FUSION OF MULTIPLE MULTIBAND IMAGES WITH COMPLEMENTARY SPATIAL AND SPECTRAL RESOLUTIONS

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ABSTRACT

We consider fusing an arbitrary number of multiband, i.e., panchromatic, multispectral, or hyperspectral, images of the same scene. Using the well-known forward observation and linear mixture models, a vector total-variation penalty, and appropriate constraints, we cast this problem as a convex optimization problem. The total-variation penalty helps cope with the potential ill-posedness of the underlying inverse problem by exploiting the prior knowledge that natural images are mostly piecewise smooth in the spatial domain and comprise relatively few archetypical signatures, ie endmembers, in the spectral domain. We solve the formulated convex but non-smooth optimization problem using the alternating direction method of multipliers. Our experiments with multiband images constructed from real hyperspectral datasets demonstrate the performance advantages of the proposed algorithm over the existing algorithms, which need to be used in tandem to fuse more than two multiband images.

Index Terms—ADMM, forward observation model, linear mixture model, multiband image fusion, total variation.

1. INTRODUCTION

Finely-resolved hyperspectral images are in great demand by various computer vision and remote sensing applications [1]-[4]. However, limitations in light intensity as well as efficiency of the current sensors impose a trade-off between the spatial resolution, spectral sensitivity, and the signal-to-noise ratio (SNR) of existing spectral imagers [5]. As a results, typical spectral imaging systems can capture multiband images of high spatial resolution at a small number of spectral bands or multiband images of high spectral resolution with a reduced spatial resolution. One way to overcome this fundamental limitation is to capture multiple multiband images of the same scene that have complementary spatial and spectral resolutions and fuse them together synergistically.

Initial multiband image fusion algorithms were developed to fuse a panchromatic image (with a single band) and a multispectral image (with typically 3 to 10 bands), which are geometrically co-registered. The associated inverse problem was named pansharpening [6]-[8]. Some of the algorithms developed for pansharpening have been successfully extended to perform hyperspectral pansharpening [9], i.e., to fuse a panchromatic image and a hyperspectral image (with typically more than 10 bands). Recently, significant research effort has been expended to fuse two co-registered multispectral and hyperspectral images [35], which is essentially different from (hyperspectral) pansharpening.

The spatial or spectral degradation of an observed multiband image with respect to the target image can be expressed as a linear transformation, which induces a forward observation model. In addition, the spectrum of each pixel in a typical hyperspectral image is usually a linear mixture of a relatively few spectral signatures, called endmembers. This is because hyperspectral image data often reside in a subspace with a dimension that is much smaller than the number of the spectral bands, i.e., the dimension of the data space [10]-[12].

Many recent works on multiband image fusion that deal with fusing a multispectral image with a hyperspectral image of the same scene employ the abovementioned forward observation and linear mixture models, e.g., [13]-[18]. They generally cast the task of multiband image fusion as the reconstruction of a target image from two spatially and spectrally downgraded versions of it following the forward observation model. If the endmembers are known or extracted from the observed images, the problem boils down to estimating the endmember abundances of the target image. The estimate of the target image can then be obtained by mixing the extracted endmembers and the estimated abundances.

When the number of spectral bands in the multispectral image is smaller than the number of endmembers, the linear inverse problem associated with the fusion of the multispectral image with a hyperspectral image is ill-posed and requires some form of regularization. Natural images are known to mostly consist of smooth segments with few abrupt changes corresponding to the edges and object boundaries [19]-[21]. Therefore, penalizing the total-variation of the target image is an effective way of regularizing the multiband image fusion problems [14], [22], [23].

To the best of our knowledge, all existing multiband image fusion algorithms are designed to fuse a *pair* of multiband images with complementary characteristics. Therefore, fusing more than two multiband images using the existing algorithms can only be realized by performing a hierarchical procedure that combines multiple fusion processes possibly implemented via different algorithms. For instance, in order to fuse a panchromatic, a multispectral, and a hyperspectral image of a scene, one can first fuse the panchromatic and multispectral images, then fuse the resultant multispectral image with the hyperspectral image. Such an approach is potentially slow and inaccurate since it may require several runs of different algorithms and may suffer from propagation and accumulation of errors.

