CHANNEL DEPENDENT CODEBOOK DESIGN IN SPATIAL MODULATION

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ABSTRACT

In this paper, we present a modulation design based on Spatial Modulation for the uplink in IoT applications. The proposed modulation design uses a Tabu search based deterministic heuristic to adapt the modulation link based on channel information fed back by the receiver. Our approach allows adaptivity to rate and energy constraints.

We numerically validate the proposed method on a scenario with full channel state information available at the transceiver, showing clear performance gains compared to simpler heuristics and channel independent codebook designs.

Index Terms—Energy efficiency, Spatial Modulation, Codebook design

1. INTRODUCTION

Internet of things (IoT) have been driving a lot of attention from industrial actors in the recent years. Since IoT applications are envisioned to have the highest wireless traffic in the future communication networks [1], it is necessary to design a modulation scheme fit for them. Power is one the main concerns for IoT applications since some devices would be battery powered. Hence we need to consider modulations that are energy efficient. In that aspect Spatial Modulation (SM) [2, 3] has been developed for multi-antenna systems by using the choice of a single transmitting antenna to encode information and hence lowering the power cost of the modulation. It is to be noted that antenna spacing is crucial in the sense that the channel for each transmit antenna should be different enough to be informative. Given the spatial constraints of many IoT applications, this is unpractical. However upcoming antenna technologies allows pattern reconfigurability in the directivity/polarization space with low switching costs [4]. Such devices make it possible to design SM schemes based on the pattern choice rather than the antenna index.

In that context, we study codebook design strategies suitable for quantized channel information available at the receiver. The quantization strategies associated to spatial modulation are however outside the scope of this paper. Since we hold the focus on IoT like scenarii, the system is considered overdetermined with more receiving antennas than available patterns at the transmitter. Since most of the low-cost RF switching systems are degrading through time, we modify the system model by limiting the switching rate including lifetime maximization concerns in our modulation design.

In this paper, we propose a heuristic for codebook building based on Tabu search [5]. Moreover, the proposed design heuristic integrates flexibility in terms of rate and energy constraints.

Notations: Vectors are denoted as bold face letters and matrices are bold face capital letters. Calligraphic capital letters denote discrete sets and |.| denotes the cardinal of this set. $(.)^{T}$ de-

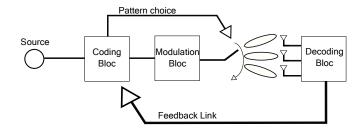


Fig. 1. Uplink chain using SM-PS

notes the transpose operator while $(.)^{H}$ denotes the Hermitian operator (transpose-conjugate). ||.|| denotes the 2-norm of the Hilbert space while $||.||_{F}$ is the Frobenius norm and eventually we write $[\![i;j]\!] = \{i, i+1, \ldots, j\}.$

2. SYSTEM MODEL

We recall the system model introduced for classical Spatial Modulation [2] with a slight difference in formulation, we refer to the spatial information as patterns where classical SM refers to antennas. We consider a transmitter with P patterns available and a constellation alphabet $\Omega \in \mathbb{C}$. The receiver has N_r antennas. Fig. 1 models the communication system for Spatial Modulation using Pattern Switching (SM-PS).

For each pattern *i* there is a different channel response $\mathbf{h}_i \in \mathbb{C}^{N_r}$ caused by the different propagation conditions (LOS, number and distances of reflective/diffractive clusters, etc.). Let $\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_P]$, then the received signal is [2]

$$\mathbf{y} = \sqrt{\rho} \alpha \mathbf{H} \, \mathbf{x}_i + \boldsymbol{w} \in \mathbb{C}^{N_r} \tag{1}$$

where $\alpha \in \Omega$ is the coded symbol, $\mathbf{x}_i = [\delta(i, 1), \dots, \delta(i, P)]^T$ is the pattern selection vector, ρ is the SNR and \boldsymbol{w} the AWGN noise vector. Due to low cost hardware interfaces in IoT devices, it is necessary to constrain the switching to occur only between blocks of τ symbol periods, either to avoid transient behaviour while transmitting either to prolong device lifetime by limiting the number of switching per transmissions.

