BAYESIAN SPARSE SIGNAL DETECTION EXPLOITING LAPLACE PRIOR

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ABSTRACT

In this paper, we consider the problem of sparse signal detection with compressed measurements in a Bayesian framework. Multiple nodes in the network are assumed to observe sparse signals. Observations at each node are compressed via random projections and sent to a centralized fusion center. Motivated by the fact that reliable detection of the sparse signals does not require complete signal reconstruction, we propose two computationally efficient methods for constructing decision statistics for detection. First, using the Laplace prior directly to impose sparsity as widely considered in Bayesian Compressive Sensing (BCS), we develop an average likelihood ratio based detection method where the average is taken over the Laplace probability density function. Second, we exploit a three-stage hierarchical prior on the signal and construct decision statistics based on the noisy reconstruction (partial estimates) of the signals. Experimental results show that both average likelihood-based detection method and noisy-reconstruction based methods outperform most of the state-of-the-art algorithms.

Index Terms— Sparse signal detection, Bayesian compressive sensing, Laplace prior, multiple measurement vectors

1. INTRODUCTION

Detection of a sparse signal in the presence of noise from compressed measurements is one of the important inference tasks in signal processing and has many applications including sensor networks, cognitive radio networks, and radar networks. This problem has been investigated using the theories developed for sparse signal reconstruction [1–3]. These works have focused on deriving performance bounds on the probability of detection [1, 4–8], developing detection algorithms [2, 3, 6, 9–12], and design of measurement matrices [13–15].

Most of the available detection algorithms use a deterministic sparse signal model. In this work, we consider the Bayesian Compressive Sensing (BCS) framework. The Laplace density as a sparsity prior, as has been well discussed in the literature [16, 17], is used to model the sparse signals. Our goal is to develop robust detection algorithms with better detection performance and with less computational complexity compared than the existing works in the literature [3, 7]. In [7], the authors have modeled sparse signals as a random process and an approach to detect the sparse signals based on a single compressive measurement vector is proposed without reconstructing the signal. In this work, we consider a more general multiple measurement vectors (MMVs) based detection problem that employs signal model that is different from [7].

The sparsity prior of the signals is exploited in different ways to develop detection algorithms. First, considering the Laplace prior on the signals, we develop the average likelihood ratio test where the average is taken over Laplace probability density function (pdf). Though this method is computationally efficient, averaging the signal over the Laplace density may result in some loss in detection performance. Second, we propose two methods based on partial signal reconstruction to improve the detection performance compared to the first approach. In these methods, we use a three-stage hierarchical prior similar to [16]. The first two stages of the three-stage hierarchical prior result in the Laplace prior on the signal. Based on this framework, we propose a detection method where the energy of the partial estimate of signals obtained using Multitask BCS [17], is used to compute the decision statistics. Next, we aim to reduce the computational complexity by proposing a projection based detection method. In this approach, the detection decision is made based on the energy of the signal projected on the subspace spanned by the column vectors of measurement matrix indexed by the estimated support set. Experimental results show that likelihood-based detection method and projection-based method have better (or at least comparable) detection performance with less computational requirements when compared to the state-of-the-art methods.

2. PROBLEM FORMULATION

We consider a distributed network with P nodes that observe the sparse signals. The observation model at the p-th node under hypothesis $\mathcal{H}_1$, (the signal is present) and $\mathcal{H}_0$ (the signal is absent) is given by

$$\begin{aligned}\mathcal{H}_1 : \ \ \boldsymbol{z}_p &= \boldsymbol{x}_p + \boldsymbol{\eta}_p \\ \mathcal{H}_0 : \ \ \boldsymbol{z}_p &= \boldsymbol{\eta}_p\end{aligned} \tag{1}$$

