OPTIMAL POWER AND BIT ALLOCATION FOR GRAPH SIGNAL INTERPOLATION

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ABSTRACT

We study centralized interpolation of bandlimited graph signals at a fusion center (FC), when sampled data are transmitted over rate-constrained links. In such a scenario, the performance of the reconstruction task is inevitably affected by several sources of errors such as observation noise and quantization due to source encoding. In this paper, we propose two strategies for optimally selecting transmission powers, quantization bits, and the sampling set, with the aim of interpolating a graph signal with guaranteed performance. Numerical results validate the proposed approach for interpolation of bandlimited graph signals under communication constraints.

Index Terms— Graph signal processing, interpolation, sampling on graphs, probabilistic quantization.

1. INTRODUCTION

Over the last few years, there was a surge of interest in developing novel analysis methods for graph signals, thus leading to the research field known as graph signal processing (GSP), see, e.g., [1, 2]. The goal of GSP is to extend classical processing tools to the analysis of signals defined over an irregular discrete domain, typically represented by a graph [2, 3]. A fundamental task in GSP is to infer the values of a graph signal by interpolating the samples collected from a known set of vertices. A first seminal contribution to sampling/interpolation theory in GSP is given by [4]; the approach was then extended in [5, 6]. The work in [7]provides conditions that guarantee unique reconstruction of signals spanned over a subset of vectors composing the graph Fourier basis. Reference [8] creates a conceptual link between uncertainty principle and sampling of graph signals. Another valid approach is the so called aggregation sampling [9], which involves successively shifting a signal using the adjacency matrix and aggregating the values at a given node. The work in [10] proposes efficient methods to select the sampling set based on powers of the variation operator. Greedy sampling strategies with provable performance were proposed in [11]. There exist also randomized sampling strategies, e.g., [12], [13], which are based on a smart design of the sampling probability distribution. Finally, adaptive interpolation methods that are capable to handle dynamic graph signals were proposed in [14–17].

The goal of this work is to propose an optimization strategy that finds optimal radio parameters (i.e., powers and number of coding bits) to be used over fading communication links between graph nodes and the FC, in order to ensure graph signal interpolation with guaranteed reconstruction performance. Similar approaches for distributed estimation/detection problems were proposed in [18], [19]. More specifically, in this paper we find the sampling set and the optimal radio resource allocation that minimizes the sum of the transmit powers, while enforcing an upper bound on the mean-square error of a graph signal interpolation task. The resulting problem turns out to be non-convex, and two alternative strategies are proposed to cope with such issue.

2. BACKGROUND

We consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consisting of a set of N nodes $\mathcal{V} = \{1, 2, ..., N\}$, along with a set of weighted edges $\mathcal{E} = \{a_{ij}\}_{i,j \in \mathcal{V}}$, such that $a_{ij} > 0$, if there is a link from node *j* to node *i*, or $a_{ij} = 0$, otherwise. A signal *x* over a graph \mathcal{G} is defined as a mapping from the vertex set to the set of complex numbers, i.e., $x : \mathcal{V} \to \mathbb{C}$. The graph \mathcal{G} is endowed with a graph-shift operator S defined as an $N \times N$ matrix whose entry (i, j), denoted with S_{ij} , can be non-zero only if i = j or the link $(j, i) \in \mathcal{E}$; common choices for **S** are the adjacency matrix [2], the Laplacian [1], and its generalizations [10]. We assume that S admits the decomposition $S = UAU^{H}$ for some eigenvector matrix $\mathbf{U} = [u_1, \ldots, u_N]$ and diagonal matrix Λ , such that $\mathbf{SS}^{H} = \mathbf{S}^{H} \mathbf{S}$. The Graph Fourier Transform (GFT) of a signal x is defined as its projection onto the set of eigenvectors $\{u_i\}_{i=1,\dots,N}$ [1], i.e., $GFT(x) = \mathbf{U}^H x$. Perfect recovery of a graph signal from its samples is possible if x is *bandlimited* in the graph frequency domain, i.e., it can be expressed as:

$$\boldsymbol{x} = \mathbf{U}_{\mathcal{F}} \boldsymbol{s}_{\mathcal{F}},\tag{1}$$

where $\mathbf{U}_{\mathcal{F}} \in \mathbb{C}^{N \times |\mathcal{F}|}$ represents the collection of graph Fourier vectors associated with a subset of frequency indices \mathcal{F} , and $s_{\mathcal{F}} \in \mathbb{C}^{|\mathcal{F}|}$ are the corresponding graph signal's frequency coefficients. In this context, \mathcal{F} denotes the support of the signal in the graph Fourier domain.