Taking into account this additional parameter, the system model is updated as

$$\mathbf{Y} = \sqrt{\rho} \mathbf{H} \, \mathbf{x}_i \boldsymbol{\alpha}^{\mathrm{T}} + \mathbf{W} \in \mathcal{M}_{N_r, \tau}(\mathbb{C}), \tag{2}$$

where $\boldsymbol{\alpha}$ is a vector of Ω^{τ} and \mathbf{W} the noise matrix.

For this new system model, we derive two decoding criteria for the "spatial code" $\mathcal{A} = \llbracket 1; P \rrbracket \times \Omega^{\tau}$.

2.1. Maximum Likelihood Decoding Criterion

By stacking the columns of \mathbf{Y} , (2) can be alternatively written

$$\mathbf{y} = \sqrt{\rho} \boldsymbol{\alpha} \otimes \mathbf{h}_i + \mathbf{w} \in \mathbb{C}^{N_r \tau}, \tag{3}$$

where \otimes denotes the Kronecker product $\boldsymbol{\alpha} \otimes \mathbf{h}_i = [s_1 \mathbf{h}_i^{\mathsf{T}}; \cdots; s_\tau \mathbf{h}_i^{\mathsf{T}}]^{\mathsf{T}}$. For Gaussian noise $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C})$, the likelihood function of $\mathbf{y}|(i, \boldsymbol{\alpha})$ is obtained from the multivariate Gaussian distribution

$$p(\mathbf{y}|(i,\boldsymbol{\alpha})) = \frac{1}{\pi^{N_r} \det(\mathbf{C})} e^{-(\mathbf{y} - \sqrt{\rho}\boldsymbol{\alpha} \otimes \mathbf{h}_i)^{\mathsf{H}} \mathbf{C}^{-1} (\mathbf{y} - \sqrt{\rho}\boldsymbol{\alpha} \otimes \mathbf{h}_i)}.$$

Assuming uncorrelated noise, $\mathbf{C} = \mathbf{I}$ and

$$p(\mathbf{y}|(i, \boldsymbol{\alpha})) = rac{1}{\pi^{N_r}} \exp\left(-\|\mathbf{y} - \sqrt{\rho}\boldsymbol{\alpha} \otimes \mathbf{h}_i)\|^2\right).$$

The maximum likelihood estimator (MLE) of the spatial symbol (i, α) is then obtained as

$$\widehat{(i, \boldsymbol{\alpha})} = \arg \max_{(j, \boldsymbol{\beta}) \in \mathcal{A}} \| \mathbf{y} - \sqrt{\rho} \boldsymbol{\beta} \otimes \mathbf{h}_j \|$$
(4)

The pairwise error probability (PEP) associated with this decoder is [3]

$$P((j, \boldsymbol{\beta}) \to (i, \boldsymbol{\alpha})) = P(\|\mathbf{y} - \sqrt{\rho}\boldsymbol{\alpha} \otimes \mathbf{h}_i\| \le \|\mathbf{y} - \sqrt{\rho}\boldsymbol{\beta} \otimes \mathbf{h}_j\|)$$

$$P\left((j,\boldsymbol{\beta})\to(i,\boldsymbol{\alpha})\right)=Q\left(\sqrt{\frac{\rho}{2}}\|\boldsymbol{\beta}\otimes\mathbf{h}_{j}-\boldsymbol{\alpha}\otimes\mathbf{h}_{i}\|\right)$$
(5)

This formulation induces a distance in \mathcal{A} as

$$d_{\mathcal{A}}((i, \boldsymbol{\alpha}), (j, \boldsymbol{\beta})) = \|\boldsymbol{\beta} \otimes \mathbf{h}_j - \boldsymbol{\alpha} \otimes \mathbf{h}_i\|$$

The PEP gives through the union bound an upper bound on the probability of error of the decoder as [3]

$$P_e \leq \frac{1}{P|\Omega|^{\tau}} \sum_{(i,\boldsymbol{\alpha})\in\mathcal{A}} \sum_{(j,\boldsymbol{\beta})\neq(i,\boldsymbol{\alpha})} P\left((j,\boldsymbol{\beta})\to(i,\boldsymbol{\alpha})\right).$$
(6)

However the complexity of an exhaustive search induced by this criterion can be high since it requires $P \times |\Omega|^{\tau}$ computations of an L_2 norm in $\mathbb{C}^{N_r \tau}$.

