \text{ s.t. } |x_n| = 1, n = 1, 2, \cdots, N. \tag{14}$$

In the k^{th} (inner) iteration, we update $\mathbf{x}$ as follows:

$$\mathbf{x}^{(i,k)} = \exp(j \arg(\mathbf{D}_y \mathbf{x}^{(i,k-1)})). \tag{15}$$

• **Optimization of $\mathbf{y}$ for fixed $\mathbf{x}$:**

The optimization of $\mathbf{y}$ for fixed $\mathbf{x}$ is formulated as follows:

$$\min_{\mathbf{y}} \mathbf{y}^H \mathbf{B}_x \mathbf{y}, \text{ s.t. } |y_n| = 1, n = 1, 2, \cdots, N, \tag{16}$$

where

$$\mathbf{B}_x = \sum_{l=-N+1}^{N-1} \sum_{p=-P}^{P} \mathbf{C}_l^H \mathrm{Diag}^H(\mathbf{f}_p)\mathbf{x}\mathbf{x}^H \mathrm{Diag}(\mathbf{f}_p)\mathbf{C}_l. \quad (17)$$

Similarly, we can tackle the optimization problem in (16) iteratively. Specifically, the solution in the k^{th} (inner) iteration is given by

$$\mathbf{y}^{(i,k)} = \exp(j \arg(\mathbf{D}_x \mathbf{y}^{(i,k-1)})), \quad (18)$$

where $\mathbf{D}_x = \mu_x \mathbf{I} - \mathbf{B}_x$ and μ_x is a positive constant to ensure $\mathbf{D}_x \succ \mathbf{0}$.

4. NUMERICAL EXAMPLES

Consider two identical FMCW radar systems with the same carrier frequency of $f_c = 24$ GHz. The bandwidth of the chirp signal is $B = 150$ MHz. The sweep time is $T_{\text{chirp}} = 50$ μs. The number of periods within a CPI is $N = 256$.

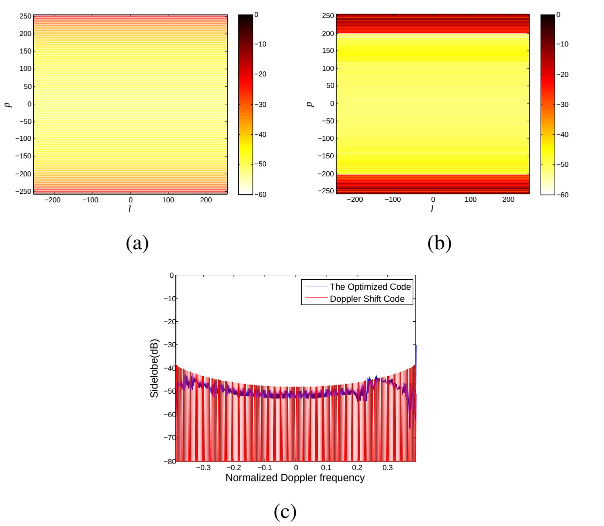

Fig. 3. Discrete periodic cross-ambiguity functions. (a) Doppler-shifting. (b) The optimized coding scheme. (c) The zero-delay cut. $N = 256$. $P = 200$. $N_f = 512$.

Fig. 3(a) and Fig. 3(b) show the discrete periodic cross-ambiguity functions associated with the Doppler-shifting, and the optimized codes, respectively, where $P = 200$ and $N_f = 512$ (which implies that the maximum Doppler frequency of interest should be lower than 3906.25 Hz, corresponding to the maximum relative radial velocity to be 87.9 km/h), and we initialize our algorithm with randomly generated codes (for $\mathbf{x}$ and $\mathbf{y}$, respectively,) in the optimized coding scheme. Fig. 3(c) compares their discrete periodic cross-ambiguity functions at the zero-delay cut. We can observe that, both coding schemes achieve very low sidelobes in the desired area. Thus, they can be used to effectively suppress the interference. Moreover, the peak side lobe (PSL) corresponding to the optimized codes is approximately 3.55 dB lower than that of the Doppler-shifting, within the desired range of Doppler frequency of interest (Interestingly, if we fix $\mathbf{y} = \mathbf{1}_N$ and we can only optimize $\mathbf{x}$, we obtain similar results, which corresponds to a more practical coding method, since no coordination between the two radar systems is needed.)

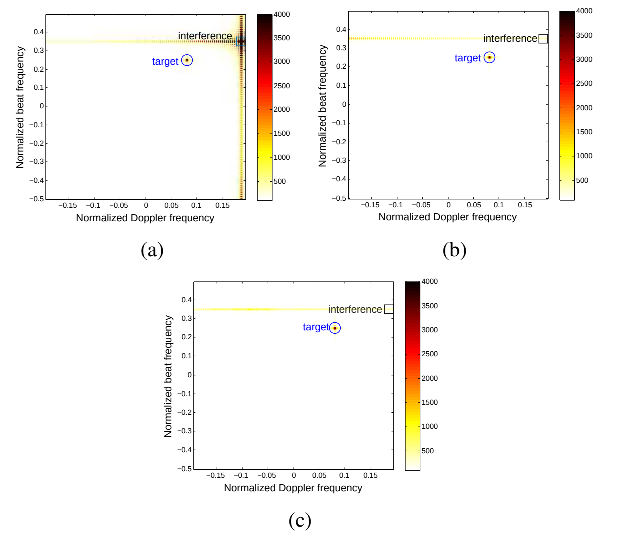

Fig. 4. The range-Doppler image with and without slow-time coding. (a) Without coding. (b) Doppler-shifting code. (c) The optimized coding scheme.

Next we apply the coding schemes to mitigate the mutual interference for two identical automotive radar systems operating in a typical scenario: the range of target and interference are at 50 m and 140 m, respectively. The speeds associated with them are 36.432 km/h and 84.42 km/h. The signal-to-noise ratios (SNR) are 30 dB and 60 dB, respectively. The sampling frequency is $f_s = 4$ MHz. $M = 100$ samples are collected for each period. Fig. 4(a) shows the range-Doppler image of the scenario without slow-time coding. We can observe that, the power of the interference is much stronger than that of the target such that false alarm happens. When our slow-time coding schemes are applied, the interference power level is significantly reduced and the target can be easily detected without suffering from false alarm problems.

5. CONCLUSIONS

We have proposed two slow-time coding schemes to mitigate the mutual interference in two identical or similar FMCW radar systems. We have presented efficient methods to construct the codes. We have shown that both coding schemes can be used to reduce the interference power level significantly. We have demonstrated that the second coding scheme, obtained via a cyclic optimization method, can achieve a lower PSL than the first more intuitive coding scheme.

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