# FAST PROJECTION-BASED SOLVERS FOR THE NON-CONVEX QUADRATICALLY CONSTRAINED FEASIBILITY PROBLEM

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## ABSTRACT

Quadratically constrained quadratic programming (QCQP) forms an important class of optimization tasks in various engineering disciplines. Fast identification of a feasible point under low computational complexity load is critical for several approximation techniques which have been developed to solve non-convex QCQPs. This paper introduces two projection-based techniques to compute feasible points of non-convex QCQPs with low computational complexity footprints: The first one employs successive projection mappings, while the second one builds on a composition of successive and averaged projection steps. Extensive experiments on synthetically generated instances of non-convex quadratically constrained feasibility problems demonstrate that the simple successive-projection based technique compares favorably against state-of-the-art feasible point pursuit methods which capitalize on successive convex approximation, parallel projections and computationally demanding interior-point techniques.

***Index Terms***— QCQP, non-convex, feasibility, projections.

## 1. INTRODUCTION

This paper studies the following feasibility problem:

$$\text{Find } \mathbf{x}_* \in \bigcap\nolimits_{k=1}^{K} C_k \neq \emptyset\,, \tag{P}$$

where

$$C_k := \left\{ \mathbf{x} \in \mathbb{R}^D \,\middle|\, \mathbf{x}^\top \mathbf{Q}_k \mathbf{x} - 2\mathbf{b}_k^\top \mathbf{x} - c_k \leq 0 \right\} \tag{1}$$

for a given symmetric $D \times D$ matrix $\mathbf{Q}_k$, $D \times 1$ vector $\mathbf{b}_k$, scalar $c_k$ and positive integers $D, K \in \mathbb{Z}_{>0}$. To keep the exposition simple, all quantities are considered to be real-valued ones. Since $\{\mathbf{Q}_k\}_{k=1}^K$ are only required to be symmetric, the quadratic constraints $\{C_k\}_{k=1}^K$ are in general non-convex; *cf.* Fig. 1. Notice that (1) accommodates also quadratic equality constraints, since $\mathbf{x}^\top \mathbf{Q}_k \mathbf{x} - 2\mathbf{b}_k^\top \mathbf{x} - c_k = 0$ can be viewed as $\mathbf{x}^\top \mathbf{Q}_k \mathbf{x} - 2\mathbf{b}_k^\top \mathbf{x} - c_k \leq 0$ and $-\mathbf{x}^\top \mathbf{Q}_k \mathbf{x} + 2\mathbf{b}_k^\top \mathbf{x} + c_k \leq 0$. Equality constraints will be explored in Sec. 3.

Task (P) is a special case of the QCQP problem: $\min_{\mathbf{x}} \mathbf{x}^\top \mathbf{Q}_0 \mathbf{x} - 2\mathbf{b}_0^\top \mathbf{x} - c_0 \leq 0$ such that (s.t.) $\mathbf{x} \in \cap_{k=1}^K C_k \neq \emptyset$, for given matrix $\mathbf{Q}_0$, vector $\mathbf{b}_0$ and scalar $c_0$. QCQPs are in general NP-hard, except for special cases, where, for example, all $\{\mathbf{Q}_k\}_{k=0}^K$ are positive semidefinite [6].

Non-convex QCQPs form an important class of optimization problems with applications which span from transmit beamforming in wireless networks [16] and portfolio risk management in financial engineering [10] to power system state estimation [22]. Several methods have been proposed to solve non-convex QCQPs such as **(i)** the (prevailing) semi-definite relaxation (SDR) approach, where matrix lifting together with rank relaxation are used to approximate the original problem by a convex semi-definite program [6]; **(ii)** the reformulation linearization technique (RLT) [1]; where redundant non-linear constraints and linearization steps are introduced to generate an approximate convex optimization problem; and **(iii)** successive convex approximation (SCA) [5, 17, 21, 23], where each SCA iterate is obtained by solving a convex optimization task, constructed locally around the previous SCA iterate.