2.2. Sequential Hybrid Decoding Criterion

In order to reduce the complexity of the decoder we propose to first decode the pattern used \hat{i} and then decode the modulated symbol assuming \hat{i} . The pattern decoding writes

$$\widehat{i} = \arg \max_{j \in [1;P]} \frac{\|\mathbf{h}_{j}^{\mathsf{H}}\mathbf{Y}\|_{F}}{\|\mathbf{h}_{j}\|}$$
(7)

This decoder picks the pattern that has the 'maximum collinearity' with the columns of **Y**, since angular information in \mathbb{C}^{N_r} is $|\cos(\measuredangle(\mathbf{h}_i, \mathbf{h}_j))| = \frac{|\mathbf{h}_i^H \mathbf{h}_j|}{\|\|\mathbf{h}_i\|\|\|\mathbf{h}_j\|}$, maximized when the two vectors are collinear. This is equivalent to a maximum likelihood decoder where the constellation symbols are relaxed to $\alpha \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. The modulated symbol is then decoded as the maximum likelihood symbol conditioned on $i = \hat{i}$

$$\widehat{\boldsymbol{\alpha}} = \arg\max_{\boldsymbol{\beta} \in \Omega^{\tau}} \| \mathbf{y} - \sqrt{\rho} \boldsymbol{\beta} \otimes \mathbf{h}_{\widehat{\imath}} \|.$$
(8)

Since there does not seem to be any close form expressions of the pairwise error probability for the pattern estimator we can only write the union bound as

$$P_{e} \leq \frac{1}{P|\Omega|^{\tau}} \sum_{(i,\alpha)\in\mathcal{A}} \left[\left(\sum_{j\neq i} P\left(j \to i\right) \right) + \sum_{\beta\neq\alpha} P\left(\beta \to \alpha | i\right) \right],$$
(9)
where $P\left(\beta \to \alpha | i\right) = Q\left(\sqrt{\frac{P}{2}} d_{\mathcal{A}}((i,\alpha), (i,\beta))\right).$

3. CHANNEL DEPENDENT CODEBOOK DESIGN

In regards of the diversity of the upcoming IoT applications, we want to add flexibility in the design of the modulation, i.e. the codebook. Since the probability of error derived for both decoders is linked with the distance d_A , the fitness of a subset S of A in terms of decodability (probability of error) is quantified by the one to one distances of its elements. However in such communication system design it is highly improbable that there is full channel state information available at the transceiver. Hence in this paper we develop a codebook design paradigm that fits for both full and quantized versions of the channel information available at the transceiver.

Channel adaptive codebook design have been widely explored in the context of MIMO systems [6, 7], but with tools unfit to low complexity devices (in terms of computation abilities and memory). In this section we present channel estimate dependent codebook for our specific spatial modulation model.

3.1. Best Codebook Under Objective

We aim at minimizing the probability of error of a codebook S under different type of constraints. The first type of constraint is related to the rate and is a function of the cardinality of the codebook.

$$\operatorname{Rate}(\mathcal{S}) = \frac{\log_2(|\mathcal{S}|)}{\tau \times \text{ sample period bits/s.}}$$