In this paper, we propose an algorithm that can simultaneously fuse an arbitrary number of multiband images. We utilize the forward observation and linear mixture models to effectively model the data and reduce the dimensionality of the problem. Assuming matrix normal distribution for the perturbations in the observed images, the maximum-likelihood estimation of the endmember abundances of the target image amounts to solving a weighted least-squares problem. We regularize this problem by adding a vector total-variation penalty term and constraining the abundances to be nonnegative and add up to one for each pixel. The total-variation penalty serves two major purposes. First, it helps tackle the likely illposedness of the maximum-likelihood estimation problem. Second, it allows for the incorporation of the prior knowledge that the natural images are mostly piecewise smooth and have few sharp variations. The nonnegativity and sum-to-one constraints on the endmember abundances ensure that the abundances have physically plausible values. They also implicitly promote sparsity in the estimated abundances. We solve the formulated convex but non-smooth optimization problem using the alternating direction method of multipliers (ADMM) [24], [25]. Simulation results show that the proposed algorithm outperforms several combinations of the existing algorithms, which need be cascaded to carry out fusion of multiple (more than two) multiband images.

2. DATA MODEL

2.1. Forward observation model

Let us denote the target multiband image with *L* spectral bands and *N* pixels by $\mathbf{X} \in \mathbb{R}^{L \times N}$. We wish to recover \mathbf{X} from *K* observed multiband images $\mathbf{Y}_k \in \mathbb{R}^{L_k \times N_k}$, k = 1, ..., K, that are spatially or spectrally downgraded and corrupted versions of \mathbf{X} . We assume that \mathbf{Y}_k , k = 1, ..., K, are geometrically corregistered and are related to \mathbf{X} via the following forward observation model

 $\mathbf{Y}_k = \mathbf{R}_k \mathbf{X} \mathbf{B}_k \mathbf{S}_k + \mathbf{P}_k$

 $L_k \leq L$ and $N_k = N/D_k^2$ with D_k being the spatial downsampling ratio of the *k*th image;

 $\mathbf{R}_k \in \mathbb{R}^{L_k \times N}$ is the spectral response of the sensor producing \mathbf{Y}_k ;

 $\mathbf{B}_k \in \mathbb{R}^{N \times N}$ is a band-independent spatial blurring matrix that represents a two-dimensional convolution with a blur kernel corresponding to the point-spread function of the sensor producing \mathbf{Y}_k ;

 $\mathbf{S}_k \in \mathbb{R}^{N \times N_k}$ is a sparse matrix with N_k ones and zeros elsewhere that implements a two-dimensional uniform downsampling of ratio D_k on both spatial dimensions (e.g., horizontal and vertical) and satisfies $\mathbf{S}_k^{\mathsf{T}} \mathbf{S}_k = \mathbf{I}_N$;

 $\mathbf{P}_k \in \mathbb{R}^{L_k \times N_k}$ is an additive perturbation representing the noise or error associated with the observation of \mathbf{Y}_k .

2.2. Linear mixture model

Under some mild assumptions, multiband images of natural scenes can be suitably described by a linear mixture model [1]. Specifically, the spectrum of each pixel can often be expressed as a linear mixture of a few archetypal spectral signatures known as endmembers. The number of endmembers, denoted by M, is usually much smaller than the spectral dimension of a hyperspectral image, i.e, $M \ll L$. Therefore, if we arrange Mendmembers corresponding to **X** as columns of the matrix $\mathbf{E} \in \mathbb{R}^{L \times M}$, we can factorize **X** as

$$\mathbf{X} = \mathbf{E}\mathbf{A} + \mathbf{P} \tag{2}$$

where $\mathbf{A} \in \mathbb{R}^{M \times N}$ is the matrix of endmember abundances and $\mathbf{P} \in \mathbb{R}^{L \times N}$ is a perturbation matrix that accounts for any possible inaccuracy or mismatch in the linear mixture mode. Every column of **A** contains the fractional abundances of the endmembers at a pixel. The fractional abundances of each pixel are nonnegative and often assumed to add up to one.