3. RATE-CONSTRAINED INTERPOLATION OF GRAPH SIGNALS

Consider a bandlimited signal x defined over the graph \mathcal{G} . A subset \mathcal{S} of nodes samples the graph signal at its location and, according to (1), the measurements are described by:

$$y_i = x_i + v_i = \boldsymbol{u}_{\mathcal{F},i}^H \boldsymbol{s}_{\mathcal{F}} + v_i, \quad i \in \mathcal{S},$$
(2)

where $u_{\mathcal{F},i}^{H}$ is the *i*-th row of matrix $\mathbf{U}_{\mathcal{F}}$, and v_i is zero-mean, uncorrelated noise with variance σ_i^2 . Let us recast the scalar observations in (2) using the vector model:

$$\boldsymbol{y}_{\mathcal{S}} = \mathbf{P}_{\mathcal{S}}(\boldsymbol{x} + \boldsymbol{v}) = \mathbf{P}_{\mathcal{S}}\mathbf{U}_{\mathcal{F}}\boldsymbol{s}_{\mathcal{F}} + \mathbf{P}_{\mathcal{S}}\boldsymbol{v}, \qquad (3)$$

where $\mathbf{P}_{\mathcal{S}} \in \mathbb{R}^{|\mathcal{S}| \times N}$ is a sampling matrix whose rows are indicator functions for nodes in \mathcal{S} , $\boldsymbol{y}_{\mathcal{S}} = \{y_i\}_{i \in \mathcal{S}}$, and $\boldsymbol{v} = \{v_i\}_{i=1}^N$. In a centralized scheme, the measurements are sent to an FC, which interpolates using the best linear unbiased estimator (BLUE) [20] as:

$$\widehat{\boldsymbol{x}} = \mathbf{U}_{\mathcal{F}} \left(\mathbf{U}_{\mathcal{F}}^{H} \mathbf{P}_{\mathcal{S}}^{T} \left(\mathbf{P}_{\mathcal{S}} \mathbf{C}_{v} \mathbf{P}_{\mathcal{S}}^{T} \right)^{-1} \mathbf{P}_{\mathcal{S}} \mathbf{U}_{\mathcal{F}} \right)^{-1} \times \\ \times \mathbf{U}_{\mathcal{F}}^{H} \mathbf{P}_{\mathcal{S}}^{T} \left(\mathbf{P}_{\mathcal{S}} \mathbf{C}_{v} \mathbf{P}_{\mathcal{S}}^{T} \right)^{-1} \boldsymbol{y}_{\mathcal{S}}, \tag{4}$$

where $C_v = \text{diag}\{\sigma_1^2, \dots, \sigma_N^2\}$. As a measure of quality of signal reconstruction provided by (4), we will use the mean-square error (MSE), which is given by [20]:

$$MSE = Tr \left\{ \left(\mathbf{U}_{\mathcal{F}}^{H} \mathbf{P}_{\mathcal{S}}^{T} \left(\mathbf{P}_{\mathcal{S}} \mathbf{C}_{v} \mathbf{P}_{\mathcal{S}}^{T} \right)^{-1} \mathbf{P}_{\mathcal{S}} \mathbf{U}_{\mathcal{F}} \right)^{-1} \right\}$$
$$= Tr \left\{ \left(\sum_{i \in \mathcal{S}} \boldsymbol{u}_{\mathcal{F},i} \boldsymbol{u}_{\mathcal{F},i}^{H} / \sigma_{i}^{2} \right)^{-1} \right\},$$
(5)

where the second equality in (5) comes from the uncorrelatedness of observation noise. The above estimation scheme can be applied only when observations can be collected without any distortion. Typically, such an assumption is unrealistic when the communication links between the fusion center and sensor nodes are rate-constrained and affected by fading/pathloss. Thus, we propose an interpolation scheme where each sensor performs a local quantization of y_i in (2) and generates a message $m_i(y_i, b_i)$ of b_i bits, where the quantizer $Q_i : y_i \to m_i(y_i, b_i)$ has to be designed. Each message $m_i(y_i, b_i)$ is then transmitted to the FC through a separate AWGN channel in order to perform the final interpolation.