The second constraint is the energy constraint and is represented by a cost function J over the symbol set. This cost function can represent a variety of costs associated to energy dissipation in the device. It can vary from the symbol energy $||\alpha||^2$ to more complex functions taking into account processing energy costs as what has been developed for base stations in Green Radio projects such as [8]. Hence the energy constraint can be seen as a restriction of the vector alphabet to the symbols which have a cost lower than a limitation parameter γ , such set is denoted $C_{\gamma}(\Omega^{\tau}) = \{\alpha \in \Omega^{\tau} | J(\alpha) \leq \gamma\}$. The constrained spatial modulation space is $\mathcal{A}_{\gamma} = [\![1; P]\!] \times C_{\gamma}(\Omega^{\tau})$ From the upper bounds on error probabilities derived in this document, we can extract that a way to minimize the probability of error is to maximize the worst of the one to one distances of its elements. In particular we want to find

$$\widehat{\mathcal{S}} = \arg \max_{\mathcal{S} \subset \mathcal{A}_{\gamma} ||\mathcal{S}|=N} \min_{a \neq b \in \mathcal{S}} d_{\mathcal{A}}(a, b).$$
(10)

3.2. Heuristic for Pseudo-Optimal Codebook Building

Even if in this context we feed back quantized channel information for which ideally the best codebook corresponding to each quantized channel can be computed in an off-line fashion, memory restrictions in low costs device makes the storage of such off-line computed codewords impossible. In that regard we want to use combinatorial optimization methods to adapt the modulation to the channel

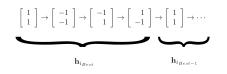


Fig. 2. Toy example of S_0 building with $\Omega = \{1, -1\}, \tau = 2$

realisation. For such a method to be valid, the proposed optimization technique shall be deterministic since both encoder and decoder need to be able to compute the same codebook. We propose a Tabu type heuristic [5] for which the stopping criterion is a fixed number of iterations, we hence have no guarantees on the optimality of the built codebook other than that it will have greater or equal fitness (here min dist) than the algorithm starting point, i.e. the *initial solution*.

3.2.1. Initial solution

We define an order relation \succeq on the pattern space $[\![1; P]\!]$ such that $i \succeq j \Leftrightarrow |\![\mathbf{h}_i|\!] \ge |\![\mathbf{h}_j|\!]$. Based on that ordering, we successively fill in the patterns $i_{Best} \succeq i_{Best-1} \succeq \cdots \succeq i_{Worst}$ with symbols of Ω^{τ} until the cardinality of N is reached. Ω^{τ} is allocated in such a way that the one to one distance between symbols is maximized when the set is not fully explored. Fig. 2 illustrates how the initial solution is sequentially built.

This starting point is extracted from the intuition that it is likely that the optimal solution set contains more elements using the patterns that have high SNR since for the same pair of constellation symbols $\alpha, \beta \in \Omega^{\tau}$ used on the pattern they will have higher distances.

The codebook obtained in such way, the *initial solution*, is denoted S_0 . When $\tau = 1$ and $N \leq |\Omega|$, it is verified that such solution, corresponding to the best antenna selection in classical Spatial Modulation is optimal even though it may not be unique [9].

3.2.2. Codebook building heuristic

We build a deterministic heuristic based on Tabu search [5]. Starting from the *initial solution* S_0 , we perform local search in order to improve the set fitness. In order to introduce the notion of locality and neighbourhood we define mutations of the solution set S as

$$\mu(\mathcal{S}, b \to a) = \{b\} \cup \mathcal{S} \setminus \{a\}$$

The neighbourhood of $\mathcal S$ induced by μ is then

$$\mathcal{N}_{\mu}(\mathcal{S}) = \left\{ \mathcal{S}' \left| \exists a \in \mathcal{S}, b \in \overline{\mathcal{S}}, \mathcal{S}' = \mu(\mathcal{S}, b \to a) \right.
ight\}.$$

To reduce the notion of locality and improve the speed of the algorithm, introduce the notation

$$(a,b) = ((i,\boldsymbol{\alpha}), (j,\boldsymbol{\beta})) = \arg\min_{a\neq b\in\mathcal{S}} \, d_{\mathcal{A}}(a,b),$$

and define two refinements of the mutation:

- Inner mutation: The symbol α of a is replaced by another symbol δ ∈ Ω^τ leading the mutation to be μ_I(S, δ) = {(i, δ)} ∪ S \{(i, α)}.
- Outer mutation: The pattern *i* of *a* is replaced by another pattern *l* ∈ [[1; *P*]] leading the mutation to be μ_O(S, *l*) = {(*l*, α)} ∪ S\{(*i*, α)}.