2.3. Fusion model

Substituting (2) into (1) gives

$$\mathbf{Y}_k = \mathbf{R}_k \mathbf{E} \mathbf{A} \mathbf{B}_k \mathbf{S}_k + \mathbf{\check{P}}_k \tag{3}$$

where the aggregate perturbation of the kth image is

$$\mathbf{P}_k = \mathbf{P}_k + \mathbf{R}_k \mathbf{P} \mathbf{B}_k \mathbf{S}_k.$$

Instead of estimating the target multiband image X directly, we consider estimating its abundance matrix A from the observations Y_k , k = 1, ...K, given the endmember matrix E. We can then obtain an estimate of the target image by multiplying the estimated abundance matrix by the endmember matrix. This way, we substantially reduce the dimensionality of the fusion problem and consequently its computational complexity. In addition, by estimating A then X, we attain an unmixed fused image and obviate any additional unmixing, if demanded in a later processing stage. However, this approach requires the prior knowledge of the endmember matrix E. The columns of this matrix can be selected from a library of known spectral signatures, such as the U.S. Geological Survey digital spectral library [26], or extracted from the observed multiband images.

3. ALGORITHM

3.1. Optimization problem

Given the observed images \mathbf{Y}_k , k = 1, ..., K, modeled by (3) and under the realistic assumption that the aggregate perturbations \mathbf{P}_k , k = 1, ..., K, have independent matrix normal distributions with identity column-covariance matrices and row-covariance matrices of Λ_k , it can be shown that the maximum-likelihood estimate of \mathbf{A} is the solution of the following weighted least-squares problem

$$\min_{\mathbf{A}} \frac{1}{2} \sum_{k=1}^{K} \left\| \mathbf{\Lambda}_{k}^{-1/2} (\mathbf{Y}_{k} - \mathbf{R}_{k} \mathbf{E} \mathbf{A} \mathbf{B}_{k} \mathbf{S}_{k}) \right\|_{\mathrm{F}}^{2}.$$
(4)

Since (4) is prone to ill-posedness, we regularize it in two ways. First, we add an isotropic vector total-variation penalty term to the objective function. We denote this penalty by $\|\nabla \mathbf{A}\|_{2,1}$ where $\|\cdot\|_{2,1}$ is the $\ell_{2,1}$ -norm operator that returns the sum of ℓ_2 -norms of all the columns of its matrix argument and $\nabla \mathbf{A} = [(\mathbf{A}\mathbf{D}_h)^{\mathsf{T}}, (\mathbf{A}\mathbf{D}_v)^{\mathsf{T}}]^{\mathsf{T}} \in \mathbb{R}^{2M \times N}$ with \mathbf{D}_h and \mathbf{D}_v being discrete differential matrix operators that, respectively, yield the horizontal and vertical first-order backward differences (gradients) of the row-vectorized image that they multiply from the right. Second, we constrain all columns of \mathbf{A} to be

(1)

nonnegative and sum to one. We symbolize this constraint, which forces the columns of **A** to reside on the unit (M - 1)simplex, by adding the below indicator function to the objective function:

$$u(\mathbf{A}) = \begin{cases} 0 & \mathbf{A} \in \{\mathbf{A} | \mathbf{A} \ge 0, \mathbf{1}_M^\top \mathbf{A} = \mathbf{1}_N^\top \} \\ +\infty & \mathbf{A} \notin \{\mathbf{A} | \mathbf{A} \ge 0, \mathbf{1}_M^\top \mathbf{A} = \mathbf{1}_N^\top \} \end{cases}$$

where $\mathbf{A} \ge 0$ means all the entries of \mathbf{A} are greater than or equal to zero. Consequently, we modify (4) as

$$\min_{\mathbf{A}} \frac{1}{2} \sum_{k=1}^{K} \left\| \mathbf{\Lambda}_{k}^{-1/2} (\mathbf{Y}_{k} - \mathbf{R}_{k} \mathbf{E} \mathbf{A} \mathbf{B}_{k} \mathbf{S}_{k}) \right\|_{\mathrm{F}}^{2} + \alpha \| \nabla \mathbf{A} \|_{2,1} + \iota(\mathbf{A})$$
(5)

where $\alpha \geq 0$ is the regularization parameter.

