PARAMETER ESTIMATION OF HEAVY-TAILED RANDOM WALK MODEL FROM INCOMPLETE DATA

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ABSTRACT

This paper proposes a novel and structured framework for parameter estimation from incomplete time series data under heavy-tailed random walk model. Traditionally, maximum likelihood estimation (MLE) for Gaussian random walk model from incomplete data has been considered. However, it is not applicable in many practical applications that follow some heavy-tailed random walk model. We first model a random walk model with Student-t residuals. Then we develop an MLE-based stochastic expectation maximization (EM) algorithm. The algorithm provides tractable E and M steps, which are easy to implement with simple updates and fast convergence. The simulation results illustrate the improved performance over the benchmarks.

***Index Terms*—** random walk, missing data, heavy-tailed, stochastic EM, Gibbs sampling, Student-t

1. INTRODUCTION

In the recent era of data deluge, many applications collect and process large amount of time series data for inference, learning, parameter estimation and decision making. Time series data play an important role in data analysis and its applications span almost all disciplines of science, engineering, and social science. Based on the applications various models are proposed for analyzing the time series data [1]. Of specific interest here is the famous random walk model. In practice many real world applications of time series data follow a random walk model. For example, the evolution of stock log-prices and traded volume in financial applications [2, 3]; gene association, genetic interactions [4], brain data [5, 6], and biological networks; movement of animals, micro-organisms, and cells [7].

In all such applications, issues with missing values frequently occur in the data observation or recording process. Various reasons that can lead to missing values are: values may not be measured, values may be measured but get lost, or values may be measured but are considered unusable as in the case of outliers[8]. Some real world cases are: some stocks may suffer a lack of liquidity resulting in no transaction and hence no price recorded, and observation devices like sensors breakdown during the measurement, weather or other conditions disturb sample taking schemes.

The expectation maximization (EM) type algorithms are state-of-art techniques for parameter estimation from data with missing values. In this direction, many variants of EM algorithms have been proposed to handle specific challenges of missing data. For example, the stochastic EM allows to tackle the problem posed by intractability of expected complete data log-likelihood. It has also been quite popular to curb the curse of dimensionality [9, 10], since its computation complexity is lower than the EM algorithm. On the other hand, regularized EM algorithms are used to enforce certain structures in parameter estimates like: sparsity, low-rank, and network structure [11, 12, 13, 10, 3].

Traditionally, the parameter estimation for time series models, for example, a random walk model or an autoregressive (AR) model, from data with missing values has been considered under Gaussian noise. However, many real-world data sets often follow heavy-tailed distributions. For example financial time series [14, 15], brain fMRI [5, 16], animal movement data [17], and black-swan events in animal population [18]. The estimation based on Gaussian model will provide unreliable estimates under such situations. Therefore, it is desirable to consider time series models under some heavy-tailed distribution, like the Student-t distribution. Unfortunately, the parameter estimation in such a case will become much more complicated. The objective of the current paper is to deal with this challenge and develop an efficient framework for parameter estimation from incomplete data under the heavy-tailed time series models.

In this paper, we first introduce the random walk model under the Student-t distribution, which is a commonly used heavy-tailed distribution. Then the MLE problem formulation given the incomplete data is presented. Finally, motivated by the usefulness of EM-type algorithm for time-series data, we propose a stochastic EM framework for the parameter estimation from incomplete data. The algorithm enjoys cheap iterates, fast convergence, and also provides reliable estimates. Here for ease of exposition and page limitation, we only illustrate the univariate random walk model, the idea is

This work was supported by the Hong Kong RGC 16208917 research grant.

very general and can be extended to the multivariate case and generalized innovation models like AR(p) model.