These two mutations reduce the topology of the considered problem to a neighbourhood induced by the elements that are closest to each other. Splitting the mutation in two types is not so to say necessary but it is used to reduce the cardinality and drives the tabu search heuristic by splitting the exploration into two phases.

Algorithm 1: Codebook Building Heuristic
Data: $C_{\gamma}(\Omega^{\tau})$ the set of energy constrained symbols,
$\{\widetilde{\mathbf{h}}_i\}$ the channel estimates,
mindist the fitness function,
N the cardinality of the target set,
tabusize the maximum size of the Tabu list,
N_{it} the maximal number of iterations.
Result: S_{Best} the best codebook obtained after N_{it}
iterations.
Initialize S_0 , Tabu set $\mathcal{T} \leftarrow \emptyset$, $it = 0$;
$\mathcal{S}_{Best} \leftarrow \mathcal{S}_0; \mathcal{S} \leftarrow \mathcal{S}_0;$
while $it \leq N_{it}$ do
Tabu exploration
$CandidateList \leftarrow \emptyset;$
for $\mathcal{S}' \in \mathcal{N}_{\mu}(\mathcal{S})$ do
if $\mathcal{S}' \notin \mathcal{T}$ then
$ $ CandidateList $\leftarrow S';$
end
end
$\mathcal{S}_{Candidate} \leftarrow Best(CandidateList);$
$\mathcal{S} \leftarrow \mathcal{S}_{Candidate};$
if $mindist(S_{Candidate}) > mindist(S_{Best})$ then
$\mathcal{S}_{Best} \leftarrow \mathcal{S}_{Candidate};$
end
$\mathcal{T} \leftarrow \mathcal{T} \cup \{\mathcal{S}_{Candidate}\};$
if $ \mathcal{T} > tabusize$ then
Remove the $ \mathcal{T} $ – <i>tabusize</i> oldest members of \mathcal{T} ;
end
Update μ ; //Alternate between μ_I and μ_O .
$it \leftarrow it + 1;$
end

A brief description of the search heuristic can be found in Algorithm 1. The determinism is guaranteed by the choice of S_0 as well as the neighbourhood uniquely determined by the choice of the mutation μ . It is to be noted that since the set fitness depends mostly on two of its elements, it is highly likely that the mutated element *d* is not achieving the minimal distance in the set. This indicates that the set fitness is not a good metric for choosing Best(CandidateList) in Algorithm 1. For a candidate solution $S' = \mu(S, c \to a)$ we use the metric

$$m_{\mathcal{S}}(c \to a) = \min_{b \in \mathcal{S} \setminus \{a\}} d_{\mathcal{A}}(c, b)$$

to qualify the goodness of the mutation and allow a fairer comparison between candidate solutions. Such metric requires N-1 distances computation while the fitness requires to compute all the one to one distances in the set. The fitness is only computed once $S_{Candidate}$ have been chosen.

4. NUMERICAL RESULTS

4.1. Empirical Guidelines for Heuristic Implementation

Algorithm 1 describes the skeleton of the optimization heuristic however following its structure may not be sufficient to provide fast enough convergence towards 'good enough' solutions either because the neighbourhood has high cardinality Ω^{τ} and then evaluating \mathcal{N}_{μ_I} may take prohibitively high computation time, mostly because the minimal distance is not uniquely achieved.

Sticking to the low number of iterations required by the low complexity application, it appears that the heuristic would be more of a Tabu assisted hill climbing (approximately greedy search) and hence we aim at using empirical observations on the topology to tune and speed up the heuristic. The guidelines are listed as follow:

- Due to the small number of iterations the choice of the initial solution S_0 is crucial, however we see in the following subsection that the proposed initial solution seems to perform well.
- The mutation µ_I can be modified to have a smaller neighbourhood, by picking the τ successive best mutations of α ∈ Ω^τ allocating the best option dimension by dimension.
- Due to the structure of S_0 it is more advantageous to use the outer mutation μ_O to spread across different patterns the members of the pairs achieving minimizing distance. We empirically noted that satisfying results are obtained, using a rate of four outer mutations between each inner mutation.
- The Tabu list mostly helps not exploring the previous state and the variable *tabusize* barely affects the heuristic since the number of solutions explored is limited. Hence a short memory size is sufficient for our application and is advantageous in order to fit the memory restrictions of low cost hardware.