3.2. ADMM iterations

To develop an efficient algorithm for fusing multiple multiband images, we use the alternating direction method of multipliers (ADMM) to solve (5). We split the problem to smaller and more manageable pieces by defining the auxiliary variables, $\mathbf{U}_k \in \mathbb{R}^{M \times N}$, k = 1, ..., K, $\mathbf{V} \in \mathbb{R}^{2M \times N}$, and $\mathbf{W} \in \mathbb{R}^{2M \times N}$ $\mathbb{R}^{M \times N}$, and changing (5) into

$$\min_{\mathbf{A},\{\mathbf{U}_k\},\mathbf{V},\mathbf{W}} \frac{1}{2} \sum_{k=1}^{K} \left\| \mathbf{\Lambda}_k^{-1/2} (\mathbf{Y}_k - \mathbf{R}_k \mathbf{E} \mathbf{U}_k \mathbf{S}_k) \right\|_{\mathrm{F}}^2 + \alpha \|\mathbf{V}\|_{2,1} + \iota(\mathbf{W})$$

subject to: $\mathbf{U}_k = \mathbf{A} \mathbf{B}_k, \mathbf{V} = \nabla \mathbf{A}, \mathbf{W} = \mathbf{A}.$ (6)

bject to:
$$\mathbf{U}_k = \mathbf{A}\mathbf{B}_k, \mathbf{V} = \nabla \mathbf{A}, \mathbf{W} = \mathbf{A}.$$
 (6)

Then, we write the augmented Lagrangian function associated with (6) as

$$\mathcal{L}(\mathbf{A}, \mathbf{U}_{1}, ..., \mathbf{U}_{K}, \mathbf{V}, \mathbf{W}, \mathbf{F}_{1}, ..., \mathbf{F}_{K}, \mathbf{G}, \mathbf{H}) = \frac{1}{2} \sum_{k=1}^{K} \| \mathbf{\Lambda}_{k}^{-1/2} (\mathbf{Y}_{k} - \mathbf{R}_{k} \mathbf{E} \mathbf{U}_{k} \mathbf{S}_{k}) \|_{\mathrm{F}}^{2} + \alpha \| \mathbf{V} \|_{2,1} + \iota(\mathbf{W}) + \frac{\mu}{2} \sum_{k=1}^{K} \| \mathbf{A} \mathbf{B}_{k} - \mathbf{U}_{k} - \mathbf{F}_{k} \|_{\mathrm{F}}^{2} + \frac{\mu}{2} \| \nabla \mathbf{A} - \mathbf{V} - \mathbf{G} \|_{\mathrm{F}}^{2} + \frac{\mu}{2} \| \mathbf{A} - \mathbf{W} - \mathbf{H} \|_{\mathrm{F}}^{2}$$
(7)

where $\mathbf{F}_k \in \mathbb{R}^{M \times N}$, k = 1, ..., K, $\mathbf{G} \in \mathbb{R}^{2M \times N}$, and $\mathbf{H} \in \mathbb{R}^{M \times N}$ are the scaled Lagrange multipliers and $\mu \ge 0$ is the penalty parameter.

Using the ADMM, we minimize the augmented Lagrangian function (7) in an iterative manner. At each iteration, we alternate the minimization with respect to the main latent variable A and the auxiliary variables; then, we update the scaled Lagrange multipliers. Hence, we compute the iterates as

$$\mathbf{A}^{(n)} = \underset{\mathbf{A}}{\operatorname{argmin}} \sum_{k=1}^{K} \left\| \mathbf{A} \mathbf{B}_{k} - \mathbf{U}_{k}^{(n-1)} - \mathbf{F}_{k}^{(n-1)} \right\|_{\mathrm{F}}^{2} + \left\| \nabla \mathbf{A} - \mathbf{V}^{(n-1)} - \mathbf{G}^{(n-1)} \right\|_{\mathrm{F}}^{2} + \left\| \mathbf{A} - \mathbf{W}^{(n-1)} - \mathbf{H}^{(n-1)} \right\|_{\mathrm{F}}^{2} (8) \mathbf{U}^{(n)} = \underset{\mathbf{A}}{\operatorname{argmin}} \frac{1}{2} \left\| \mathbf{A}^{-1/2} (\mathbf{V} - \mathbf{P} - \mathbf{F} \mathbf{H} - \mathbf{C}) \right\|_{\mathrm{F}}^{2}$$