3.1. Probabilistic Quantization

Suppose that [-A, A] is the signal range that sensors can observe. We consider a uniform quantizer that divides the range [-A, A] into intervals of length $\Delta = 2A/(2^b-1)$, and rounds the obserbations in (2) to the neighboring endpoints of these small intervals in a probabilistic manner [21], [18]. Then, if

 $l\Delta < y < (l+1)\Delta$, with $l \in \{-2^{b-1}, \ldots, 0, \ldots, 2^{b-1}\}$, then y is quantized to m(y, b) according to:

$$m(y,b) = l\Delta + \alpha\Delta, \tag{6}$$

where α is a Bernoulli random variable such that

$$\mathbb{E}\{\alpha\} = \operatorname{Prob}\{\alpha = 1\} = (y - l\Delta)/\Delta \in [0, 1].$$

According to (6), the quantized *i*-th observation, i.e., $m_i(y_i, b_i)$, can be equivalently written as:

$$m_i(y_i, b_i) = m(x_i + v_i, b_i) = x_i + v_i + q(y_i, b_i), \quad (7)$$

where $q(y_i, b_i) = (\alpha - \mathbb{E}\{\alpha\})\Delta$ denotes the quantization noise. In particular, it is possible to show that $m_i(y_i, b_i)$ in (7) is an unbiased estimator of x_i , and

$$\mathbb{E}|m_i(y_i, b_i) - x_i|^2 \le \sigma_i^2 + \frac{A^2}{(2^{b_i} - 1)^2}$$
(8)

is an upper bound on the estimation variance [18,21].

3.2. Graph Signal Interpolation

Our goal is to construct a linear interpolator of x from $\{m_1, \ldots, m_N\}$ such that the MSE is minimized. Let us assume that each sensor can send information to the FC, and that sampling (i.e., selection of the sensing/transmitting nodes) will be performed by a following optimization step. In particular, we consider the (quasi-)BLUE given by:

$$\widehat{\boldsymbol{x}} = \mathbf{U}_{\mathcal{F}} \left(\mathbf{U}_{\mathcal{F}}^{H} \left(\mathbf{C}_{v} + \mathbf{C}_{q} \right)^{-1} \mathbf{U}_{\mathcal{F}} \right)^{-1} \mathbf{U}_{\mathcal{F}}^{H} \left(\mathbf{C}_{v} + \mathbf{C}_{q} \right)^{-1} \boldsymbol{m},$$
(9)

where $\boldsymbol{m} = \{m_1, \ldots, m_N\}$, $\mathbf{C}_q = \text{diag}\{\sigma_{q_1}^2, \ldots, \sigma_{q_N}^2\}$, with $\sigma_{q_i}^2 = A^2/(2^{b_i} - 1)^2$. Notice that $\hat{\boldsymbol{x}}$ in (9) is an unbiased estimator of \boldsymbol{x} since every m_i is an unbiased quantization of x_i . Furthermore, combining (5) and (8), it holds that:

$$MSE \leq Tr\left\{ \left(\sum_{i=1}^{N} \frac{\boldsymbol{u}_{\mathcal{F},i} \boldsymbol{u}_{\mathcal{F},i}^{H}}{\sigma_{i}^{2} + \frac{A^{2}}{(2^{b_{i}} - 1)^{2}}} \right)^{-1} \right\}.$$
 (10)

Now, we assume that the channel between each sensor and the FC is corrupted with additive white Gaussian noise whose double-sided power spectrum density is given by $N_0/2$. Furthermore, we denote by h_i the channel coefficient between sensor *i* and the FC. If sensor *i* sends b_i bits with quadrature amplitude modulation with constellation size 2^{b_i} at a bit error probability BER_i, then the total amount of required transmission power [18, 22, 23], is given by:

$$p_{i} = \frac{2B_{s}N_{f}N_{0}G_{d}}{h_{i}^{2}} \left(\ln\frac{2}{\text{BER}_{i}}\right)(2^{b_{i}}-1), \quad (11)$$

where B_s denotes the sampling rate, N_f is the receiver noise figure, and G_d s a system constant defined in the same way as in [22,23]. In the sequel, for the sake of simplicity, we assume that the bit error probability of each transmission is made sufficiently small such that errors have a negligible effect on the MSE in (10). Thus, letting $c_i = \frac{2B_s N_f N_0 G_d}{h_i^2} \left(\ln \frac{2}{\text{BER}_i} \right)$, and using (11) in (10), we obtain:

$$\text{MSE} \leq \text{Tr} \left\{ \left(\sum_{i=1}^{N} \frac{\boldsymbol{u}_{\mathcal{F},i} \boldsymbol{u}_{\mathcal{F},i}^{H}}{\sigma_{i}^{2} + \frac{A^{2} c_{i}^{2}}{p_{i}^{2}}} \right)^{-1} \right\}.$$
 (12)

In the next section, we will illustrate how to optimize the transmission powers and bits [cf. (11)] with the aim of interpolating a graph signal with guaranteed MSE performance.

