PRIMA: PROBABILISTIC RANKING WITH INTER-ITEM COMPETITION AND MULTI-ATTRIBUTE UTILITY FUNCTION

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ABSTRACT

This paper proposes PRIMA: Probabilistic Ranking with Inter-item competition and Multi-Attribute utility function, which ranks items based on their probabilities of being a user's best choice. This framework is particularly important in E-commerce applications for making recommendations, predicting sales, and developing pricing strategies. To achieve mathematical tractability, it uses the weight-based multi-attribute utility function to address the inter-attribute tradeoff, where the weight reflects a user's personal preference for each attribute. The proposed work updates the weight from a user's past transactions using the concept of marginal rate of substitution from microeconomics, addresses the interitem competition, and computes the items' probabilities of being a user's best choice. Real user test results show that the proposed framework achieves comparable ranking accuracy to the state-of-the-art work with significant improvements in model simplicity and mathematical tractability.

1. INTRODUCTION

Recent advances in networking provide people with more choices when shopping online, while they are also overwhelmed by the avalanche of products/services available. For the continuous prosperity of E-commerce, it is crucial to learn users' personal preferences and to understand their decisions.

To offer better and personalized services, in addition to recommending new products/services such as the recommender systems in [1–4], another equally important problem is: given a list of sellers offering the same or similar products/services, rank them according to their match to the user's personal interest [5]. Here, the critical issues to address include: 1) the *inter-attribute tradeoff*, e.g., the tradeoff between price and reputation; and 2) the *inter-item competition*, that is, the existence of similar and competitive items may reduce one item's probability of being selected [6]. This is the focus of this work.

Multi-criteria decision making theory was proposed to use an explicit utility function to describe relationship among multiple conflicting attributes [7]. The most commonly used is the weight-based multi-attribute function, such as the weighted arithmetic mean [8] and the weighted geometric mean in the Cobb-Douglas model in microeconomics [9]. Note that although these methods are mathematically tractable, the utility function itself can only address the interattribute tradeoff but not the inter-item competition.

To consider the competition among similar items, items should be ranked based on their probabilities of being selected but not their utilities, such as the work in [6] and [10]. However, the work in [6] reduced information in the pricereputation 2D plane into a 1D visual angle, and this information loss might result in incomplete user preference modeling and cause performance loss. The work in [10] used the concepts of indifference curve and marginal rate of substitution in microeconomics [9] to model users' preference and achieved higher ranking accuracy, while the model was very complicated and computationally expensive.

In this work, we propose PRIMA, which combines the utility-based method [8] and the indifference curve-based method [10]. The novelty of PRIMA lies in its ability to greatly simplify the analysis of users' preferences on the premise of comparable ranking accuracy to the state-of-theart work. We use the weight-based function to compute the items' utilities, where the weight reflects the user's personal preference on the inter-attribute tradeoff. We extend the estimation method in [10] to estimate the weight in our utility function. Based on the utility function and the estimated weight, we analyze the competition among items, and propose a simpler and more elegant model to compute the probability that an item is the user's best choice. In addition to personalized ranking, this proposed model can also provide important guidelines on seller pricing strategies, market demand analysis, and how rating manipulation impacts profits and sales.

2. THE PROPOSED FRAMEWORK

2.1. Problem Formulation

In this work, we consider a user query of a certain product in an online shopping platform, and there are a list of matching items for sale. Assume that all items have two conflicting attributes, price and seller reputation, and our work can be extended to the scenario with three or more attributes. For an item, let P and R be its price and seller reputation, respectively. Let $P \in [P_{MIN}, P_{MAX}]$ where P_{MIN} and P_{MAX} are the lower and upper bounds of the price, respectively, and $R \in [R_{MIN}, R_{MAX}]$, where R_{MIN} and R_{MAX} are the lower and upper bounds of the reputation, respectively. Following the work in [10], we use the linear normalization function $p = (P_{MAX} - P)/(P_{MAX} - P_{MIN})$ and $r = (R - R_{MIN})/(R_{MAX} - R_{MIN})$ to normalize P and R into the range [0, 1], respectively. Note that after normalization, for both attributes p and r, a larger value indicates a higher preference of the user.