2. HEAVY-TAILED RANDOM WALK WITH MISSING DATA

Consider a univariate time series $Y_1, Y_2, \ldots, Y_T$ that follows a Student-t random walk model:

$$Y_t - Y_{t-1} \overset{i.i.d.}{\sim} t\left(\mu, \sigma^2, \nu\right), \tag{1}$$

where $\nu > 0$. From (1), the pdf of Y_t given μ, σ^2, ν and $Y_{t-1} = y_{t-1}$ is

$$\begin{aligned} &p\left(Y_t = y_t | \mu, \sigma^2, \nu, Y_{t-1} = y_{t-1}\right) \\ =&f_t\left(y_t; \mu + y_{t-1}, \sigma^2, \nu\right) \\ =&\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\sigma\Gamma\left(\frac{\nu}{2}\right)}\left(1 + \frac{\left(y_t - y_{t-1} - \mu\right)^2}{\nu\sigma^2}\right)^{-\frac{\nu+1}{2}}, \end{aligned} \tag{2}$$

where $f_t(\cdot)$ denotes the pdf of Student-t distribution.

In practice, certain sample y_t may be missing due to various reasons, then we denote $Y_t = \mathsf{NA}$ (not available). Suppose we have an observation of this time series with D missing blocks as follows:

$$\begin{aligned} &y_1, \ldots, y_{t_1}, \mathsf{NA}, \ldots, \mathsf{NA}, y_{t_1+n_1+1}, \ldots y_{t_d}, \mathsf{NA}, \ldots, \mathsf{NA}, \\ &y_{t_d+n_d+1}, \ldots, y_{t_D}, \mathsf{NA}, \ldots, \mathsf{NA}, y_{t_D+n_D+1}, \ldots, y_T, \end{aligned}$$

where, in the d-th missing block, there are n_d missing samples $y_{t_d+1}, \ldots, y_{t_d+n_d}$, which are bounded by two observed data y_{t_d} and $y_{t_d+n_d+1}$. We set for convenience $t_0 = 0$ and $n_0 = 0$. Let us denote the set of the indexes of the observed y_t's by C_{obs}, the set of the indexes of the missing y_t's by C_{mis}.

Ignoring the marginal distribution of y_1, the log-likelihood of the observed data $l\left(\{y_t\}_{t \in C_{obs}} | \mu, \sigma^2, \nu\right)$ is the log of the product of the pdf of every observed sample conditional on all the preceding observed data:

$$\begin{aligned} &l^{obs}\left(\{y_t\}_{t \in C_{obs}} | \mu, \sigma^2, \nu\right) \\ =&\log\left(\prod_{d=0}^{D} \prod_{t=t_d+n_d+2}^{t_{d+1}} f_t\left(y_t; \mu + y_{t-1}, \sigma^2, \nu\right)\right) \\ &+ \log\left(\prod_{d=1}^{D} \int\!\!\cdots\!\!\int \prod_{t=t_d+1}^{t_d+n_d+1} f_t\left(y_t; \mu + y_{t-1}, \sigma^2, \nu\right) \mathsf{d}y_{t_d+1} \cdots \mathsf{d}y_{t_d+n_d}\right) \end{aligned} \tag{3}$$

Then the MLE problem for μ, σ^2, and ν can be formulated as

$$\underset{\mu, \sigma^2, \nu > 0}{\mathsf{maximize}} \quad l^{obs}\left(\{y_t\}_{t \in C_{obs}} | \mu, \sigma^2, \nu\right) \quad . \tag{4}$$

The integral in (3) has no closed-form expression; thus, the objective function is very complicated and we cannot solve the optimization problem directly. In order to deal with this, we resort to the EM framework, which circumvents such difficulty by optimizing a sequence of simpler approximations of the original objective function instead.

3. STOCHASTIC EM FOR PARAMETER ESTIMATION

The EM algorithm is a very general iterative algorithm to solve MLE problems with missing data or latent variables when the optimization problem cannot be solved directly. The EM iteration alternates between an expectation (E) step, which computes the expectation of the complete data log-likelihood with respect to the posterior distribution of latent data given the observed data and the current estimates for the parameters, and a maximization (M) step, which computes the parameters maximizing the expected complete data log-likelihood. The key to the success of the EM algorithm is to consider some appropriate data as latent variables so that the expected complete data log-likelihood is easy to optimize.

