GREEDY PURSUITS BASED GRADUAL WEIGHTING STRATEGY FOR WEIGHTED ℓ_1-MINIMIZATION

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***Abstract*—In Compressive Sensing (CS) of sparse signals, standard ℓ_1-minimization can be effectively replaced with Weighted ℓ_1-minimization (Wℓ_1) if some information about the signal or its sparsity pattern is available. If no such information is available, Re-Weighted ℓ_1-minimization (ReWℓ_1) can be deployed. ReWℓ_1 solves a series of Wℓ_1 problems, and therefore, its computational complexity is high. An alternative to ReWℓ_1 is the Greedy Pursuits Assisted Basis Pursuit (GPABP) which employs multiple Greedy Pursuits (GPs) to obtain signal information which in turn is used to run Wℓ_1. Although GPABP is an effective fusion technique, it adapts a binary weighting strategy for running Wℓ_1, which is very restrictive. In this article, we propose a gradual weighting strategy for Wℓ_1, which handles the signal estimates resulting from multiple GPs more effectively compared to the binary weighting strategy of GPABP. The resulting algorithm is termed as Greedy Pursuits assisted Weighted ℓ_1-minimization (GP-Wℓ_1). For GP-Wℓ_1, we derive the theoretical upper bound on its reconstruction error. Through simulation results, we show that the proposed GP-Wℓ_1 outperforms ReWℓ_1 and the state-of-the-art GPABP.**

***Index Terms*—Weighted ℓ_1-minimization, greedy pursuits assisted basis pursuit**

I. INTRODUCTION

Compressive Sensing (CS) ensures the reconstruction of a sparse signal $x \in \mathbb{R}^n$ from its measurement vector $y \in \mathbb{R}^m$ of the form $y = \Phi x + v \in \mathbb{R}^m$, where $\Phi \in \mathbb{R}^{m \times n}$ is a known CS matrix with $m \ll n$ and v is the measurement noise [1]-[2]. Let K denote the signal sparsity level (i.e. there are only $K \ll n$ significant entries in x). Typically, $m = O(K \log(n))$ [3]. A sensing matrix Φ is said to obey the Restricted Isometry Property (RIP) if there exists a constant $\delta_K \in [0, 1]$ satisfying

$$(1 - \delta_K)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq (1 + \delta_K)\|x\|_2^2 \tag{1}$$

for all K-sparse vectors $x \in \mathbb{R}^n$. Note that $\|.\|_2$ stands for the vector ℓ_2-norm. The sparse signal can be reconstructed from y using CS reconstruction algorithms. They are broadly classified as convex relaxation methods (such as Basis Pursuit or ℓ_1-minimization [4]) and Greedy Pursuit (GP) algorithms (such as Orthogonal Matching Pursuit (OMP) [5], Subspace Pursuit (SP) [6], Iterative Hard Thresholding (IHT) [7] and Backtracking-based Adaptive Orthogonal Matching Pursuit (BAOMP) [8]). Among these two classes, GPs are known for their faster convergence. Applications of CS include image/video compression [9]-[10], radar echo recovery [11] and ECG signal reconstruction [12].

Motivation and Relation to Prior Work: In reconstruction of sparse signals, standard ℓ_1-minimization can be effectively replaced with Weighted ℓ_1-minimization (Wℓ_1) if some information about the signal is known apriori [13]-[16]. If no such information is available, Re-Weighted ℓ_1-minimization (ReWℓ_1) can be deployed [17]. As ReWℓ_1 solves a series of Wℓ_1 problems, its complutational complexity is high [18]. The Greedy Pursuits Assisted Basis Pursuit (GPABP) algorithm [19], an effective alternative to ReWℓ_1 in no prior information scenario, employs multiple GPs to obtain signal information which in turn is used to run Wℓ_1. However, GPABP adapts a binary weighting strategy to determine the weight vector for Wℓ_1, which is very restrictive [19]-[20]. Given the support estimates resulting from multiple GPs, a gradual weighting could be adapted. This article proposes a gradual weighting strategy for Wℓ_1, which handles the signal estimates resulting from multiple GPs more effectively compared to the binary weighting strategy of GPABP. The resulting algorithm is termed as Greedy Pursuits assisted Weighted ℓ_1-minimization (GP-Wℓ_1).