Let U(p, r) be the user's utility function, where a higher utility value indicates a higher preference of the user. We assume that U(p, r) is continuously differentiable and its second order derivative exists. Following the study in microeconomics [9], we assume that all users are rational and have consistent behaviors, and consider utility functions that satisfy the following assumptions:

1. Monotonicity: when one attribute (either price or reputation) is fixed, U is an increasing function of the other attribute, that is, $\partial U/\partial p > 0$ and $\partial U/\partial r > 0$.

2. Diminishing value: users receive diminishing additional level of satisfaction with the increase of a certain attribute's value. That is, with one attribute value fixed, when the other attribute value increases, the additional level of satisfaction that the user obtains diminishes. Mathematically, we have $\partial^2 U/\partial p^2 < 0$ and $\partial^2 U/\partial r^2 < 0$.

In this work, we consider skyline items only, whose attribute values are not all worse than another item [11]. That is: given two items $s_i = (p_i, r_i)$ and $s_j = (p_j, r_j)$, we have either $\{p_i > p_j, r_i < r_j\}$ or $\{p_i < p_j, r_i > r_j\}$. Otherwise, if $\{p_i < p_j, r_i < r_j\}$, or equivalently $\{P_i > P_j, R_i < R_j\}$, item s_i has a higher price but a lower reputation than s_j , which will never be chosen by a rational user.

Consider a market with N skyline items $S = \{s_i = (p_i, r_i)\}$, and without loss of generality, let $p_1 > p_2 > \cdots > p_N$ and $r_1 < r_2 < \cdots < r_N$.

2.2. Important Concepts, Ideas and Performance Metrics

In the price-reputation plane, an *indifference curve* (IC) connects points having the same utility value. We assume that the indifference curves are continuously differentiable. *Marginal rate of substitution* (MRS) describes a user's tradeoff between two attributes, which is the rate at which a user is willing to give up some amount of one good (normalized reputation in our work) to obtain the increase of another good (normalized price) while maintaining the same level of utility [9]. Then MRS at a given point is the slope of the indifference curve at that point [12].

In our work, we propose a novel probabilistic ranking

model, which combines the utility-based work and the indifference curve-based method. We use a multi-attribute utility function to compute the items' utilities, where a personalized weight indicates the user's tradeoff between price and reputation. In particular,

- Given a few past transaction records, our proposed framework first extracts the user's preference and estimates the range of the personalized weight based on the estimated MRS range.
- Then given a new market with N skyline items, we use the estimated weight range to estimate each item's probability of being the user's best choice, and rank them in the descending order of their probabilities.

Same as in [10], the *performance metrics* we use are

- ranking quality (rq): for an item set S with N skyline items, let s_b be the user's true preferred choice and v_b be its ranking position. Then the ranking quality is defined as $rq = (N - v_b)/(N - 1)$. If our proposed method ranks the user's preferred item s_b as the top one with $v_b = 1$, then rq = 1. If the user's preferred choice is ranked the last with $v_b = N$, then rq = 0.
- success rate (*sr*): it is the frequency that our proposed work ranks the user's favorite choice in the first place.

3. PROBABILISTIC RANKING WITH MULTI-ATTRIBUTE UTILITY FUNCTION

3.1. The Utility Function

Based on the discussion in Section 2.1, we consider simple separable utility functions, that is, those functions that can be written as $U(p,r) = u_1(p) + u_2(r)$ [13]. More complicated utility functions will be investigated in our future work. There are many functions that satisfy the above requirements, and in this work, we use

$$U = \alpha \ln p + \beta \ln r \tag{1}$$

to illustrate how to estimate and analyze user preference. Note that (1) is equivalent to the Cobb-Douglas model $U = p^{\alpha}r^{\beta}$ [9], which is often used for theoretical analysis in microeconomics but rarely used in real applications.