Interestingly, the Student-t distribution can be regarded as a Gaussian mixture [19]. Since $Y_t - Y_{t-1} \sim t\left(\mu, \sigma^2, \nu\right)$, we have

$$Y_t - Y_{t-1} | \mu, \sigma^2, \tau_t \sim \mathcal{N}\left(\mu, \sigma^2/\tau_t\right), \tau_t \sim Gamma\left(\nu/2, \nu/2\right),$$

where the pdf of the above Gamma distribution is

$$f_g\left(\tau_t; \frac{\nu}{2}, \frac{\nu}{2}\right) = \frac{\left(\frac{\nu}{2}\right)^{\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)} \tau_t^{\frac{\nu}{2}-1} \exp\left(-\frac{\nu}{2}\tau_t\right). \tag{5}$$

Therefore, we apply the EM algorithm to the above optimization problem by regarding $\{\tau_t\}$ and $\{y_t\}_{t \in C_m}$ as latent variables. A detailed description of the (E)-step and (M)-step is given as follows.

E step

In this part, we need to compute expected complete data log-likelihood. The log-likelihood of the complete data $\{y_t\}$ and $\{\tau_t\}$ is

$$\begin{aligned} &l\left(\{y_t\}, \{\tau_t\} | \mu, \sigma^2, \nu\right) \\ =&\sum_{t=2}^{T}\left\{\log\left(f_N\left(y_t; \mu + y_{t-1}, \frac{\sigma^2}{\tau_t}\right) f_g\left(\tau_t; \frac{\nu}{2}, \frac{\nu}{2}\right)\right)\right\} \\ =&\sum_{t=2}^{T}\left\{-\frac{\tau_t}{2\sigma^2}\left(y_t - y_{t-1} - \mu\right)^2 - \log(\sigma) \right. \\ &\left. - \frac{\nu}{2}\tau_t + \frac{\nu}{2}\log\left(\frac{\nu}{2}\right) + \frac{\nu+1}{2}\log(\tau_t) - \log\left(\Gamma\left(\frac{\nu}{2}\right)\right)\right\} \end{aligned} \tag{6}$$

where $f_N(\cdot)$ is the pdf for the Gaussian distribution.

If there are no missing values in the time series $\{y_t\}$, then only $\{\tau_t\}$ are latent variables in (6). The posterior distribution of $\{\tau_t\}$ is simple. The corresponding expected complete log-likelihood has a closed-from expression and is easy to optimize.

However, when missing values occur in the time series, both $\{y_t\}_{t\in C_m}$ and $\{\tau_t\}$ are latent variable. There are more variables in the complete data log-likelihood. And there is even no closed-form expression for the pdf of the posterior distribution of $\{\tau_t\}$ and $\{y_t\}_{t\in C_{mis}}$. Therefore, it is very difficult to compute the expectation of complete data log-likelihood. To solve the unavailability of an exact expression for the expected complete data log-likelihood, some stochastic EM methods have been proposed to approximate the expected complete data log-likelihood [20, 21, 10]. Firstly, they draw realizations of latent variables from the posterior distribution. Then they approximate the expected complete data log-likelihood of the complete data log-likelihood of these realizations. Finally, they update the parameters as a combination of the current estimates and the maximizer of the approximated expected complete data log-likelihood.

Nevertheless, since the closed-form expression for the pdf of the posterior distribution of $\{\tau_t\}$ and $\{y_t\}_{t\in C_{mis}}$ is unavailable, it is hard to directly draw random samples from the posterior distribution. Therefore, we resort to Gibbs sampling, which, instead of drawing the all components of the latent variables jointly, draws realizations of each component sequentially based on its distribution conditional on all the other components. Lemmas 1 and 2 give the conditional distribution of each latent variable in our model.