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II. PRIOR WORKS

This section briefly describes Wℓ_1, ReWℓ_1 and GPABP. The actual support of x, $T \subset \{1, 2, ..., n\}$, is defined as the set of indices i where $x(i)$ is non-zero. In [13], Wℓ_1 was proposed for reconstructing sparse signals whose prior information is available in the form of partial support. The partial support, say $T_k \subset \{1, 2, ..., n\}$, is defined as the set of indices i where $x(i)$ is estimated to be non-zero. The Wℓ_1 problem is formulated as

$$\hat{x} = \arg\min_{\tilde{x}} \|\tilde{x}\|_{1,w} \text{ s.t. } \|\Phi\tilde{x} - y\|_2 \leq \epsilon \tag{2}$$

where $\hat{x}$ is the reconstructed signal, $w \in [0, 1]^n$ and $\|\tilde{x}\|_{1,w} := \sum_i w(i)|\tilde{x}(i)|$ is the weighted ℓ_1 norm with $w(i) = \omega \in [0, 1)$ whenever $i \in T_k$, and $w(i) = 1$ otherwise. Note that, in (2), ϵ is the error tolerance (due to the presence of noise in y). A similar problem to Wℓ_1 was reported in [14] but it assumed a probabilistic prior on the support. If no prior information is available, ReWℓ_1 can be applied [17]. ReWℓ_1 solves a series of Wℓ_1 problems where the weights for the next iteration (say $w^{(t+1)}$) are computed from the value of the current solution (say $\hat{x}^{(t)}$) as follows,

$$w(i)^{(t+1)} = \frac{1}{|\hat{x}(i)^{(t)}| + \tau} \text{ for } i = 1, 2, ..., n. \tag{3}$$

The parameter $\tau > 0$ provides stability. In the initial iteration of ReWℓ_1, $\hat{x}^{(0)}$ is obtained by solving (2), fixing $\omega = 1$.

The GPABP algorithm [19], tailored specially for the no prior information scenario, employs multiple GPs to form $\tilde{T}_k$ as $\tilde{T}_k = \bigcap_{i=1}^{L} \hat{T}_i$, where L is the number of GPs and $\hat{T}_i$ is the support set estimated by the i^{th} GP. Then, $\tilde{T}_k$ is used to run Modified Basis Pursuit (Mod-BP) [15] as follows,

$$\hat{x} = \arg\min_{\tilde{x}} \|\tilde{x}_{\tilde{T}_k^c}\|_1 \text{ s.t. } \|\Phi\tilde{x} - y\|_2 \leq \epsilon \tag{4}$$

where $\tilde{T}_k^c$ is the set compliment of $\tilde{T}_k$, $\tilde{x}_{\tilde{T}_k^c}$ is the subset of $\tilde{x}$ formed by extracting the entries of $\tilde{x}$ corresponding to the indices in $\tilde{T}_k^c$ and $\|.\|_1$ stands for the vector ℓ_1-norm. The GPABP algorithm was shown to outperform fusion-based CS reconstruction algorithms such as fusion-of-algorithms for CS [21] and committee machine approach for CS [22].

Algorithm 1 Proposed GP-Wℓ_1

Require: Φ, y, K, L and ϵ

1: *for* i=1:L; $\hat{T}_i$ = GP$_i$(Φ, y, K); *end*

2: Weight vector: $w' = [w'(1)\ w'(2)\ ....\ w'(n)]$ where

$$w'(i) = 1 - \frac{G_i}{L}, \quad i = 1, 2, ..., n.$$

3: Obtain $\hat{x}$ using Wℓ_1:

$$\hat{x} = \arg\min_{\tilde{x}} \|\tilde{x}\|_{1,w'} \text{ s.t. } \|\Phi\tilde{x} - y\|_2 \leq \epsilon.$$