$$\mathbf{U}_{k}^{(n)} = \underset{\mathbf{U}_{k}}{\operatorname{argmin}} \frac{1}{2} \left\| \mathbf{A}_{k}^{-1/2} (\mathbf{Y}_{k} - \mathbf{R}_{k} \mathbf{E} \mathbf{U}_{k} \mathbf{S}_{k}) \right\|_{\mathrm{F}}^{2} + \frac{\mu}{2} \left\| \mathbf{A}^{(n)} \mathbf{B}_{k} - \mathbf{U}_{k} - \mathbf{F}_{k}^{(n-1)} \right\|_{\mathrm{F}}^{2}, k = 1, ..., K \quad (9)$$

$$\mathbf{V}^{(n)} = \underset{\mathbf{V}}{\operatorname{argmin}} \alpha \|\mathbf{V}\|_{2,1} + \frac{\mu}{2} \|\nabla \mathbf{A}^{(n)} - \mathbf{V} - \mathbf{G}^{(n-1)}\|_{\mathrm{F}}^{2}$$
(10)

$$\mathbf{W}^{(n)} = \underset{\mathbf{W}}{\operatorname{argmin}} \ \iota(\mathbf{W}) + \frac{\mu}{2} \| \mathbf{A}^{(n)} - \mathbf{W} - \mathbf{H}^{(n-1)} \|_{\mathrm{F}}^{2}$$
(11)

$$\mathbf{F}_{k}^{(n)} = \mathbf{F}_{k}^{(n-1)} - (\mathbf{A}^{(n)}\mathbf{B}_{k} - \mathbf{U}_{k}^{(n)}), \quad k = 1, ..., K \mathbf{G}^{(n)} = \mathbf{G}^{(n-1)} - (\nabla \mathbf{A}^{(n)} - \mathbf{V}^{(n)}) \mathbf{H}^{(n)} = \mathbf{H}^{(n-1)} - (\mathbf{A}^{(n)} - \mathbf{W}^{(n)})$$

where superscript (n) denotes the value of an iterate at iteration number $n \ge 0$. We repeat the iterations until convergence or a maximum allowed number of iterations is reached.

3.3. Solution of subproblems

The solution of (8) is

$$\mathbf{A}^{(n)} = \left[\sum_{k=1}^{K} \left(\mathbf{U}_{k}^{(n-1)} + \mathbf{F}_{k}^{(n-1)}\right) \mathbf{B}_{k}^{\mathsf{T}} + \mathbf{Q}_{1}^{(n-1)} \mathbf{D}_{h}^{\mathsf{T}} \right. \\ \left. + \mathbf{Q}_{2}^{(n-1)} \mathbf{D}_{v}^{\mathsf{T}} + \mathbf{W}^{(n-1)} + \mathbf{H}^{(n-1)} \right] \\ \times \left(\sum_{k=1}^{K} \mathbf{B}_{k} \mathbf{B}_{k}^{\mathsf{T}} + \mathbf{D}_{h} \mathbf{D}_{h}^{\mathsf{T}} + \mathbf{D}_{v} \mathbf{D}_{v}^{\mathsf{T}} + \mathbf{I}_{N}\right)^{-1}$$

where we define $\mathbf{Q}_{1}^{(n)}$ and $\mathbf{Q}_2^{\mathsf{c}}$ as

$$\begin{bmatrix} \mathbf{Q}_1^{(n-1)} \\ \mathbf{Q}_2^{(n-1)} \end{bmatrix} = \mathbf{V}^{(n-1)} + \mathbf{G}^{(n-1)}.$$