Lemma 1. *Given the current estimates, the mixture weights $\{\tau_t\}$ and all the samples except y_t, the conditional distribution of Y_t for $t \in C_{mis}$ is*

$$\begin{aligned} & Y_t|\mu^{(k)}, \left(\sigma^{(k)}\right)^2, \nu^{(k)}, \mathbf{Y}_{-t}, \{\tau_t\} \\ \sim & \mathcal{N}\left(\frac{\tau_t\left(\mu^{(k)}+y_{t-1}\right)+\tau_{t+1}\left(y_{t+1}-\mu^{(k)}\right)}{\tau_t+\tau_{t+1}}, \frac{\left(\sigma^{(k)}\right)^2}{\tau_t+\tau_{t+1}}\right) \end{aligned} \tag{7}$$

where $\mathbf{Y}_{-t}$ is the set of all the samples except y_t.

Lemma 2. *Given the current estimates, all the samples, and all the mixture weights except τ_t, the conditional distribution of τ_t is*

$$\begin{aligned} & \tau_t|\mu^{(k)}, \left(\sigma^{(k)}\right)^2, \nu^{(k)}, \{y_t\}, \mathcal{T}_{-t} \\ \sim & \textit{ Gamma}\left(\frac{\nu^{(k)}+1}{2}, \frac{\left(\sigma^{(k)}\right)^{-2}\left(y_t-\mu^{(k)}-y_{t-1}\right)^2+\nu^{(k)}}{2}\right) \end{aligned} \tag{8}$$

where $\mathcal{T}_{-t}$ is the set of all the mixture weights except τ_t.

The conditional distributions are tractable; thus, with the help of Gibbs sampling, we can draw realizations of latent variables from its posterior distribution easily. As it is suggested in [21], when the maximization step is straightforward to implement from the computational point of view, one may use only one realization in each iteration. Here, at the iteration k, we only use one realization of $\left\{y_t^{(k)}\right\}_{t\in C_{mis}}$ and $\left\{\tau_t^{(k)}\right\}$ to approximate the expected complete data log-likelihood as in [10]. The resulting log-likelihood of the simulated complete data is

$$\begin{aligned} & l\left(\left\{y_t^{(k)}\right\}, \left\{\tau_t^{(k)}\right\}|\mu, \sigma^2, \nu\right) \\ = & \sum_{t=2}^{T}\left\{-\frac{\tau_t^{(k)}}{2\sigma^2}\left(y_t^{(k)}-\mu-y_{t-1}^{(k)}\right)^2-\log(\sigma)\right. \\ & \left. -\frac{\nu}{2}\tau_t^{(k)}+\frac{\nu}{2}\log\left(\frac{\nu}{2}\right)+\frac{\nu-1}{2}\log\left(\tau_t^{(k)}\right)-\log\left(\Gamma\left(\frac{\nu}{2}\right)\right)\right\} \end{aligned} \tag{9}$$

where $y_{t\in C_{obs}}^{(k)} = y_t$.

M step

First, we need to find the maximizer of the approximation of the expected complete data log-likelihood. The optimization of μ and σ^2 are decoupled with the optimization of ν in (9). Setting gradients with respect to μ and $1/\sigma^2$ equal to 0 gives

$$\sum_{t=2}^{T}\frac{\tau_t^{(k)}\left(y_t^{(k)}-\mu-y_{t-1}^{(k)}\right)}{\sigma^2}=0, \tag{10}$$

$$\sum_{t=2}^{T}\frac{\tau_t^{(k)}\left(y_t^{(k)}-\mu-y_{t-1}^{(k)}\right)^2-\sigma^2}{2}=0. \tag{11}$$