Both RLT and SCA need a feasible point as initialization. Moreover, in the case where all $\{\mathbf{Q}_k\}_{k=1}^K$ are indefinite, SDR (with randomization) often fails to provide a feasible solution [18, Sec. I]. Identifying, thus, a feasible point is a critical step for success in SDR, RLT and SCA. To this end, feasible point pursuit (FPP)-SCA was proposed in [18] specifically for this task. FPP-SCA uses SCA with auxiliary slack variables to approximate the feasibility problem by a sequence of convex subproblems. The algorithm works with any choice of initialization, as the slack variables guarantee that each SCA subproblem is feasible at every step. Notwithstanding, FPP-SCA comes at the expense of increased computational complexity, since it requires solving a sequence of convex optimization problems via computationally demanding interior-point methods.

A consensus (C)ADMM algorithm, designed for general non-convex QCQPs, can be also used to solve (P) [9]. The per-iteration computational complexity of CADMM is much lower than that of FPP-SCA, but CADMM is memory intensive since it uses local copies of the global optimization variable (one for each constraint). To surmount the large memory footprint of CADMM, [13] reformulated the optimization criterion employed by FPP-SCA and applied computationally light first-order methods (FOMs) to solve (P). The work in [13] follows that of [12] where FOMs have been used on a special class of non-convex QCQPs. FOMs employed in [13] to solve (P) include the classical gradient descent (GD) method, its stochastic gradient descent (SGD) version, the popular stochastic variance reduced gradient (SVRG) approach [11, 24], and a custom-made stochastic subgradient descent (SSGD-)SCA method which solves a specific reformulation of (P) (*cf.* [13, Sec. IV.B]).

Following the success in solving convex feasibility problems [2], projection-based methods have been attracting recently considerable interest also for non-convex feasibility tasks [3, 4, 7, 8, 14, 15, 19, 20, 25]. Apart from the generic [7], [3, 4, 8, 14, 15, 19, 20, 25] are based on the classical method of alternating projections (MAP), where iterates are alternatingly projected onto two closed sets $C_1, C_2$, in a cyclic fashion, to generate a sequence that, under certain assumptions and a proper choice of the starting point, converges to a point in $C_1 \cap C_2$

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(more details in Sec. 2). Parallel projection methods operating on multiple sets $\{C_k\}_{k=1}^K$ also stem from MAP and appropriate product-space formulations [15, 25].

Motivated by the previous line of research, this paper explores projection-based solvers for the specific feasibility problem (P). Two algorithms are introduced: The first one is based on relaxed successive projections mappings (Alg. 1), while the second one combines averaging with successive projections (Alg. 2). Extensive tests on synthetically generated feasibility scenarios and initializations underline the rich potential of the advocated projection-based solvers. Having all eigen-decompositions of $\{\mathbf{Q}_k\}_{k=1}^K$ cached in memory, Alg. 1 compares favorably against all competing methods in computation time and in averaged feasibility success rate, even in cases where *both* inequality and equality constraints appear in (P), or, whenever the number of constraints exceeds the dimensionality $D$.

## 2. PROJECTION-BASED FEASIBILITY SOLVERS

Prior to introducing the algorithms, further details on projection-based solvers are provided to shed more light on the complications that (P) entails. However, notation comes first. Given a point $\mathbf{x} \in \mathbb{R}^D$ and the closed set $C_k$, the distance of $\mathbf{x}$ from $C_k$ is defined as $d(\mathbf{x}, C_k) := \inf_{\mathbf{y}\in C_k} \|\mathbf{x}-\mathbf{y}\|_2$. The projection operator $P_{C_k}$ onto $C_k$ is the set-valued mapping $P_{C_k} : \mathbb{R}^D \rightrightarrows C_k$ defined as $P_{C_k}(\mathbf{x}) := \{\mathbf{y} \in C_k \mid d(\mathbf{x}, C_k) = \|\mathbf{x}-\mathbf{y}\|_2\}$, for any $\mathbf{x} \in \mathbb{R}^D$. To avoid confusion, any element $\mathbf{y}_{\mathbf{x}}^{(k)}$ of $P_{C_k}(\mathbf{x})$ will be indexed by the subscript $\mathbf{x}$ and the superscript $(k)$ to uniquely identify its source $P_{C_k}(\mathbf{x})$. Specifically for (1) and any $\mathbf{x} \in \mathbb{R}^D$, a $\mathbf{y}_{\mathbf{x}}^{(k)}$ in $P_{C_k}(\mathbf{x})$ can be efficiently computed via the root of a univariate polynomial equation, formed by the eigenvalues of $\mathbf{Q}_k$ [9, Sec. III.C]. The classical bisection method can be used to identify the root [9, Sec. III.C].