for $p = 1, \cdots, P$. Each $\boldsymbol{x}_p$, for $p = 1, \cdots, P$, is an unknown sparse signal which we model as a random signal. To impose sparsity, a Laplace prior is assigned to the signal. Also, signals $\boldsymbol{x}_p$ for $p = 1, \cdots, P$ are assumed to be independent of each other. The noise vectors $\boldsymbol{\eta_p}$, $p = 1, \cdots, P$ are assumed to be Gaussian with $\boldsymbol{\eta_p} \sim \mathcal{N}(\mathbf{0}, \sigma_\eta^2 \mathbf{I}_N)$. All the nodes compress their observations and report the compressed measurements to a centralized fusion center (FC). The compressed observation matrix at the FC can be represented as

$$\boldsymbol{Y} = \Phi \boldsymbol{Z} + \boldsymbol{W} \tag{2}$$

where $\boldsymbol{\Phi}$ is an $M \times N (M < N)$ projection matrix, $\boldsymbol{Z} = [\boldsymbol{z}_1, \cdots, \boldsymbol{z}_P]$, and $\boldsymbol{W} = [\boldsymbol{w}_1, \cdots, \boldsymbol{w}_P]$. Noise vectors $\boldsymbol{w}_p$, $p = 1, \cdots, P$ are assumed to be i.i.d. Gaussian distributed with mean zero and covariance matrix $\sigma_w^2 \mathbf{I}_M$. The detection problem in (1) with compressed observation in (2) reduces to

$$\begin{aligned} \mathcal{H}_1 &: \boldsymbol{Y} = \Phi \boldsymbol{X} + \boldsymbol{N} \\ \mathcal{H}_0 &: \boldsymbol{Y} = \boldsymbol{N} \end{aligned} \tag{3}$$

where $\boldsymbol{X} = [\boldsymbol{x}_1, \cdots, \boldsymbol{x}_P], \boldsymbol{N} = [\boldsymbol{n}_1, \cdots, \boldsymbol{n}_P]$ and $\boldsymbol{n}_p = \boldsymbol{\Phi}\boldsymbol{\eta}_p + \boldsymbol{w}_p$. The rows of $\boldsymbol{\Phi}$ are assumed to be orthogonal. Hence, $\boldsymbol{n}_p$ is i.i.d. Gaussian with zero mean and covariance matrix $\sigma_0^2 \mathbf{I}_M$ where $\sigma_0^2 = \sigma_\eta^2 + \sigma_w^2$.

3. SPARSE SIGNAL DETECTION WITH LAPLACE PRIOR

In this section, we solve (3) without complete reconstruction of $\boldsymbol{X}$. We consider the Laplace pdf as the sparsity prior on the unknown signals in two different ways. First, by directly using an i.i.d. Laplace pdf, the average likelihood ratio based method is proposed. Second, two detection algorithms with noisy reconstruction (partial estimate) of the signals are proposed using a three-stage hierarchical prior on the signals [16].

3.1. Likelihood Ratio Based Detection (LR-MMV)

In this subsection, we develop the likelihood ratio based approach for the detection problem when MMVs are available at a centralized FC. With Laplace prior, we have $p(\boldsymbol{x}_p) = (\lambda/2)^N \exp(-\lambda \sum_{n=1}^N |x_{p,n}|)$ where λ is a scale parameter which is assumed to be unknown. Sparse signal reconstruction from this Laplace prior does not allow tractable Bayesian analysis [16]. Since we focus only on detection, complete signal reconstruction is not necessary. We define the likelihood ratio conditioned on the sparse signal matrix, $\boldsymbol{X}$, as, $L = \frac{p(\boldsymbol{Y}|\boldsymbol{X},\lambda,\mathcal{H}_1)}{p(\boldsymbol{Y}|\mathcal{H}_0)}$. We, then, average the signals using the prior pdf for the signal to get $\Lambda_{MMV}(\lambda)$, and finally optimize it over λ. Hence, the average likelihood ratio is given by

$$\Lambda_{MMV}(\lambda) = \int L\, p(\boldsymbol{X}|\lambda)\, d\boldsymbol{X} = \int \frac{p(\boldsymbol{Y}|\boldsymbol{X},\lambda,\mathcal{H}_1)p(\boldsymbol{X}|\lambda)d\boldsymbol{X}}{p(\boldsymbol{Y}|\mathcal{H}_0)}$$