In (1), α and β are positive weights describing a user's preference. For instance, $\alpha > \beta$ means that the user put more emphasis on price and less on reputation. Without loss of generality, for each user, we let $\alpha + \beta = 1$, and rewrite (1) as

$$U = \alpha \ln p + (1 - \alpha) \ln r, \tag{2}$$

where $\alpha \in [0, 1]$ is a critical factor in our model that describes the user's personal preference.



Fig. 1. MRS range estimation with (a) one transaction record, and (b) multiple transactions.

3.2. Parameter Estimation

From (2), α is a critical factor in our model that describes the user's personal preference, and it is of crucial importance to accurately estimate its value. Following the work in [10], we estimate the range of α based on past transactions.

We first briefly introduce the work in [10], which estimates the MRS.

- Given one past transaction where the user chose item s_b as his/her best choice from an item set S, S is divided into two subsets: S₊ = {s_{b+1}, ..., s_N} including points above the best choice s_b; and S₋ = {s₁, ..., s_{b-1}} including points below s_b, as shown in Fig. 1(a). Then the initial upper bounds {k_{si}} for items in S₊ and the lower bounds {k_{si}} for items in S₋ are estimated.
- Given multiple past transactions, for an item s_i , all the other items in the past transactions are divided into four subsets as shown in Fig. 1(b). Items in S_i^{II} are used to refine the lower bound of MRS at s_i with $\underline{k}_{s_i} = \max_{s_j \in S_i^{II} \cup s_i} \{\underline{k}_{s_j}\}$ and items in S_i^{IV} are used to refine the uper bound of MRS with $\overline{k}_{s_i} = \min_{s_j \in S_i^{IV} \cup s_i} \{\overline{k}_{s_j}\}$. Then, the MRS range $[\underline{k}_{s_i}, \overline{k}_{s_i}]$ at s_i is obtained.

In our work, based on the estimated MRS, we investigate how to estimate the parameter α . Given an item s_i , or equivalently, a point (p_i, r_i) in the 2D price-reputation (p-r) plane, let k_i be the true MRS at that point. When the user's utility function is $U(p, r) = \alpha \ln p + (1 - \alpha) \ln r$, we have

$$k_{i} = \frac{\mathrm{dr}}{\mathrm{dp}} \Big|_{s_{i}} = -\frac{\frac{\partial U}{\partial p}}{\frac{\partial U}{\partial r}} \Big|_{p_{i},r_{i}} = -\frac{\alpha}{1-\alpha} \cdot \frac{r_{i}}{p_{i}}, \quad (3)$$

that is,
$$\alpha = \frac{kp}{kp-r} \Big|_{k_i, p_i, r_i} = \frac{k_i p_i}{k_i p_i - r_i}.$$
 (4)

Eq. (3) and (4) show that at a given point in the p - r plane, there is a one-to-one mapping between the MRS and

the parameter α . In addition, if we have *accurate* information of the MRS at one point in the 2D plane, it is sufficient to determine α .

Unfortunately, we do not have accurate MRS at any point, but only the estimated lower and upper bounds of MRS. Thus, given a point (p_i, r_i) , we can only estimate the range $I_i = [\underline{\alpha}_i, \overline{\alpha}_i]$ of α , where $\underline{\alpha}_i$ and $\overline{\alpha}_i$ are the lower and upper bounds of α , respectively. From Eq. (4), given (p_i, r_i) , α is a decreasing function of k_i , and therefore, we have

$$\overline{\alpha}_{i} = \frac{\underline{k}_{i}p_{i}}{\underline{k}_{i}p_{i} - r_{i}} \quad \text{and} \quad \underline{\alpha}_{i} = \frac{\overline{k}_{i}p_{i}}{\overline{k}_{i}p_{i} - r_{i}}.$$
(5)

Note that a user's preference may be inconsistent in multiple transactions. Therefore, we introduce the following method to merge results estimated from different points with consistency check.