From them, we can get a closed-form maximizer of (9) as follows

$$\mu_1^{(k+1)}=\frac{\sum_{t=2}^{T}\tau_t^{(k)}\left(y_t^{(k)}-y_{t-1}^{(k)}\right)}{\sum_{t=2}^{T}\tau_t^{(k)}}, \tag{12}$$

$$\left(\sigma_1^{(k+1)}\right)^2=\frac{\sum_{t=2}^{T}\tau_t^{(k)}\left(y_t^{(k)}-\mu^{(k+1)}-y_{t-1}^{(k)}\right)^2}{T-1}. \tag{13}$$

The optimal $\nu_1^{(k+1)}$ can be found by a one-dimensional search. Next, we update the estimates of the parameters by a linear combination of the current estimate and the maximizer:

$$\boldsymbol{\theta}^{(k+1)}=\boldsymbol{\theta}^{(k)}+\gamma^{(k)}\left(\boldsymbol{\theta}_1^{(k+1)}-\boldsymbol{\theta}^{(k)}\right), \tag{14}$$

where $\gamma^{(k)}=\frac{1}{k}$, $\boldsymbol{\theta}^{(k)}=\left\{\mu^{(k)}, \left(\sigma^{(k)}\right)^2, \nu^{(k)}\right\}$, and $\boldsymbol{\theta}_1^{(k+1)}=\left\{\mu_1^{(k+1)}, \left(\sigma_1^{(k+1)}\right)^2, \nu_1^{(k+1)}\right\}$. The resulting stochastic algorithm is summarized in Algorithm 1.

Note that the log-likelihood of the incomplete data usually has more than one stationary points. Therefore, the algorithm may converge to a local maximum depending on the initial point. The estimation result and the convergence speed can be improved by choosing a proper initialization, and it can be obtained by having domain specific knowledge. This is also observed inour analysis, as detailed in next section.

Algorithm 1 Stochastic algorithm

1. Initialize $\mu^{(0)}$ and $\sigma^{(0)}$ as an arbitrary number, $\nu^{(0)}$ as an arbitrary positive number, and $k = 0$.
2. Draw one realization $\left\{y_t^{(k)}\right\}_{t \in C_{mis}}$ and $\left\{\tau_t^{(k)}\right\}$ via Gibbs sampling method:

- Generate some initial values for $\{y_t\}_{t \in C_{mis}}$ randomly.
- Sample $\left\{\tau_t^{(k)}\right\}$ and $\left\{y_t^{(k)}\right\}_{t \in C_m}$ according to Lemma 2 and Lemma 1.

3. Calculate the maximizer of the log-likelihood of the simulated complete dataset $\left\{y_t^{(k)}\right\}$and $\left\{\tau_t^{(k)}\right\}$and update parameter estimates according to (12) (13) and (14).
4. Return to step 2 or stop if convergence is determined.

4. SIMULATIONS

In this section, we present some numerical results of applying the proposed algorithm to estimate the parameters of Student-t random walk model from incomplete data. We also compare our approach with the traditional estimation approaches, MLE of Gaussian random walk model from incomplete data and MLE of Student-t random walk model by ignoring the missing values (only use the available differences between two adjacent samples). Note the Gaussian distribution is a special case of the Student-t distribution with $\nu = +\infty$. The estimation performance is quantified by the normalized square errors (NSEs) defined as $\mathsf{NSE}_\mu = \frac{|\hat{\mu}-\mu_{true}|}{|\mu_{true}|}$, $\mathsf{NSE}_{\sigma^2} = \frac{|\hat{\sigma}^2-\sigma^2_{true}|}{|\sigma^2_{true}|}$, and $\mathsf{NSE}_\nu = \frac{|\hat{\nu}-\nu_{true}|}{|\nu_{true}|}$. The stopping criteria for Algorithm 1 is $\frac{|\mu^{(k+1)}-\mu^{(k)}|}{|\mu^{(k)}|} < 10^{-5}$, $\frac{|(\sigma^{(k+1)})^2-(\sigma^{(k)})^2|}{(\sigma^{(k)})^2} < 10^{-5}$, and $\frac{|\nu^{(k+1)}-\nu^{(k)}|}{|\nu^{(k)}|} < 10^{-5}$.