$$= \int \frac{\beta(\lambda)\exp(-(\frac{\sum_{p=1}^P \|\boldsymbol{y}_p - \boldsymbol{\Phi}\boldsymbol{x}_p\|_2^2}{2\sigma_0^2} - \lambda \sum_{n=1}^N \sum_{p=1}^P |x_{p,n}|))}{\exp(-(\frac{\sum_{p=1}^P \|\boldsymbol{y}_p\|_2^2}{2\sigma_0^2}))} d\boldsymbol{X}$$

$$= \int \beta(\lambda)\exp(-\frac{\sum_{p=1}^P (\boldsymbol{x}_p^T \boldsymbol{\Phi}^T \boldsymbol{\Phi} \boldsymbol{x}_p - 2\boldsymbol{y}_p^T \boldsymbol{\Phi}\boldsymbol{x}_p + 2\sigma_0^2 \lambda \mathbf{1}^T |\boldsymbol{x}_p|)}{2\sigma_0^2}) d\boldsymbol{X},$$

where $\beta(\lambda) = (\lambda/2)^{NP}$, and $|\boldsymbol{x}_p| = [|x_{p,1}|, \cdots, |x_{p,N}|]^T$. When the elements of the $\boldsymbol{\Phi}$ matrix are random variables with zero mean and the rows of $\boldsymbol{\Phi}$ are orthogonal, we approximate $\boldsymbol{\Phi}^T\boldsymbol{\Phi} \approx \frac{M}{N}\mathbf{I}_N$. Now, $\Lambda_{MMV}(\lambda)$ can be written as

$$\Lambda_{MMV}(\lambda) = \beta(\lambda) \int \exp(-\frac{\sum_{p=1}^P (\frac{M}{N}\boldsymbol{x}_p^T \boldsymbol{x}_p - 2\boldsymbol{y}_p^T \boldsymbol{\Phi}\boldsymbol{x}_p + 2\sigma_0^2 \lambda \mathbf{1}^T |\boldsymbol{x}_p|)}{2\sigma_0^2}) d\boldsymbol{X} = \prod_{p=1}^P \Lambda_p, \tag{4}$$

where $\Lambda_p = (\lambda/2)^N \int d\boldsymbol{x}_p \exp(-\frac{\frac{M}{N}\boldsymbol{x}_p^T\boldsymbol{x}_p - 2\boldsymbol{v}_p^T\boldsymbol{x}_p + 2\sigma_0^2\lambda\mathbf{1}^T|\boldsymbol{x}_p|}{2\sigma_0^2})$ and $\boldsymbol{v}_p$ is defined as $\boldsymbol{y}_p^T\boldsymbol{\Phi} = \boldsymbol{v}_p^T$. Let $I_{p,n} = \int dx_{p,n} \exp(-\frac{\frac{M}{N}x_{p,n}^2 - 2v_{p,n}x_{p,n} + 2\sigma_0^2\lambda|x_{p,n}|}{2\sigma_0^2})$, for $p = 1, \cdots, P$, and $n = 1, \cdots, N$. Using some algebra, Λ_p can be represented as

$$\Lambda_p = (\lambda/2)^N I_{p,1} I_{p,2} \cdots I_{p,N}, \tag{5}$$

where $I_{p,n}$ is given by,

$$\begin{aligned} I_{p,n} &= \sqrt{2\pi\sigma_0^2} \exp\left(\frac{(v_{p,n} + \sigma_0^2\lambda)^2}{2\sigma_0^2 C}\right) Q\left(\frac{(v_{p,n} + \sigma_0^2\lambda)}{\sqrt{C}\sigma_0}\right) \\ &+ \sqrt{2\pi\sigma_0^2} \exp\left(\frac{(v_{p,n} - \sigma_0^2\lambda)^2}{2\sigma_0^2 C}\right)\left(1 - Q\left(\frac{(v_{p,n} - \sigma_0^2\lambda)}{\sqrt{C}\sigma_0}\right)\right) \end{aligned} \tag{6}$$