First, for each point (p_i, r_i) , its estimated bounds should always satisfy $\underline{\alpha}_i < \overline{\alpha}_i$. Otherwise, this point is discarded.

Given multiple points, let $I = [\underline{\alpha}, \overline{\alpha}] = \bigcap_{i=1}^{N} I_i$, and we can refine our estimation by

$$\overline{\alpha} = \min_{i=1}^{N} \overline{\alpha}_i \quad \text{and} \quad \underline{\alpha} = \max_{i=1}^{N} \underline{\alpha}_i. \tag{6}$$

Here, it is obvious that the estimated ranges from multiple points $\{I_i\}$ have to overlap with each other for the above $I = [\underline{\alpha}, \overline{\alpha}]$ to be nonempty. Therefore, when merging the results from multiple points, we need to check the existence of outliers that do not overlap with others and discard them.

3.3. Probabilistic Ranking

In Section 3.2, we estimate the range of α , which describes the user's inter-attribute preference. In this section, we consider the following problem. Given a new market with Nskyline items as $S = \{s_i = (p_i, r_i)\}$, what is the probability that item s_i is the user's best choice?

To answer this question, we must analyze the competition among items. Let us first consider two items $s_i = (p_i, r_i)$ and $s_j = (p_j, r_j)$. If s_i is preferred to s_j , that is, $U(p_i, r_i) > U(p_j, r_j)$, we have

$$\alpha \ln p_i + (1 - \alpha) \ln r_i > \alpha \ln p_j + (1 - \alpha) \ln r_j, \quad (7)$$

that is,
$$\left(\ln\frac{p_i}{p_i} - \ln\frac{r_i}{r_i}\right) \cdot \alpha > -\ln\frac{r_i}{r_i}.$$
 (8)

Define

$$A_i(j) = \frac{-\ln(r_i/r_j)}{\ln(p_i/p_j) - \ln(r_i/r_j)}.$$
(9)

Note that s_i and s_j are skyline items, that is, we have either $\{p_i > p_j, r_i < r_j\}$ or $\{p_i < p_j, r_i > r_j\}$. When $\{p_i > p_j, r_i < r_j\}$, we have $p_i/p_j > 1$ and $r_i/r_j < 1$. Thus, we have $A_i(j) > 0$. Similarly, when $\{p_i < p_j, r_i > r_j\}$, we

	Coffee Maker	Itouch	Canon	Average
PRIMA	74.01%	76.43%	77.80%	76.08%
IC	78.57%	73.00%	77.75%	76.44%
MAPS	71.12%	76.12%	74.18%	73.80%

Table 1. Real user test results of ranking quality.

also have $A_i(j) > 0$. That is for any *i* and *j*, we always have $A_i(j) > 0$.

Therefore, from Eq. (9), when $\{p_i > p_j, r_i < r_j\}$, s_i is preferred to s_j when $\alpha > A_i(j)$ and when $\{p_i < p_j, r_i > r_j\}$, s_i is preferred to s_j when $\alpha < A_i(j)$.

When considering the competition among more than two items, if item s_i is the best choice for the user, that is, $U(p_i, r_i) > U(p_j, r_j)$ for all $j \neq i$, then α needs to satisfy $\underline{\alpha}_{p_i} \leq \alpha \leq \overline{\alpha}_{p_i}$, where

$$\underline{\alpha}_{p_i} = \max_{p_j < p_i} A_i(j) \text{ and } \overline{\alpha}_{p_i} = \min_{p_j > p_i} A_i(j).$$
(10)