III. Proposed Weighting Strategy for Wℓ_1

If no prior signal/support information is available, either ReWℓ_1 or GPABP can be considered. High computational complexity of ReWℓ_1 is not affordable in many practical scenarios. On the other hand, GPABP has a limitation: the weighting is only binary (i.e. ω or 1). In this article, in order to have more number of weights, we propose a novel weighting strategy which assigns the weight for each location based on the number of GPs picking that location. This gradual weighting strategy is expected to give more precise weights compared to that of the binary weighting strategy. The resulting algorithm is termed as GP-Wℓ_1. Algorithm 1 shows the step-by-step procedure in GP-Wℓ_1. The GP-Wℓ_1 algorithm employs L GPs to obtain $\hat{T}_i, i = 1, 2, ..., L$. The weight corresponding to each location i is computed as follows,

$$w'(i) = 1 - \frac{G_i}{L}, \quad i = 1, 2, ..., n, \tag{5}$$

where G_i is the number of GPs picking the location i. Then, the weight vector $w' = [w'(1)\ w'(2)\ ....\ w'(n)]$ is used to obtain $\hat{x}$ as follows,

$$\hat{x} = \arg\min_{\tilde{x}} \|\tilde{x}\|_{1,w'} \text{ s.t. } \|\Phi\tilde{x} - y\|_2 \leq \epsilon. \tag{6}$$

It can be inferred from (5) that, for a location picked by large number of GPs, the weight will be small. Fig. 1 shows the schematic of the gradual weighting strategy of GP-Wℓ_1. The set, $[1, 2, ..., n]$, is partitioned into $L+1$ disjoint subsets N_l, $l \in 0, 1, ..., L$ where N_l denotes the set of locations picked by exactly l (out of L) GPs. Larger the l, smaller will be the weight for the subset N_l. Owing to the fact that the GPs have much lesser computational complexity compared to that of the Wℓ_1, GP-Wℓ_1's computational complexity will be of the same order as that of the Wℓ_1. The fact that the reconstruction accuracy of GP-Wℓ_1 increases with an increase in L can be verified in section IV.

A. Theoretical Analysis of GP-Wℓ_1

The following theorem gives the upper bound on GP-Wℓ_1's reconstruction error.

Theorem 1: Let $x \in \mathbb{R}^n$ and let x_K be its best K-term approximation, supported on T. Assume L GPs are used for weights estimation and let $[1, 2, ..., n]$ be partitioned into $L+1$ disjoint subsets N_l, $l \in 0, 1, ..., L$ where N_l mentioned as before denotes the set of locations picked by exactly l (out of L) algorithms. Suppose that there exists an $a \in \frac{1}{K}\mathbb{Z}$ that obeys $|N_1 \cup N_2 \cup ... \cup N_L| = aK$, and the matrix Φ obeys RIP with

$$\delta_{aK} + \frac{a}{\sqrt{1-\beta'+a(1-\alpha')}}\delta_{(a+1)K} \leq \frac{a}{\sqrt{1-\beta'+a(1-\alpha')}} - 1,$$

for $\alpha' = \frac{|(N_1 \cup N_2 \cup .... \cup N_L) \cap T|}{aK}$ and $\beta' = \frac{|N_L \cap T|}{K}$. Then the solution $\hat{x}$ to (6) obeys

$$\|\hat{x} - x\|_2 \leq D_1\epsilon + D_2 \sum_{l=0}^{L-1} w_l \|x_{N_l \cap T^c}\|_1,$$

where D_1 and D_2 are constants (given in the proof below) that depend on Φ, a, α' and β'.

Proof: Let $\hat{x} = x + h$ be the minimizer of (6). Then

$$\|x + h\|_{1,w'} \leq \|x\|_{1,w'}.$$

Moreover, due to the choice of weights (given by (5)),

$$\sum_{l=0}^{L} w'(l)\|x_{N_l} + h_{N_l}\|_1 \leq \sum_{l=0}^{L} w'(l)\|x_{N_l}\|_1.$$

$$\sum_{l=0}^{L} w'(l)\|x_{N_l \cap T} + h_{N_l \cap T}\|_1 + \sum_{l=0}^{L} w'(l)\|x_{N_l \cap T^c} + h_{N_l \cap T^c}\|_1 \leq \sum_{l=0}^{L} w'(l)\|x_{N_l \cap T}\|_1 + \sum_{l=0}^{L} w'(l)\|x_{N_l \cap T^c}\|_1.$$

Using forward and reverse triangle inequalities, the above expression becomes,

$$\sum_{l=0}^{L} w'(l)\|h_{N_l \cap T^c}\|_1 \leq \sum_{l=0}^{L} w'(l)\|h_{N_l \cap T}\|_1 + 2\sum_{l=0}^{L} w'(l)\|x_{N_l \cap T^c}\|_1.$$