The method of successive projections onto closed and not necessarily convex sets of a metric space was studied in [7]. Under a boundedness constraint on the intersection $\cap_{k=1}^K C_k$, and several other technical conditions, the main result of [7] states a dichotomy: Provided that the starting point $\mathbf{x}_0$ is properly chosen, the sequence $(\mathbf{x}_n)_{n\in\mathbb{Z}_{\geq 0}}$ generated by successive projections onto $\{C_k\}_{k=1}^K$ either converges to a point in $\cap_{k=1}^K C_k$, or the cluster points of $(\mathbf{x}_n)_{n\in\mathbb{Z}_{\geq 0}}$ form a non-trivial continuum in $\cap_{k=1}^K C_k$ [7, Thm. 4.3]. Following these lines, [4] constructs two sets $C_1, C_2$ where the set of all cluster points of the sequence $(\mathbf{x}_n)_{n\in\mathbb{Z}_{\geq 0}}$, generated by MAP, is indeed a non-trivial compact continuum [4, Thm. 3.1(v)].

Recently, the local convergence properties of MAP have been studied in [3, 8, 14, 15, 19]. Considering two closed sets $C_1, C_2$, and by imposing a transversality condition at points of the intersection $C_1 \cap C_2$, *i.e.*, a condition on how $C_1, C_2$ intersect at such points, it is shown that MAP, initialized near to points where $C_1, C_2$ meet transversally, converges linearly to a point in $C_1 \cap C_2$. Such a transversality condition holds true in the case of (P), since the specific sets $\{C_k\}_{k=1}^K$ are semi-algebraic, *i.e.*, for each $C_k$ there exists a finite number of $D$-dimensional polynomials $\{\varphi_{ij}^{(k)}, \psi_{ij}^{(k)}\}_{(i,j)\in\{1,\ldots,I\}\times\{1,\ldots,J\}}$, with $I, J \in \mathbb{Z}_{>0}$, s.t. $C_k = \cup_{i=1}^I \cap_{j=1}^J \{\mathbf{x} \in \mathbb{R}^D \mid \varphi_{ij}^{(k)}(\mathbf{x}) = 0, \psi_{ij}^{(k)}(\mathbf{x}) < 0\}$. Indeed, [8, Thm. 7.3] states that, provided that $C_1$ is bounded and the starting point $\mathbf{x}_0$ is properly chosen, MAP generates a sequence whose limit point belongs to $C_1 \cap C_2$. Similar arguments can be found in [19].

All of the previous discussion on MAP provides only with local convergence results. Seeking global behavior of the sequence of iterates of projection methods, the study of [25] introduced a parallel projections method (PPM) to solve (P) in the general case where

**Algorithm 1** Relaxed successive projections method (RSPM).

**Input:** The set of constraints $\{C_k\}_{k=1}^K$, an arbitrarily fixed starting point $\mathbf{x}_0$ and a coefficient $\xi \in (0, 2)$.