From the analysis in Section 3.2, $[\underline{\alpha}, \overline{\alpha}]$ is the estimated range of the user's personal weight α . Without any other prior knowledge of α , we assume that α is uniformly distributed in the range $[\underline{\alpha}, \overline{\alpha}]$. From Eq. (10), item s_i has the largest utility and is the user's best choice if and only if α is in the range $[\underline{\alpha}_{p_i}, \overline{\alpha}_{p_i}]$. The larger the overlapping range between $[\underline{\alpha}, \overline{\alpha}]$ and $[\underline{\alpha}_{p_i}, \overline{\alpha}_{p_i}]$, the greater the probability for item s_i to be the best choice. Therefore, the probability that s_i is the user's best choice is

$$\mathcal{P}_{i} = \frac{\min\left\{\overline{\alpha}, \overline{\alpha}_{p_{i}}\right\} - \max\left\{\underline{\alpha}, \underline{\alpha}_{p_{i}}\right\}}{\overline{\alpha} - \underline{\alpha}}.$$
 (11)

Note that in (11), if $[\underline{\alpha}, \overline{\alpha}]$ and $[\underline{\alpha}_{p_i}, \overline{\alpha}_{p_i}]$ do not overlap, then $\mathcal{P}_i = 0$ and item s_i has zero probability to be selected by the user.

Eq. (11) gives the answer to the question raised at the beginning of this section. Then, the ranking problem is straightforward: sorting all items in the descending order of their probabilities to be chosen by the user. The one with the largest probability is ranked the first.

4. REAL USER TEST

In our experiment, three types of products were considered: Cuisine coffee maker DCC-1200 (price around \$100), Itouch 5th generation (price around \$200) and Canon EOS 5D Mark II camera (price around \$2000). Price and seller reputation information was collected from eBay. For each product, the collected data were processed and grouped into 15 item sets, each with $4\sim 6$ skyline items. 21 subjects were interviewed and each subject was asked to consider the given price and reputation infomation only and select one item from each group as their top 1 choice. More details of the data collection and processing are available in [14].

	Coffee Maker	Itouch	Canon	Average
PRIMA	59.86%	58.84%	62.24%	60.32%
IC	58.50%	56.80%	57.49%	57.60%
MAPS	38.10%	57.49%	46.60%	47.39%

Table 2. Real user test results of success rate.

For reputation normalization, we used $R_{MIN} = 0$ and $R_{MAX} = 10^6$ for all three products. For price normalization, we used different price ranges in the normalization: $[P_{MIN} = 75, P_{MAX} = 122]$ for coffee maker, $[P_{MIN} = 174, P_{MAX} = 251]$ for Itouch and $[P_{MIN} = 1728, P_{MAX} = 3170]$ for Canon camera. Same as in [14], the first 5 transaction records were used to estimate users' personal preferences and the remaining transactions were used to evaluate the ranking algorithms, using the metrics described in Section 2.2.

Table 1 and 2 compare the performance of PRIMA with MAPS [6] and the IC-based method [10]. First, both the work in [10] and PRIMA give higher ranking quality and success rate than MAPS. Furthemore, we observe that PRIMA achieves comparable or even better performance than the ICbased method [10]. Note that PRIMA is also much simpler and mathematically tractable.

This is because the IC-based method does not have any explicit model of the indifference curve. Therefore, it has to use a few approximations when computing each item's probability of being the user's best choice, and the probability it gets is indeed the upper bound of the true probability.

In summary, compared to the IC-based method [10], our proposed PRIMA has a much simpler math model and is mathematically tractable, while it achieves comparable or even better performance.

5. CONCLUSION

In this work, we proposed PRIMA, a novel personalized multi-attribute probabilistic ranking model. PRIMA has three components: the personalized parameter (α) in the utility function describing a user's inter-attribute tradeoff; upper and lower bound estimation of the parameter α based on the past transactions; and probabilistic ranking addressing the inter-item competition. PRIMA not only has the advantage of simplicity and mathematical tractability, but also achieves comparable accuracy to the state-of-the-art work. Using PRIMA, we can calculate each item's probability of being the user's best choice. This probability is fundamental for personalized ranking and recommendation, as well as other market analysis and pricing strategies.

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