By adding $\sum_{l=0}^{L}(1 - w'(l))\|h_{N_l \cap T^c}\|_1$ on both sides,

$$\|h_{T^c}\|_1 \leq \sum_{l=0}^{L} w'(l)\|h_{N_l \cap T}\|_1 + \sum_{l=0}^{L}(1 - w'(l))\|h_{N_l \cap T^c}\|_1 + 2\sum_{l=0}^{L} w'(l)\|x_{N_l \cap T^c}\|_1.$$

Since $w'(L) = 1 - w'(0) = 0$

$$\|h_{T^c}\|_1 \leq \sum_{l=0}^{L-1} w'(l)\|h_{N_l \cap T}\|_1 + \sum_{l=1}^{L}(1 - w'(l))\|h_{N_l \cap T^c}\|_1 + 2\sum_{l=0}^{L-1} w'(l)\|x_{N_l \cap T^c}\|_1.$$

Noting that the weights range from 0 to 1,

$$\|h_{T^c}\|_1 \leq \|h_{(N_0 \cup N_1 \cup ... \cup N_{L-1}) \cap T}\|_1 + \|h_{(N_1 \cup N_2 \cup ... \cup N_L) \cap T^c}\|_1 + 2\sum_{l=0}^{L-1} w'(l)\|x_{N_l \cap T^c}\|_1.$$

Let $\check{T} = (T - N_L) \cup (T^c - N_0)$. Then,

$$\|h_{T^c}\|_1 \leq \|h_{\check{T}}\|_1 + 2\sum_{l=0}^{L-1} w'(l)\|x_{N_l \cap T^c}\|_1. \tag{7}$$

Let us define two parameters $\alpha' = \frac{|(N_1 \cup N_2 \cup .... \cup N_L) \cap T|}{aK}$ and $\beta' = \frac{|N_L \cap T|}{K}$ such that $|\check{T}| = (1 - \beta' + a(1 - \alpha'))K$. Since for a n-length vector x, $\|x\|_1 \leq \sqrt{n}\|x\|_2$,

$$\|h_{T^c}\|_1 \leq \sqrt{(1 - \beta' + a(1 - \alpha'))K}\|h_{\check{T}}\|_2 + 2\sum_{l=0}^{L-1} w'(l)\|x_{N_l \cap T^c}\|_1. \tag{8}$$

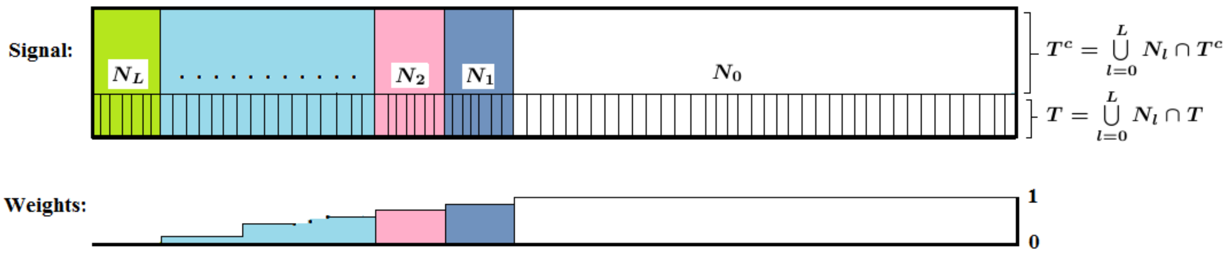

Fig. 1: Gradual Weighting Strategy of GP-Wℓ_1

As in [13], the coefficients of h_{T^c} are sorted in the order of decreasing magnitude partitioning T^c into disjoint sets T_j, $j \in \{1, 2, ...\}$ each of size aK, where $a > 1$. That is, T_1 indexes the aK largest magnitude coefficients of h_{T^c}, T_2 indexes the second aK largest magnitude coefficients of h_{T^c}, and so on. This gives

$$\|h_{T_j}\|_2 \leq (aK)^{-1/2}\|h_{T_{j-1}}\|_1. \tag{9}$$

Let $T_{01} = T \cup T_1$, then using the triangle inequality in the above gives

$$\|h_{T_{01}^c}\|_2 \leq (aK)^{-1/2}\|h_{T^c}\|_1. \tag{10}$$

Since x and $\hat{x}$ are feasible, $\|\Phi h\|_2 \leq 2\epsilon$ and

$$\|\Phi h_{T_{01}}\|_2 \leq 2\epsilon + \sum_{j>1}\|\Phi h_{T_j}\|_2 \leq 2\epsilon + \sqrt{1+\delta_{aK}}\sum_{j>1}\|h_{T_j}\|_2$$

where the last inequality follows RIP. Since $\sum_{j>1}\|h_{T_j}\|_2 = \|h_{T_{01}^c}\|_2$, using (10) in the above equation gives

$$\|\Phi h_{T_{01}}\|_2 \leq 2\epsilon + \frac{\sqrt{1+\delta_{aK}}}{\sqrt{aK}}\|h_{T^c}\|_1. \tag{11}$$