**Output:** Point $\mathbf{x}_*$ in $\cap_{k=1}^K C_k$.

1: $n = 0$.
2: **while** $\max_{k\in\{1,\ldots,K\}} d(\mathbf{x}_n, C_k) > 0$ **do**
3: Randomly reshuffle the order of constraints.
4: $\mathbf{z}_0 := \mathbf{x}_n$.
5: **for** $k = 0, 1, \ldots, K-1$ **do**
6: Choose $\mathbf{y}_k^{(k+1)} \in P_{C_{k+1}}(\mathbf{z}_k)$.
7: $\mathbf{z}_{k+1} := \xi\mathbf{y}_k^{(k+1)} + (1-\xi)\mathbf{z}_k$.
8: **end for**
9: $\mathbf{x}_{n+1} := \mathbf{z}_K$.
10: $n \leftarrow n + 1$.
11: **end while**
12: $\mathbf{x}_* := \mathbf{x}_n$.

$\{C_k\}_{k=1}^K$ are only required to be closed sets. PPM can be stated as follows: For an arbitrarily fixed starting point $\mathbf{x}_0$, compute the sequence $\mathbf{x}_{n+1} := (1/K)\sum_{k=1}^K \mathbf{y}_n^{(k)}$, where $\mathbf{y}_n^{(k)} \in P_{C_k}(\mathbf{x}_n)$ for any $n \in \mathbb{Z}_{\geq 0}$. If the feasibility condition $\cap_{k=1}^K \mathrm{Inc}(C_k) \neq \emptyset$ is satisfied, where

$$\mathrm{Inc}(C_k) := \bigcap_{\substack{\mathbf{x}\in\mathbb{R}^D,\\ \mathbf{y}_{\mathbf{x}}^{(k)}\in P_{C_k}(\mathbf{x})}} \{\mathbf{z} \in C_k \mid \langle \mathbf{x}-\mathbf{y}_{\mathbf{x}}^{(k)} \mid \mathbf{z}-\mathbf{y}_{\mathbf{x}}^{(k)}\rangle \leq 0\},$$

then it is guaranteed that PPM converges to a point which solves (P) [25]. Although the previous feasibility condition is shown to hold true in several cases of sparsity inducing non-convex constraints in [25], it does not suit the present QCQP case. A simple toy example which demonstrates that the previous feasibility condition is not necessary in the present context is as follows: Consider $\mathbb{R}^2$, with element $\mathbf{x} := (x_1, x_2) \in \mathbb{R}^2$, and define the non-convex constraints $C_1 := \{\mathbf{x} \mid -x_1^2 + x_2 - 1 \leq 0\}$, $C_2 := \{\mathbf{x} \mid -x_1^2 - x_2 - 1 \leq 0\}$. It is not hard to see that $C_1 \cap C_2 \neq \emptyset$, but $\mathrm{Inc}(C_1) \cap \mathrm{Inc}(C_2) = \emptyset$. Nevertheless, for any starting point $\mathbf{x}_0 \in \mathbb{R}^2$, MAP converges to a point in $C_1 \cap C_2$ in a *single* step.

This paper introduces the relaxed successive projections method (RSPM) in Alg. 1 to explore projection-based solvers for (P). To provide with a clear exposition, steps 4–9 of Alg. 1 are depicted in Fig. 1a. Notice that relaxation is viable here, by allowing $\xi\mathbf{y}_{\mathbf{x}}^{(k)} + (1-\xi)\mathbf{x}$ as an iterate update, where $\xi \in (0, 2)$ and $\mathbf{y}_{\mathbf{x}}^{(k)}$ is taken from $P_{C_k}(\mathbf{x})$. The range $(0, 2)$ of $\xi$ is motivated by relaxation techniques of projection mappings in the convex feasibility case [2]. Such a relaxation turns out to be beneficial in Sec. 3. To avoid potential "traps" due to the order by which constraints $\{C_k\}_{k=1}^K$ are employed, randomization of the constraints' order is introduced in step 3 of Alg. 1. It is worth noticing here that MAP becomes a special case of RSPM for $K = 2$, $\xi = 1$ and without step 3 of Alg. 1. To explore also parallel or averaged projections, the successive averaged projections method (SAPM), a hybrid method standing between RSPM and PPM of [25], is also introduced in Alg. 2, having its steps 4–9 depicted in Fig. 1b. Projections are combined in pairs before proceeding successively to the following pair of projections.