To make the computation of $A^{(n)}$ more efficient, we assume that the two-dimensional convolutions represented by \mathbf{B}_k , k =1, ..., K, are cyclic and the differential matrix operators \mathbf{D}_h and \mathbf{D}_{v} apply with periodic boundaries. Consequently, in view of the circular convolution theorem, multiplications by $\mathbf{B}_k^{\mathsf{T}}, \mathbf{D}_h^{\mathsf{T}}$, $\mathbf{D}_{v}^{\mathsf{T}}$, and $(\sum_{k=1}^{K} \mathbf{B}_{k} \mathbf{B}_{k}^{\mathsf{T}} + \mathbf{D}_{h} \mathbf{D}_{h}^{\mathsf{T}} + \mathbf{D}_{v} \mathbf{D}_{v}^{\mathsf{T}} + \mathbf{I}_{N})^{-1}$ can be performed using the fast Fourier transform and its inverse.

Equating the gradient of the cost function in (9) with respect to \mathbf{U}_k to zero results in

$$\mathbf{E}^{\mathsf{T}} \mathbf{R}_{k}^{\mathsf{T}} \mathbf{A}_{k}^{-1} \mathbf{R}_{k} \mathbf{E} \mathbf{U}_{k}^{(n)} \mathbf{S}_{k} \mathbf{S}_{k}^{\mathsf{T}} + \mu \mathbf{U}_{k}^{(n)}$$

= $\mathbf{E}^{\mathsf{T}} \mathbf{R}_{k}^{\mathsf{T}} \mathbf{A}_{k}^{-1} \mathbf{Y}_{k} \mathbf{S}_{k}^{\mathsf{T}} + \mu (\mathbf{A}^{(n)} \mathbf{B}_{k} - \mathbf{F}_{k}^{(n-1)}).$ (12)

Multiplying both sides of (12) from the right by the masking matrix $\mathbf{M}_k = \mathbf{S}_k \mathbf{S}_k^{\mathsf{T}}$ and its complement $\mathbf{I}_N - \mathbf{M}_k$ yields

$$\mathbf{U}_{k}^{(n)}\mathbf{M}_{k} = (\mathbf{E}^{\mathsf{T}}\mathbf{R}_{k}^{\mathsf{T}}\mathbf{\Lambda}_{k}^{-1}\mathbf{R}_{k}\mathbf{E} + \mu\mathbf{I}_{N})^{-1} \times [\mathbf{E}^{\mathsf{T}}\mathbf{R}_{k}^{\mathsf{T}}\mathbf{\Lambda}_{k}^{-1}\mathbf{Y}_{k}\mathbf{S}_{k}^{\mathsf{T}} + \mu(\mathbf{A}^{(n)}\mathbf{B}_{k} - \mathbf{F}_{k}^{(n-1)})\mathbf{M}_{k}] \quad (13)$$

and

$$\mathbf{U}_{k}^{(n)}(\mathbf{I}_{N}-\mathbf{M}_{k}) = \left(\mathbf{A}^{(n)}\mathbf{B}_{k}-\mathbf{F}_{k}^{(n-1)}\right)(\mathbf{I}_{N}-\mathbf{M}_{k}), \quad (14)$$

respectively. Note that we have $\mathbf{S}_k^{\dagger}\mathbf{S}_k = \mathbf{I}_N$ and \mathbf{M}_k is idempotent, i.e., $\mathbf{M}_k \mathbf{M}_k = \mathbf{M}_k$. Summing both sides of (13) and (14) gives the solution of (9) for k = 1, ..., K as

$$\begin{aligned} \mathbf{U}_{k}^{(n)} &= \mathbf{U}_{k}^{(n)} \mathbf{M}_{k} + \mathbf{U}_{k}^{(n)} (\mathbf{I}_{N} - \mathbf{M}_{k}) \\ &= (\mathbf{E}^{\mathsf{T}} \mathbf{R}_{k}^{\mathsf{T}} \mathbf{\Lambda}_{k}^{-1} \mathbf{R}_{k} \mathbf{E} + \mu \mathbf{I}_{N})^{-1} \\ &\times \left[\mathbf{E}^{\mathsf{T}} \mathbf{R}_{k}^{\mathsf{T}} \mathbf{\Lambda}_{k}^{-1} \mathbf{Y}_{k} \mathbf{S}_{k}^{\mathsf{T}} + \mu (\mathbf{A}^{(n)} \mathbf{B}_{k} - \mathbf{F}_{k}^{(n-1)}) \mathbf{M}_{k} \right] \\ &+ (\mathbf{A}^{(n)} \mathbf{B}_{k} - \mathbf{F}_{k}^{(n-1)}) (\mathbf{I}_{N} - \mathbf{M}_{k}). \end{aligned}$$