First, we randomly generate a time series with $T = 200$ from a random walk model with $\mu_{true} = 1$, $\sigma^2_{true} = 0.5$, $\nu_{true} = 3$. Then we randomly delete 40 samples and get an incomplete data set. Finally, we apply above three different approaches to estimate μ, σ^2 and ν from this incomplete data set.

One interesting thing to note here is that, when we initialize our algorithm with the estimates from Gaussian random walk, the algorithm converges much faster and the final estimates are significantly improved, in comparison to random initialization. Figure 1 compares the estimation errors of parameters versus iterations using a random initial point and the Gaussian estimates as initial $\mu^{(0)}$ and $\sigma^{(0)}$, and a random $\nu^{(0)}$. We can see the algorithm converges in 100 iterations, where each iteration just needs one run of Gibbs sampling, and also the final estimation error is much smaller. This also testifies our argument that for heavy-tailed data the traditional methods for Gaussian distribution are too inefficient, and significant performance gain can be achieved by designing the algorithm under heavy-tailed model.

For a generalized performance analysis, we test our algorithms with different initializations, and compare the final estimation result with the estimation results of two existing methods. Figure 2 shows the final estimation results of three different approaches. Here we have tried 20 different random ν_0 's for Algorithm 1. In all cases, our estimates are most reliable since we not only consider heavy tail, but also make full use of the data.

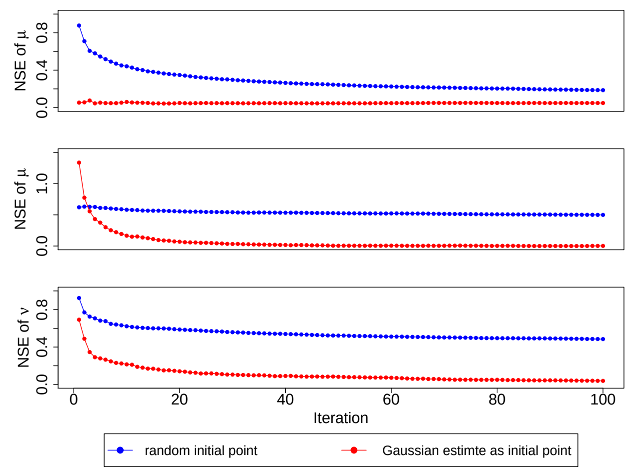

Fig. 1. Estimation errors of parameters versus iterations.

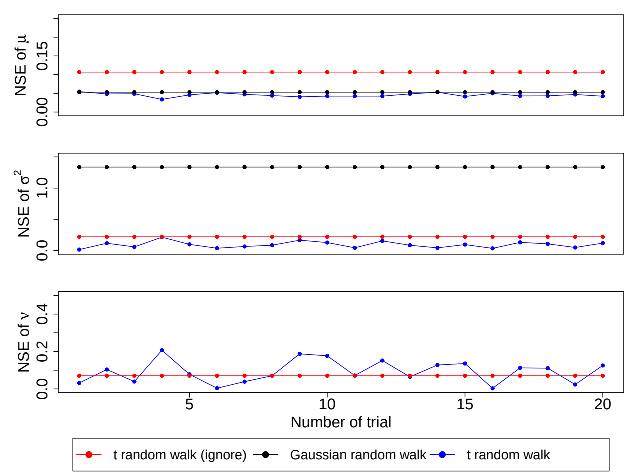

Fig. 2. Final estimation errors of three different estimation approaches.

5. CONCLUSION

In this paper, we have considered heavy-tailed random walk time series with missing values and developed an efficient algorithm to solve the parameter estimation problem. Numerical simulations show that the proposed algorithm can achieve a good estimation result with low computation cost.

6. REFERENCES

[1] H. Lütkepohl, *New introduction to multiple time series analysis*. New York: Springer Science & Business Media, 2005.