4. OPTIMAL POWER AND BIT ALLOCATION

The proposed allocation strategy aims at minimizing the sum of powers transmitted by all the sensors under the constraint that the MSE in (12) is lower than a prescribed value γ . Note that, since the values of $\{b_i\}$ are integers, the transmitted powers in (11) can assume only discrete values. In the sequel, to avoid complex integer programming formulations, we suppose that the variables $\{b_i\}$ and, consequently, $\{p_i\}$ assume real values. The optimization problem can then be cast as:

$$\min_{\boldsymbol{p}} \sum_{i=1}^{N} p_i$$
subject to
$$\operatorname{Tr}\left\{ \left(\sum_{i=1}^{N} \frac{\boldsymbol{u}_{\mathcal{F},i} \boldsymbol{u}_{\mathcal{F},i}^{H}}{\sigma_i^2 + \frac{A^2 c_i^2}{p_i^2}} \right)^{-1} \right\} \leq \gamma \qquad (13)$$

$$p_i \geq 0, \quad i = 1, \dots, N,$$

where $p = \{p_1, \ldots, p_N\}$. It is interesting to notice that, when $p_i = 0$ [i.e., $b_i = 0$, cf. (11)] as a result of the optimization in (13), sensor *i* does not transmit and its measurement is not included in the evaluation of the MSE in (12). This means that problem (13) is capable to perform automatic selection of the sampling set S, i.e., the set of nodes that send data to the FC. In fact, numerical results show that we can obtain sparse power vectors p, as an interesting by-product of the proposed allocation strategy. Moreover, the problem formulation in (13) can easily incorporate unobservable nodes belonging to some set, say, e.g., S_{un} , by simply adding the equality constraints $p_i = 0$ for all $i \in S_{un}$.

Unfortunately, problem (13) is non-convex due to the non-convex constraint on the MSE. Thus, to handle the nonconvexity of (13), in the sequel we propose two different methods. The first approach exploits successive convex ap-

Algorithm 1: SCA method for Problem (15)

Start with $z[1] \in C$. Then, for $k \ge 1$, repeat until convergence the following steps:

S.1)
$$\widehat{\boldsymbol{z}}[k] = \operatorname*{arg\,min}_{\boldsymbol{z}\in\mathcal{C}} \widetilde{f}(\boldsymbol{z};\boldsymbol{z}[k])$$

S.2) $\boldsymbol{z}[k+1] = \boldsymbol{z}[k] + \alpha[k] (\widehat{\boldsymbol{z}}[k] - \boldsymbol{z}[k])$

proximation (SCA) methods [24], whose aim is to find local optimal solutions of (13). Let us first perform a change of variables to recast (13) in a form more amenable for SCA optimization. In particular, let us define

$$z_i = \frac{1}{\sigma_i^2 + \frac{A^2 c_i^2}{p_i^2}}, \quad i = 1, \dots, N.$$
(14)

Thus, using (14) in (13) and letting $z = \{z_1, \ldots, z_N\}$, we obtain the following equivalent non-convex problem:

$$\min_{\boldsymbol{z}} \sum_{i=1}^{N} c_i \sqrt{\frac{z_i}{1 - z_i \sigma_i^2}}$$
subject to $\operatorname{Tr} \left\{ \left(\sum_{i=1}^{N} z_i \, \boldsymbol{u}_{\mathcal{F},i} \boldsymbol{u}_{\mathcal{F},i}^H \right)^{-1} \right\} \leq \gamma$ (15)
 $0 \leq z_i < \frac{1}{\sigma_i^2}, \quad i = 1, \dots, N.$