The terms $(\mathbf{E}^{\mathsf{T}}\mathbf{R}_{k}^{\mathsf{T}}\boldsymbol{\Lambda}_{k}^{-1}\mathbf{R}_{k}\mathbf{E} + \mu\mathbf{I}_{N})^{-1}$ and $\mathbf{E}^{\mathsf{T}}\mathbf{R}_{k}^{\mathsf{T}}\boldsymbol{\Lambda}_{k}^{-1}\mathbf{Y}_{k}\mathbf{S}_{k}^{\mathsf{T}}$ do not change during the iterations and can be precomputed.

The subproblem (10) can be decomposed pixelwise and its solution is linked to the so-called Moreau proximity operator of the $\ell_{2,1}$ -norm given by column-wise vector-soft-thresholding [27]. Therefore, defining

$$\mathbf{Z}^{(n)} = \nabla \mathbf{A}^{(n)} - \mathbf{G}^{(n-1)},$$

the *j*th column of $\mathbf{V}^{(n)}$, denoted by $\mathbf{v}_i^{(n)}$, is given in terms of the *j*th column of $\mathbf{Z}^{(n)}$, denoted by $\mathbf{z}_i^{(n)}$, as

$$\mathbf{v}_{j}^{(n)} = \left(\max\left\{ \left\| \mathbf{z}_{j}^{(n)} \right\|_{2} - \alpha/\mu, 0 \right\} / \left\| \mathbf{z}_{j}^{(n)} \right\|_{2} \right) \mathbf{z}_{j}^{(n)}.$$

The solution of (11) is the value of the proximity operator of the indicator function $\iota(\mathbf{W})$ at $\mathbf{A}^{(n)} - \mathbf{H}^{(n-1)}$, which is the projection of $\mathbf{A}^{(n)} - \mathbf{H}^{(n-1)}$ onto the unit (M-1)-simplex. We implement this projection employing the algorithm proposed in [28].

Table 1. The values of the performance metrics for assessing the fusion quality as well as the runtimes of the considered algorithms.

	ERGAS	SAM (°)	$Q2^n$	time (s)
proposed	1.378	1.454	0.969	47.01
BDSD & HySure	2.268	2.228	0.923	62.58
BDSD & R-FUSE-TV	2.276	2.238	0.923	62.10
MTF-GLP-HPM & HySure	2.034	2.256	0.938	62.78
MTF-GLP-HPM & R-FUSE-TV	2.044	2.265	0.938	62.20

Botswana

4. SIMULATIONS

To examine the performance of the proposed algorithm in comparison with the state-of-the-art, we carry out experiments with fusing three multiband images, viz. a panchromatic image, a multispectral image, and a hyperspectral image. We create the multiband images of the experiments using crops of two publicly available hyperspectral datasets, namely, Botswana [29] and Washington DC Mall [30]. After cropping, the former dataset has 400×240 pixels and 145 spectral bands and the latter datasets as the ground truth (reference image) for evaluating the fusion performance. Further simulation results using additional datasets can be found in [31].

We generate the hyperspectral image by applying a Gaussian blur filter with a kernel size of 5×5 and a variance of 1.28 to the reference image followed by downsampling with a ratio of 4 in both horizontal and vertical directions for all bands. For the multispectral image, we use a Gaussian blur filter with a kernel size of 3×3 and a variance of 0.64 and downsampling with a ratio of 2 in both horizontal and vertical directions for all bands of the reference image. Afterwards, we downgrade the resulting image spectrally by applying the spectral responses of the Landsat 8 multispectral sensor [32]. We create the panchromatic image from the reference image using the panchromatic band of the Landsat 8 sensor without applying any spatial blurring or downsampling. We add zero-mean Gaussian white noise to each band of the produced multiband images such that the band-specific signal-to-noise ratio (SNR) is 30 dB for the multispectral and hyperspectral images and 40 dB for the panchromatic image.