Using RIP on the R.H.S. of the above we get

$$\sqrt{1-\delta_{(a+1)K}}\|h_{T_{01}}\|_2 \leq 2\epsilon + \frac{\sqrt{1+\delta_{aK}}}{\sqrt{aK}}\|h_{T^c}\|_1. \tag{12}$$

Since T_1 contains the largest aK coefficients of h_{T^c}, $\|h_{\check{T}}\|_2 \leq \|h_{T_{01}}\|_2$, and therefore (8) becomes

$$\begin{aligned}\|h_{T^c}\|_1 \leq & \sqrt{(1-\beta'+a(1-\alpha'))K}\|h_{T_{01}}\|_2 \\ & + 2\sum_{l=0}^{L-1} w'(l)\|x_{N_l\cap T^c}\|_1.\end{aligned} \tag{13}$$

Combining (12) and (13) gives

$$\|h_{T_{01}}\|_2 \leq \frac{2\epsilon + 2\frac{\sqrt{1+\delta_{aK}}}{\sqrt{aK}}\sum_{l=0}^{L-1} w'(l)\|x_{N_l\cap T^c}\|_1}{\sqrt{1-\delta_{(a+1)K}} - \frac{\sqrt{1-\beta'+a(1-\alpha')}}{\sqrt{a}}.\sqrt{1+\delta_{aK}}}. \tag{14}$$

Finally, using the relation $\|h\|_2 \leq \|h_{T_{01}}\|_2 + \|h_{T_{01}^c}\|_2$, and substituting (14), (10) and (13) in it gives

$$\|h\|_2 \leq D_1\epsilon + D_2\sum_{l=0}^{L-1} w'(l)\|x_{N_l\cap T^c}\|_1 \tag{15}$$

where the constants

$$D_1 = \frac{2\left(1+\frac{\sqrt{1-\beta'+a(1-\alpha')}}{\sqrt{a}}\right)}{\sqrt{1-\delta_{(a+1)K}} - \frac{\sqrt{1-\beta'+a(1-\alpha')}}{\sqrt{a}}\sqrt{1+\delta_{aK}}} \tag{16}$$

and

$$D_2 = \frac{2\frac{\sqrt{1-\delta_{(a+1)K}}+\sqrt{1+\delta_{aK}}}{\sqrt{aK}}}{\sqrt{1-\delta_{(a+1)K}} - \frac{\sqrt{1-\beta'+a(1-\alpha')}}{\sqrt{a}}\sqrt{1+\delta_{aK}}}, \tag{17}$$

with the condition that the denominator is positive, equivalently

$$\begin{aligned}\delta_{aK} + & \frac{a}{\sqrt{1-\beta'+a(1-\alpha')}}\delta_{(a+1)K} \\ & \leq \frac{a}{\sqrt{1-\beta'+a(1-\alpha')}} - 1. \qquad \square\end{aligned}$$

Remark 1: If x is exactly sparse (i.e. $\|x_{N_l\cap T^c}\|_1 = 0$) and y is noiseless (i.e. $\epsilon = 0$), then $\|h\|_2 = 0$ which implies the reconstruction is exact.