$C = \frac{M}{N}$, and $Q(z) = 1/(\sqrt{2\pi}) \int_{-\infty}^z exp(-t^2/2)dt$. Using Equations (4), (5) and (6), Λ_{MMV} can be expressed as

$$\Lambda_{MMV}(\lambda) = \beta(\lambda) \prod_{p=1}^P \prod_{n=1}^N I_{p,n}. \tag{7}$$

Next, the goal is to find $\hat{\lambda}$ that maximizes $\Lambda_{MMV}(\lambda)$, i.e., $\hat{\lambda} = \arg\max_{\lambda} \Lambda_{MMV}(\lambda)$. As we will observe in an example presented in Section 4, detection performance appears to be insensitive to the choice of λ. Therefore, we carry out performance analysis in Section 4 for a given value of λ.

Algorithm 1 Multi-task BCS based Sparse Signal Detection (LBCS-MT)

Inputs : $\boldsymbol{\Phi}$, $\boldsymbol{Y} = [\boldsymbol{y}_1, \cdots, \boldsymbol{y}_P]$
Outputs : Decision statistic Λ_{MT}, Detection Decision

Initialize $\zeta_j = 0$, for $j = 1, \cdots, N$, and $\lambda = 0$. Set $k = 0$.
While $k \leq R$
  Select a particular ζ_j^k out of $\zeta^k = [\zeta_1^k, \cdots, \zeta_N^k]$.
  If $A < 0$ and $\zeta_j^k = 0$, add ζ_j^k to the model.
  else if $A < 0$ and $\zeta_j^k > 0$ then find ζ_j^{k+1} using (10).
  else if $A > 0$, the prune ζ_j^k and set $\zeta_j^{k+1} = 0$
  end if
  Update $\boldsymbol{\mu}_p = \boldsymbol{\Sigma}_p \boldsymbol{\Phi}^T \boldsymbol{y}_p, \boldsymbol{\Sigma}_p = [\boldsymbol{\Phi}^T\boldsymbol{\Phi} + \boldsymbol{Z}]^{-1}$
  Update $\boldsymbol{s}_{p,j}, \boldsymbol{q}_{p,j}$, and $\boldsymbol{g}_{p,j}$
  Update λ as in Equations (9).
 $k = k + 1$
end While
Detection decision:
If $\Lambda_{MT} = \frac{1}{RP}\sum_{r=1}^R \sum_{p=1}^P \mu_{p,r}^2 \geq \theta, \boldsymbol{\mathcal{H}}_1$ is true, otherwise $\boldsymbol{\mathcal{H}}_0$ is true where θ is the threshold.

3.2. Partial Estimate based Detection

In this subsection, we exploit the three-stage hierarchical prior [17] to partially estimate the signals to construct decision statistics. The pdf of the p-th measurement vector, $\boldsymbol{y}_p$, under $\mathcal{H}_1$ is $p(\boldsymbol{y}_p|\boldsymbol{x}_p, \sigma_0^2) = \mathcal{N}(\boldsymbol{y}_p|\Phi\boldsymbol{x}_p, \sigma_0^2\mathbf{I}_M)$. The prior pdf of $\boldsymbol{x}_p$ is $p(\boldsymbol{x}_p|\boldsymbol{\zeta},\sigma_0^2) = \prod_{j=1}^{N} \mathcal{N}(x_{p,j}|0,\zeta_j\sigma_0^2)$. σ_0^2 and $\boldsymbol{\zeta}$ follows a Gamma distribution as, $p((\sigma_0^2)^{-1}|a,b) = \mathrm{G}((\sigma_0^2)^{-1}|a,b)$, $p(\zeta_j|\lambda) = \mathrm{G}(\zeta_j|1,\lambda/2) = \frac{\lambda}{2}\exp(-\frac{\lambda\zeta_j}{2})$, where $\mathrm{G}(\alpha|a,b) = \frac{b^a}{\Gamma(a)}\alpha^{a-1}\exp(-b\alpha)$ and $\Gamma(a)$ is the Gamma function. The parameter λ follow a Gamma distribution, i.e., $p(\lambda) = \Gamma(\lambda|0,0)$. The conditional pdf of $\boldsymbol{x}_p$ given $(\boldsymbol{\zeta},\lambda)$ is [17]