The current multiband image fusion algorithms available in the literature are designed to fuse two images at a time. In order to compare the performance of the proposed algorithm with that of the existing ones, we consider fusing the abovementioned three multiband images in two stages using existing algorithms for pansharpening and hyperspectral-multispectral fusion. At the first stage, we fuse the panchromatic image with the multispectral one using two algorithms called the banddependent spatial detail (BDSD) [33] and the modulationtransfer-function generalized Laplacian pyramid with high-pass modulation (MTF-GLP-HPM) [34]. In [6], where several pansharpening algorithms are studied, it is shown that the BDSD and MTF-GLP-HPM algorithms exhibit the best performance among all the considered ones. After performing the pansharpening, at the second stage, we fuse the pansharpened multispectral image with the hyperspectral image using the algorithms proposed in [14] and [17], [18], which are called HySure and R-FUSE-TV, respectively. These algorithms are based on total-variation regularization and are among the

Washington DC Mall						
	ERGAS	SAM (°)	$Q2^n$	time (s)		
proposed	2.276	2.533	0.975	59.52		
BDSD & HySure	4.039	4.767	0.923	79.68		
BDSD & R-FUSE-TV	4.141	4.787	0.921	78.41		
MTF-GLP-HPM & HySure	4.240	4.809	0.916	79.28		
MTF-GLP-HPM & R-FUSE-TV	4.354	4.827	0.913	78.13		

best performing and most efficient multispectral-hyperspectral fusion algorithms currently available [9], [35]. Using two different algorithms at each of the two stages results in four combined solutions.

We use three performance metrics for assessing the quality of a fused image with respect to its reference image. They are the relative dimensionless global error in synthesis (ERGAS) [36], spectral angle mapper (SAM) [37], and $Q2^n$ [38]. We extract the endmembers (columns of **E**) from the hyperspectral image using the VCA algorithm [39]. We utilize the SUnSAL algorithm [40] together with the extracted endmembers to unmix the hyperspectral image and obtain its abundance matrix. Then, we upscale the resulting abundance matrix by a factor of four and apply two-dimensional spline interpolation on each of its rows (abundance bands) to generate the initial estimate **A**⁽⁰⁾. We initialize the proposed algorithm as well as the HySure and R-FUSE-TV algorithms by **A**⁽⁰⁾.

In the experiments with both considered datasets, we tune the values of the parameters in the HySure and R-FUSE-TV algorithms to yield the best possible performance. Additionally, in order to use the BDSD and MTF-GLP-HPM algorithms to their best potential, we provide them with the true point-spread function, i.e., the blurring kernel used to generate the multispectral images. In the proposed algorithm, we use $\mu = 1.5 \times 10^3$ and $\alpha = 9$ with both considered datasets.

To assess the quality of the images fused using the proposed algorithm and the considered benchmarks, we present the values of the performance metrics resulting from experiments with both considered datasets in Table 1. The table also includes the time taken by each algorithm to produce the fused images. According to the results given in Table 1, the proposed algorithm significantly outperforms the considered benchmarks. It is also evident from the required processing times that the computational (time) complexity of the proposed algorithm is lower than those of its contenders.

5. CONCLUSION

We proposed a new multiband image fusion algorithm that can simultaneously fuse an arbitrary number of multiband images. We utilized the forward observation model together with the linear mixture model to cast the fusion problem as a reduced-dimension linear inverse problem. We used a vector total-variation penalty as well as nonnegativity and sum-to-one constraints on the endmember abundances to regularize the associated maximum-likelihood estimation problem. We developed the proposed algorithm by solving the consequent optimization problem using the alternating direction method of multipliers. Our experiments using two real hyperspectral datasets substantiated the advantages of the proposed algorithm.

6. REFERENCES

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