Remark 2: For theorem 1 to hold, it is sufficient if Φ satisfies

$$\delta_{(a+1)K} < \frac{a-(1-\beta'+a(1-\alpha'))}{a+(1-\beta'+a(1-\alpha'))}. \tag{18}$$

Compared to the standard ℓ_1-minimization [2] which requires Φ to satisfy $\delta_{(a+1)K} < \frac{a-1}{a+1}$, GP-W$\ell_1$ has a weaker requirement provided

$$\beta' > a(1-\alpha'). \tag{19}$$

As per the definitions of a, α', and β', the above condition holds when the ingredient GPs in GP-Wℓ_1 (i.e., GP$_i$ in step 1 of algorithm 1) estimate the signal support such that

$$|(N_1\cup N_2\cup ...\cup N_L)\cap T| + |N_L\cap T| > |N_1\cup N_2\cup ...\cup N_L|. \tag{20}$$

There are two favourable scenarios for the above condition to hold. First, every GP in GP-Wℓ_1 should pick most of the correct non-zero locations. This will lead to the LHS of (20) being close to $2K$. Second, all GPs should estimate nearly the same signal support. More importantly, the union of wrong locations picked by GPs (i.e. $(N_1\cup N_2\cup ...\cup N_L)\cap T^c$) should be a small-sized set. This will lead to the RHS of (20) being close to K. Consider a typical reconstruction example where $n = 50$, $L = 3$ and $T = \{2, 4, 6, 8, 10, 12\}$. Let the signal estimates resulting from L GPs be $\hat{T}_1 = \{2, 4, 6, 8, 10, 11\}$, $\hat{T}_2 = \{2, 4, 6, 8, 12, 15\}$ and $\hat{T}_3 = \{2, 4, 6, 8, 11, 12\}$. This will result in $N_1 = \{10, 15\}$, $N_2 = \{11, 12\}$ and $N_3 = \{2, 4, 6, 8\}$. Therefore, $|N_L\cap T| = 4$, $|(N_1\cup N_2\cup ...\cup N_L)\cap T| = 6$ and $|N_1\cup N_2\cup ...\cup N_L| = 8$. Since the condition in (20) holds, for this reconstruction scenario, GP-Wℓ_1 has a more favorable RIP requirement compared to that of the standard ℓ_1-minimization.

IV. Experimental Results

Signal reconstruction is performed using three methods: ReWℓ_1 [18], GPABP [19], and the proposed GP-Wℓ_1. Both GPABP and GP-Wℓ_1 involve four GPs (OMP [5], SP [6], IHT [7] and BAOMP [8]). The *cvx* solver [23] is used for the implementation of Wℓ_1 in all