$$p(\boldsymbol{x}_p|\boldsymbol{\zeta},\lambda) = \frac{\Gamma(a+N/2)\left[1+\frac{1}{2b}(\boldsymbol{x}_p-\boldsymbol{\mu}_p)^T\boldsymbol{\Sigma}_p^{-1}(\boldsymbol{x}_p-\boldsymbol{\mu}_p)\right]}{\Gamma(a)(2\pi b)^{N/2}|\boldsymbol{\Sigma}_p|^{N/2}},$$

where $|.|$ is a determinant operator, $\boldsymbol{\mu}_p = \boldsymbol{\Sigma}_p\boldsymbol{\Phi}^T\boldsymbol{y}_p$, $\boldsymbol{\Sigma}_p = [\boldsymbol{\Phi}^T\boldsymbol{\Phi}+\boldsymbol{Z}]^{-1}$, and $\boldsymbol{Z} = \mathrm{diag}(1/\zeta_1,\cdots,1/\zeta_N)$. We estimate hyperparameters $(\boldsymbol{\zeta},\lambda)$ by maximizing the logarithm of $p(\boldsymbol{\zeta},\lambda,\boldsymbol{y}_1,\cdots,\boldsymbol{y}_P)$. Let $\mathcal{L}(\boldsymbol{\zeta},\lambda) \triangleq \log p(\boldsymbol{\zeta},\lambda,\boldsymbol{y}_1,\cdots,\boldsymbol{y}_P)$. From [17], $\mathcal{L}(\boldsymbol{\zeta},\lambda) = \mathcal{L}(\boldsymbol{\zeta})+\mathcal{L}'$, where $\mathcal{L}(\boldsymbol{\zeta})$ contains all the terms with $\boldsymbol{\zeta}$ from $\mathcal{L}(\boldsymbol{\zeta},\lambda)$ and $\mathcal{L}'$ contains all the remaining terms. Also,

$$\mathcal{L}(\boldsymbol{\zeta}) = \mathcal{L}(\boldsymbol{\zeta}_{\setminus j}) + l(\zeta_j), \tag{8}$$

where $\mathcal{L}(\boldsymbol{\zeta}_{\setminus j})$ is the total contribution of $\boldsymbol{\zeta}$ except ζ_j in $\mathcal{L}(\boldsymbol{\zeta})$ and $l(\zeta_j)$ is only due to ζ_j and is given by $l(\zeta_j)$= $\frac{-1}{2}\sum_{p=1}^{P}(M+2a)\log\left(1-\frac{\zeta_j q_{p,j}^2/g_{p,j}}{1+\zeta_j s_{p,j}}\right)+\log(1+\zeta_j s_{p,j})+\lambda\zeta_j$, where $s_{p,j} \triangleq \boldsymbol{\Phi}_j^T\boldsymbol{C}_{p,-j}^{-1}\boldsymbol{\Phi}_j$, $g_{p,j} \triangleq \boldsymbol{y}_p^T\boldsymbol{C}_{p,-j}^{-1}\boldsymbol{y}_p + 2b$, $q_{p,j} = \boldsymbol{\Phi}_j^T\boldsymbol{C}_{p,-j}^{-1}\boldsymbol{y}_p$, $\boldsymbol{C}_p = \boldsymbol{I}+\boldsymbol{\Phi}_p\boldsymbol{Z}^{-1}\boldsymbol{\Phi}_p^T, \boldsymbol{Z} = \mathrm{diag}(1/\zeta_1,\cdots,1/\zeta_N)$, and $\boldsymbol{C}_{p,-j}$ is $\boldsymbol{C}_p$ with contribution of $\boldsymbol{\Phi}_j$ removed. As in [17], the hyperparameters can be updated as

$$\lambda = \frac{N-1}{\sum_j \zeta_j}, \tag{9}$$

$$\zeta_j^{-1} \approx \begin{cases} \frac{-B\pm\sqrt{B^2-4AC}}{2A}, & A<0 \\ \infty, & \text{otherwise.} \end{cases} \tag{10}$$