[2] E. F. Fama, "Random walks in stock market prices," *Financial analysts journal*, vol. 51, no. 1, pp. 75–80, 1995.

[3] R. Samci Karadeniz, "Modelling share prices as a random walk on a markov chain," Ph.D. dissertation, Department of Mathematics, 2017.

[4] K. C. Chipman and A. K. Singh, "Predicting genetic interactions with random walks on biological networks," *BMC bioinformatics*, vol. 10, no. 1, p. 17, 2009.

[5] D. Alexander, G. Barker, and S. Arridge, "Detection and modeling of non-gaussian apparent diffusion coefficient profiles in human brain data," *Magnetic Resonance in Medicine*, vol. 48, no. 2, pp. 331–340, 2002.

[6] B. J. He, J. M. Zempel, A. Z. Snyder, and M. E. Raichle, "The temporal structures and functional significance of scale-free brain activity," *Neuron*, vol. 66, no. 3, pp. 353–369, 2010.

[7] E. A. Codling, M. J. Plank, and S. Benhamou, "Random walk models in biology," *Journal of the Royal Society Interface*, vol. 5, no. 25, pp. 813–834, 2008.

[8] R. J. Little and D. B. Rubin, *Statistical Analysis with Missing Data*, 2nd ed. Hoboken, N.J.: John Wiley & Sons, 2002.

[9] S. F. Nielsen *et al.*, "The stochastic em algorithm: estimation and asymptotic results," *Bernoulli*, vol. 6, no. 3, pp. 457–489, 2000.

[10] G. DiCesare, "Imputation, estimation and missing data in finance," Ph.D. dissertation, University of Waterloo, 2006.

[11] M. J. Wainwright, "Structured regularizers for high-dimensional problems: Statistical and computational issues," *Annual Review of Statistics and Its Application*, vol. 1, pp. 233–253, 2014.

[12] X. Yi and C. Caramanis, "Regularized em algorithms: A unified framework and statistical guarantees," in *Advances in Neural Information Processing Systems*, 2015, pp. 1567–1575.

[13] G. McLachlan and T. Krishnan, *The EM algorithm and extensions*. New York: John Wiley & Sons, 2007, vol. 382.

[14] R. S. Tsay, *Analysis of financial time series*. John Wiley & Sons, 2005, vol. 543.

[15] S. T. Rachev, *Handbook of heavy tailed distributions in finance: Handbooks in finance*. Amsterdam, Netherlands: Elsevier, 2003.

[16] F. Han and H. Liu, "Eca: High dimensional elliptical component analysis in non-gaussian distributions," *Journal of the American Statistical Association*, no. just-accepted, 2016.

[17] D. W. Sims, E. J. Southall, N. E. Humphries, G. C. Hays, C. J. Bradshaw, J. W. Pitchford, A. James, M. Z. Ahmed, A. S. Brierley, M. A. Hindell *et al.*, "Scaling laws of marine predator search behaviour," *Nature*, vol. 451, no. 7182, pp. 1098–1102, 2008.

[18] S. C. Anderson, T. A. Branch, A. B. Cooper, and N. K. Dulvy, "Black-swan events in animal populations," *Proceedings of the National Academy of Sciences*, vol. 114, no. 12, pp. 3252–3257, 2017.

[19] C. Liu and D. B. Rubin, "ML estimation of the t-distribution using EM and its extensions, ECM and ECME," *Stat. Sin.*, vol. 5, no. 1, pp. 19–39, 1995.

[20] G. C. Wei and M. A. Tanner, "A monte carlo implementation of the em algorithm and the poor man's data augmentation algorithms," *Journal of the American statistical Association*, vol. 85, no. 411, pp. 699–704, 1990.

[21] B. Delyon, M. Lavielle, and E. Moulines, "Convergence of a stochastic approximation version of the em algorithm," *Annals of statistics*, vol. 27, no. 1, pp. 94–128, 1999.