Now, letting C be the feasible set of problem (15), we exploit the approach proposed in [24], and letting z[k] be the guess of the power vector at iteration k, the SCA algorithm proceeds as described in Algorithm 1. At every iteration k, given the current estimate z[k], the first step of Algorithm 1 solves a surrogate optimization problem involving a strongly convex approximant of the objective function in (15) that preserves gradient information at the point z[k], given by:

$$\widetilde{f}(\boldsymbol{z}; \boldsymbol{z}[k]) = \sum_{i=1}^{N} \frac{Ac_i}{\sqrt{z_i[k]}} \frac{z_i}{(1 - z_i[k]\sigma_i^2)^{3/2}} + \frac{\tau}{2} \|\boldsymbol{z} - \boldsymbol{z}[k]\|^2,$$
(16)

with $\tau > 0$; see [24] for details on the selection of the surrogate function $\tilde{f}(z; z[k])$. Then, the second step of Algorithm 1 generates the new point z[k+1] as a convex combination of the current estimate z[k] and the solutions $\hat{z}[k]$, exploiting the step-size sequence $\alpha[k]$. Under mild conditions on the stepsize sequence $\alpha[k]$, the sequence generated by the Algorithm 1 converges to a stationary solution of (15) [and, equivalently, of (13)], see [24, Theorem 2].

An alternative approach to SCA is to formulate the optimization problem (13) in a slightly different manner. In particular, in (13), we can minimize the squared Euclidean norm



Fig. 1: Graph topology, and sampling nodes (black squares).

of the power vector (i.e., $\sum_{i=1}^{N} p_i^2$) instead of the sum of powers. Then, applying (14) to the resulting problem, we obtain:

$$\min_{\boldsymbol{z}} \sum_{i=1}^{N} c_i^2 \frac{z_i}{1 - z_i \sigma_i^2}$$
subject to $\operatorname{Tr} \left\{ \left(\sum_{i=1}^{N} z_i \, \boldsymbol{u}_{\mathcal{F},i} \boldsymbol{u}_{\mathcal{F},i}^H \right)^{-1} \right\} \leq \gamma$ (17)
$$0 \leq z_i < \frac{1}{\sigma_i^2}, \quad i = 1, \dots, N.$$

Problem (17) is now a convex program, whose global solution can be found using efficient numerical tools [25].

5. NUMERICAL RESULTS

Let us consider a graph composed of 30 nodes, whose topology is illustrated in Fig. 1. The considered graph signal has a spectral content limited to the first five eigenvectors of the Laplacian matrix. The observation noise in (2) is zero-mean, Gaussian, with a variance $\sigma_i^2 = 10^{-4}$ for all *i*. Then, in Fig. 2 (top), we illustrate the optimal power allocation obtained using the SCA algorithm 1, considering the MSE upper-bound $\gamma = 10^{-2}$, channel gains h_i^2 as depicted in Fig. 2 (bottom), and $\tau = 10^{-4}$. The step-size rule is given by $\alpha[k] =$ $\alpha[k-1](1-\varepsilon\alpha[k-1])$, with $\alpha[1]=1$ and $\varepsilon=0.01$. For simplicity, the other parameters are set such that $c_i = 1/h_i^2$ for all *i*. The MSE constraint in (13) is always attained strictly. As we can see from Fig. 2, the method finds a sparse power vector p, whose non-zero elements are associated to nodes that have large channel gains. The set of transmitting nodes corresponds also to the sampling set, and is depicted in Fig. 1 using black squares. Finally, in Fig. 3, we compare the results obtained by the two proposed methods, i.e., the SCA algorithm 1, and the convex optimization problem in (17), in terms of temporal behavior of the sum of allocated powers, for different values of the parameter γ . In particular, we solve



Fig. 2: Optimal transmitted powers (top) and channels gain (bottom) versus node index.



Fig. 3: Sum of transmitted powers versus iteration index, for different algorithms and values of the MSE upper-bound γ .

the convex optimization problem in (17) using an SCA algorithm similar to Algorithm 1, but with a different surrogate function that is tailored to problem (17). As expected, from Fig. 3, we notice that a larger power consumption is needed when we require a larger precision of the interpolation task. Interestingly, the performance of the convexified problem in (17) is very close to the results of the SCA algorithm applied to the non-convex problem in (15).

6. CONCLUSIONS

In this paper we have proposed optimal strategies to select the sampling set and allocate radio parameters (powers/bits) over rate-constrained transmission channels, in order to meet an accuracy requirement on a graph signal interpolation task. The resulting non-convex optimization was solved using SCA techniques, having guaranteed convergence properties, or exploiting an alternative convex optimization criterion.

7. REFERENCES

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