Those guidelines were used in the implemented optimization heuristic used to produce our numerical validation of the codebook design.

4.2. Numerical Validation

In order to asses the quality of the proposed method, we conduct simulation in simplified scenarii where full channel state information is available at the transceiver. Throughout the simulations, $N_r = 8$ receiving antennas were assumed, as well as P = 4 patterns available at the transmitter. The performance is evaluated on a benchmark of 200 spatially uncorrelated Rayleigh fading channels with $\sigma_h^2 = 1$ and 10^4 realisations of an additive Gaussian noise $\boldsymbol{w} \sim \mathcal{CN}(0, \sigma^2 \mathbf{I})$ per channel realization. The heuristic proposed in Sec. 3.2 is performed with a depth of $N_{it} = 2000$ iterations. We picked a symbol set $\Omega = \{1, -1, i, -i\}$ and $\tau = 3$. The total constellation set \mathcal{A} has cardinality 296 hence the spatial constellation size N is fixed at $3/2 \cdot |\Omega^{\tau}| = 96$ elements what is already sufficient to have combinatorial explosion in terms of number of possible sets ($\sim 10^{72}$).

In Fig. 3 we evaluate the performance of the proposed codebook building heuristic in terms of spatial symbol error rate (SER) for increasing mean SNR ρ . For coherence in the simulations the mean amplitude of the channel taps of the best channel $\mathbf{h}_{i_{Best}}$ is taken to be 1 such that the mean SNR ρ is directly linked to the input power and that the Rayleigh fading statistics are preserved.

We can see that the optimized codebooks have faster decay rate in terms of SER for the ML decoder as expected from the optimization problem formulation (10) and the upper bound (6). The sequential hybrid decoder derived in Sec. 2.2 performs similarly as the ML decoder but has lower diversity order in higher SNR regimes. It is also to be noted that the initial choice allocation S_0 has better performance than the uniform allocation of symbol (24 symbols per

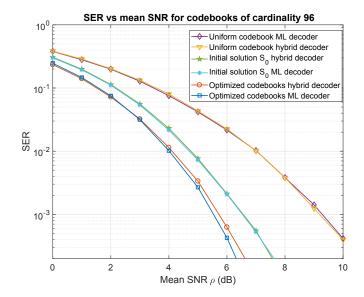


Fig. 3. Performance Evaluation for 200 different sets of P = 4Rayleigh fading uncorrelated channels and $\tau = 3$.

pattern) empirically confirming the intuition developed in Sec. 3.2.1 that the optimal solution will be more likely to have most of its symbols on the best channel in terms of SNR.

A surprising empirical observation is that the sequential decoder performs slightly better than the joint ML decoder at very low SNR. Such behaviour might be due to the fact that collinearity information on $\tau = 3$ observations of the channel realisation is less sensitive to the noise than the Euclidean distance d_A in the codebook S, and that the second decoding stage in the hybrid decoder allows proper phase retrieval, since a symbol in Ω^{τ} is similar to a rotation of $k\pi/2$, $k \in [0; 3]$ of the channel. However such behaviour cannot be investigated without a close form of the upper bound (9).

5. CONCLUSIONS

We have derived a practical channel feedback adaptive codebook building heuristic for Spatial Modulation [2] based on Tabu search [5]. Such modulation is meant to fit the energy constraints and rate adaptation of the uplink of low complexity IoT devices. We also take into account potential device lifetime maximization concerns by updating the system model and limiting the switching rate.

We provide guidelines for an efficient implementation of the Tabu search so that it provides appropriately good solutions in a short number of iterations. Finally, we validate the optimized codebook goodness under two decoding criteria.

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