where $A \triangleq \sum_{p=1}^{P}\frac{s_{p,j}+\lambda-(M+2a)q_{p,j}/g_{p,j}}{(s_{p,j}-q_{p,j}^2/g_{p,j})s_{p,j}}$, $B \triangleq \sum_{p=1}^{P}\frac{\lambda s_{p,j}+(s_{p,j}+\lambda)(s_{p,j}-q_{p,j}^2/g_{p,j})}{(s_{p,j}-q_{p,j}^2/g_{p,j})s_{p,j}}$, and $C \triangleq P\lambda$.

3.2.1. Detection of a Sparse Signal with Multitask Compressed Sensing with Laplace Priors (LBCS-MT)

In Algorithm 1, we present a method for the detection of sparse signals using the partial estimate of the signal. This method is called Multi-task BCS based sparse signal detection which is denoted as LBCS-MT. This is an adaptation of an algorithm from [17] for the detection problem. In Step 3 of the algorithm, we need to choose any ζ_j. Instead, we first compute ζ_j using (10) and choose ζ_j corresponding to the maximum value of $l(\zeta_j)$ among all $j = 1,\cdots,N$. Updates for $\boldsymbol{\Sigma}_p, \boldsymbol{\mu}_p, s_{p,j}, q_{p,j}$ and $g_{p,j}$ are evaluated using the relevance vector machine formulation as in [18]. The iterations are continued for a finite number of times namely R. The posterior mean, $\boldsymbol{\mu}_p$, is the estimate of the unknown signal, $\boldsymbol{x}_p$, at the p-th sensor and is a vector with R elements, i.e., $\boldsymbol{\mu}_p \in \mathbb{R}^R$. The average energy of the estimated signals over all the sensors is given by $\frac{1}{RP}\sum_{r=1}^{R}\sum_{p=1}^{P}\mu_{p,r}^2$ which is used as the detection statistic. The decision is made in favor of $\mathcal{H}_1$ if it is greater than the threshold (θ).

3.2.2. Decision Statistics Based on the Projection on the Estimated Support (BCSL-Prj)

Next, we aim to reduce the computational complexity of LBCS-MT. Here, we propose a method where we estimate the supports of all the signals in a single step.

Algorithm 2 Projection based sparse signal detection (BCSL-Prj)

Inputs : $\boldsymbol{\Phi}$, $\boldsymbol{Y} = [\boldsymbol{y}_1,\cdots,\boldsymbol{y}_P]$
Outputs : Decision statistic Λ_{prj}, Detection Decision

Solve for ζ_j such that $\frac{\partial \mathcal{L}(\boldsymbol{\zeta})}{\partial \zeta_j} = 0, \forall j$

Evaluate $l(\zeta_j)$, from Equation (8) $\forall \zeta_j$ from Step 2.

Arrange $l(\zeta_j)$ in descending order and choose K' indices j for the first K' largest $l(\zeta_j)$. Let $\hat{\mathcal{U}}$ be the set containing these indices

Detection decision:
If $\Lambda_{prj} = \sum_{p=1}^{P}\|\boldsymbol{\Omega}\boldsymbol{y}_p\|_2^2 \geq \theta, \boldsymbol{\mathcal{H}}_1$ is true, otherwise $\boldsymbol{\mathcal{H}}_0$ is true where θ is the threshold, where $\boldsymbol{\Omega}$ is defined in Equation (11) .

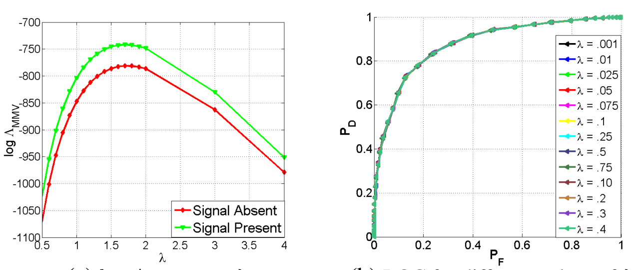
The proposed method is presented in Algorithm 2 which we refer to as BCSL-Prj. First ζ_j is estimated, assuming that all the elements of $\boldsymbol{\zeta}$, but ζ_j, are fixed, by solving $\frac{\partial \mathcal{L}(\zeta)}{\partial \zeta_j} = 0$. The value of ζ_j is given by (10). We evaluate the contribution of each ζ_j in the log likelihood, i.e., $l(\zeta_j)\ \forall \zeta_j$. We use the heuristics that the index corresponding to the largest increase in the log likelihood should be in the support set of the sparse signal. Let