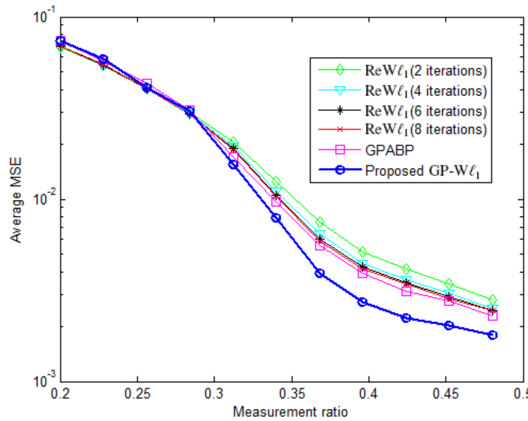

Fig. 2: Synthetic sparse signals: Average MSE versus MR

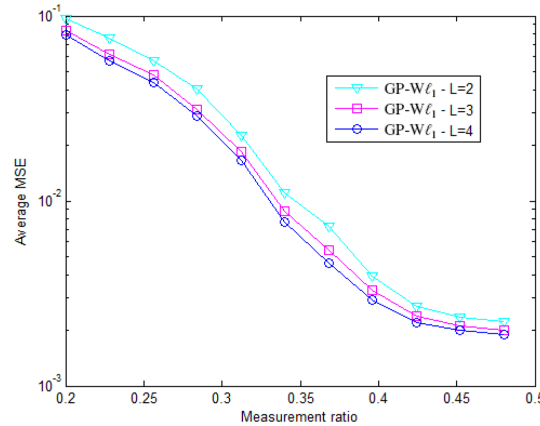

Fig. 4: Synthetic sparse signals: Effect of L on GP-Wℓ_1

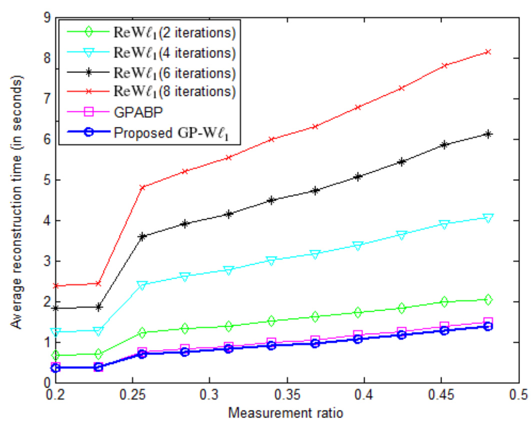

Fig. 3: Synthetic sparse signals: Average reconstruction time versus MR

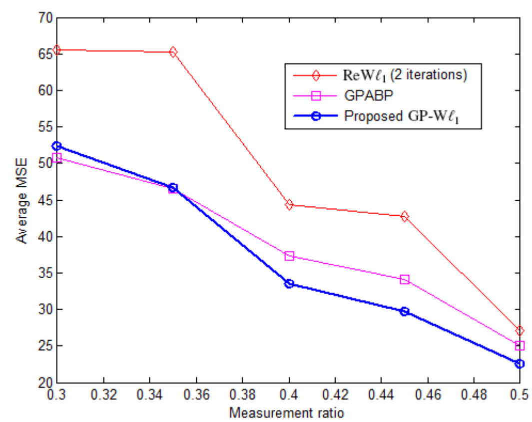

Fig. 5: Compressible ECG signals: Average MSE versus MR

three methods. For ReWℓ_1, τ is fixed as 0.1 (a value slightly smaller than the expected non-zero magnitudes of x).

Synthetic Sparse Signals: For our experiments on synthetic signals, Gaussian sparse signals of length $n = 250$ are generated. First experiment presents the reconstruction error as a function of Measurement Ratio (MR$= \frac{m}{n}$). An $m \times n$ Gaussian random measurement matrix is generated to acquire the sparse signal. In this experiment, y is corrupted by a noise such that its Signal to Measurement Noise Ratio (SMNR) is 20 dB. The SMNR is defined as

$$\text{SMNR (in dB)} = 10 \log_{10} \frac{\|x\|_2^2}{\|v\|_2^2}.$$

The constraint of Wℓ_1 is fixed as $\|\Phi\tilde{x} - y\|_2 \leq 0.001$. Sparsity level K is fixed as 30 and the MR is varied between 0.2 and 0.48. For each value of MR, 250 independent trials are performed to obtain the average results. Fig. 2 shows the average Mean Square Error (MSE $= \frac{1}{n}\|x - \hat{x}\|_2^2$) as a function of MR. For MRs greater than 0.3, GP-Wℓ_1 results in a reconstruction error that is less compared to that of the ReWℓ_1 and GPABP. Fig. 3 shows the corresponding average reconstruction time (i.e. computation time in MATLAB 7.12.0 running on a 64-bit Intel(R) Core^{TM} i5-2400 processor with 8 GB RAM). As expected, GP-Wℓ_1's convergence is much faster than that of the ReWℓ_1 and comparable to that of the GPABP.

Next experiment reports the effect of L. The experimental set-up is the same as that of the previous experiment and the GP-Wℓ_1's performance is recorded for $L = 2, 3$ and 4. In each trial, support set estimates are obtained for all four GPs mentioned above. In the case of $L < 4$, only L randomly chosen support set estimates are used for weights estimation. It can be seen from fig. 4 that the accuracy of GP-Wℓ_1 increases with an increase in L.

Compressible ECG signals: For conducting an experiment on real world signal, the leads (i.e. ECG signals) are extracted from records 100, 101, 102 and 103 from the MIT-BIH Arrhythmia database [24]. These signals are divided into chunks of $n = 250$ samples (with their amplitudes ranging from 0 to 255). The sparse representations of these signals are obtained using the discrete cosine transform. We fixed K= $\lfloor \frac{m}{\log n} \rfloor$. The measurement vector y is obtained using an $m \times n$ Gaussian random matrix Φ. Then, y is corrupted by a noise such that its SMNR is 20 dB. Fig. 5 shows the average MSE as a function of MR, for three methods (ReWℓ_1, GPABP and GP-Wℓ_1). For MRs greater than 0.35, GP-Wℓ_1 gives the least MSE.

V. Conclusion

We proposed a gradual weighting strategy for Wℓ_1, which handles the signal estimates resulting from multiple GPs more effectively compared to the binary weighting strategy of GPABP. For the proposed GP-Wℓ_1, we derived the theoretical upper bound on its reconstruction error. Experimental results showed that, in reconstruction of sparse signals with no prior information, GP-Wℓ_1 outperforms GPABP and ReWℓ_1.

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