$$\hat{\mathcal{U}} = \{j \mid l(\zeta_j) \text{ be one of the K' largest among all } l(\zeta_j), \forall j\}.$$

Let $\boldsymbol{\Phi}_{\hat{\mathcal{U}}}$ be a submatrix of $\boldsymbol{\Phi}$ which contains the columns of matrix $\boldsymbol{\Phi}$ indexed by $\hat{\mathcal{U}}$. Let $\boldsymbol{\Omega}$ be the orthogonal projection operator defined as

$$\boldsymbol{\Omega} = \boldsymbol{\Phi}_{\hat{\mathcal{U}}}(\boldsymbol{\Phi}_{\hat{\mathcal{U}}}^T\boldsymbol{\Phi}_{\hat{\mathcal{U}}})^{-1}\boldsymbol{\Phi}_{\hat{\mathcal{U}}}^T. \tag{11}$$

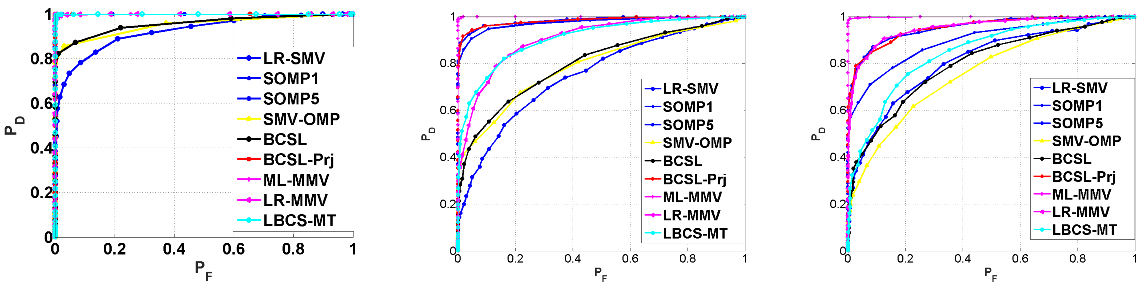
The total energy of the compressed measurements on the subspace spanned by the columns of $\boldsymbol{\Phi}$ indexed by $\hat{\mathcal{U}}$ is given by $\Lambda_{prj} = \sum_{p=1}^{P}\|\boldsymbol{\Omega}\boldsymbol{y}_p\|_2^2$ which, if is greater than the threshold, θ, the decision is made on the presence of the sparse signal. It



(a) log Λ_{MMV} vs λ (b) ROC for different values of λ

Fig. 1: Detection performance of Λ_{MMV} for different values of λ.

should be noted that LBCS-MT estimates sparse signals if we allow Algorithm 1 to converge. Instead, we run it for a finite number of iterations, R, which decreases the time complexity of the algorithm. On the other hand, BCSL-Prj runs only for a single iteration and estimates the signal. Hence, BCSL-Prj is computationally more efficient than LBCS-MT.



(a) $M/N \approx 0.49, \eta = 1.76$ dB (b) $M/N = 0.49, \eta = 13.98$ dB (c) $M/N = 0.1, \eta = 1.76$ dB

Fig. 2: Detection Performance of LR-SMV, SOMP1, SOMP5, LR-MMV, BCSL, LBCS-MT, BCSL-Prj, and ML-based methods when $N = 512$, $K = 5$ and, $R = 3$

4. NUMERICAL RESULTS

To illustrate the performance of the proposed algorithms, we consider different values of the compression ratios (M/N) and the total noise power, $\eta = 10\log_{10}(M\sigma_0^2)$ dB. We generate signals of dimension $N = 512$ with the sparsity index, $K = 5$. The sparse support set for all the signals is assumed to be the same and is selected from [1, N] uniformly. We generate the elements of the $M \times N$ measurement matrix $\mathbf{\Phi}$ and amplitudes of signals in the support set from a normal distribution with mean zero and unit variance.

The number of sensors considered in a centralized network is 4, i.e., P = 4. The single measurement vector (SMV) cases of LR-MMV and LBCS-MT are denoted as LR-SMV and BCSL, respectively. We run LR-MMV, LBCS-MT, BCSL-Prj, LR-SMV, and BCSL for 1000 Monte Carlo runs.

In the first experiment, we study the dependency of Λ_{MMV} on λ. Figure 1(a) shows Λ_{MMV} as a function of λ. It shows that the value of Λ_{MMV} when signals are present is always greater than the case when signals are absent. Because of this behavior, the detection performance with Λ_{MMV} as the decision statistic is almost the same for a wide range of λ as shown in Figure 1(b).

Table 1: Comparison of run times of LR-MMV, LBCS-MT, and BCSL-Prj in seconds to obtain the sparsity pattern when N=512, $K = 5$ and $R = 3$

Run times when $N = 512$ and $K = 5$					
$M/N \rightarrow$	0.60	0.70	0.80	0.90	1.00
LR-MMV	0.70	0.76	0.89	0.96	0.93
LBCS-MT	0.82	1.43	2.01	1.60	1.22
BCSL-Prj	0.17	2.37	2.92	2.24	3.01
SOMP5	17.26	16.25	20.25	20.86	23.19

In the second experiment, we study the detection performance of the proposed methods for different values of η and M/N. In Figure 2, we show the ROC curves for all the proposed methods. We use the detection performance of the maximum likelihood (ML) based detector, which assumes that the support set of the sparse signal is known, as a benchmark to compare the performance of the proposed algorithms. We also compare the proposed algorithms with the SOMP based method proposed in [3]. Figures 2(a) and 2(b) show the simulation results of all the proposed detection methods when η = 1.76 dB and η = 13.98 dB, respectively for $M/N \approx 0.49$. BCSL-Prj method outperforms the SOMP based method [3]. SOMP based methods assume a deterministic signal model. BCSL-Prj method consistently performs better than the SOMP1 algorithm and performs similarly as the SOMP5 algorithm. SOMP1 and SOMP5 represent the ROC curve of SOMP based detection algorithm when the decision statistic is constructed after 1 and 5 iterations, respectively. Figure 2(c) shows the detection performance when $\eta = 1.76$ dB and the number of compressed measurements is small, i.e., $M/N \approx 0.1$. The results show that LR-MMV outperforms all the other algorithms. This is because reconstruction based algorithms require a larger number of measurements to get a better estimate of the signal to provide a reliable detection performance.

Finally, we compare the time complexities of the proposed algorithms. Table 1 gives a summary of the run times required by LR-MMV, LBCS-MT, BCSL-Prj, SOMP1 and SOMP5 to make the detection decision for different values of M with N=512. The experiment is carried out in Matlab 2015b using processor Intel Xenon(R). The values in the table show the total times required by the algorithms in seconds to make the decision for 20 instances. The time needed for the proposed LR-MMV is the least and is followed by LBCS-MT and BCSL-Prj, respectively. SOMP5 has the worst time complexity. Thus, the proposed detection methods have either better detection performance or similar performance with better computational complexity compared to existing work.

5. CONCLUSION

In this paper, we have studied the problem of reliable detection of sparse signals in a distributed network with compressed measurements in a Bayesian framework. We proposed several detection methods namely LR-MMV, LBCS-MT, and BCSL-Prj. We showed that LR-MMV has better detection performance compared to the other proposed methods when the number of measurements is quite small. As the number of measurements increases, the BCSL-Prj algorithm performs better than the rest of the algorithms and the state-of-the-art algorithms. The proposed methods also provide an improvement in time complexity. In future work, we will derive bounds on the probability of error in signal detection.

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