**Algorithm 2** Successive averaged projections method (SAPM).

**Input:** The set of non-convex quadratic constraints $\{C_k\}_{k=1}^{K}$ and an arbitrarily chosen starting point $\mathbf{x}_0$.

**Output:** Point $\mathbf{x}_*$ in $\cap_{k=1}^{K} C_k$.

1: $n = 0$.
2: **while** $\max_{k\in\{1,\ldots,K\}} d(\mathbf{x}_n, C_k) > 0$ **do**
3: Randomly reshuffle the order of constraints.
4: $\mathbf{z}_0 := \mathbf{x}_n$.
5: **for** $k = 1, 2, \ldots, K-1$ **do**
6: Choose $\mathbf{y}_{k-1}^{(k)} \in P_{C_k}(\mathbf{z}_{k-1})$ and $\mathbf{y}_{k-1}^{(k+1)} \in P_{C_{k+1}}(\mathbf{z}_{k-1})$.
7: $\mathbf{z}_k := \frac{1}{2}[\mathbf{y}_{k-1}^{(k)} + \mathbf{y}_{k-1}^{(k+1)}]$.
8: **end for**
9: $\mathbf{x}_{n+1} := \mathbf{z}_{K-1}$.
10: $n \leftarrow n + 1$.
11: **end while**
12: $\mathbf{x}_* := \mathbf{x}_n$.

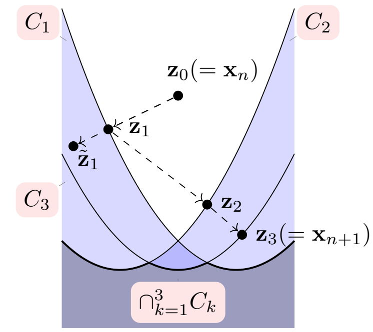
## 3. NUMERICAL TESTS

Algs. 1 and 2 are validated on synthetically generated feasibility scenarios. Competing methods are FPP-SCA [18], CADMM [9], the FOMs employed in [13], namely the classical GD and SGD, SVRG [11, 24] and SSGD-SCA [13, Sec. IV.B], as well as PPM [25]. All tests were conducted on a 32-CPU machine with Intel(R) Xeon(R) E7-4830@2.13GHz processors and 256GB of RAM. Performance is measured by the following metrics: **(i)** Computation time: The clock starts ticking once a method is initialized, and stops as soon as feasibility is achieved; and **(ii)** the average feasibility-success rate, *i.e.*, the number of cases where feasibility was achieved over the total number of employed settings. Feasibility is said to have been achieved if *all* of the constraints are satisfied. Regarding computational complexities, each constraint $C_k$ is visited once per iteration of RSPM (steps 4–9 in Alg. 1), and projections are computed by applying the bisection method to a certain uni-variate polynomial function, as in CADMM [9, Sec. III.C]. In the case of SAPM, there is a total number of $2K-2$ constraint visits per iteration (steps 4–9 in Alg. 2). Similarly to RSPM, projections in SAPM are computed via the bisection method.

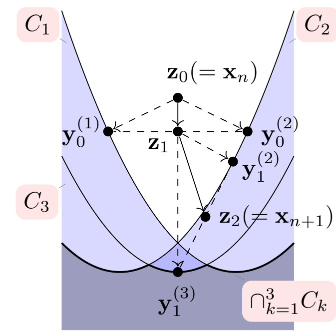
To generate a single test scenario, matrices $\{\mathbf{Q}_k\}_{k=1}^{K}$ are designed according to the following lines: **(i)** All entries of the $D \times D$ matrices $\{\mathbf{Q}'_k\}_{k=1}^{K}$ are drawn independently from the realizations of a normally distributed random variable, with mean 0 and variance 1; and **(ii)** $\mathbf{Q}_k$ is defined as the symmetric part of $\mathbf{Q}'_k$. All vectors $\{\mathbf{b}_k\}_{k=1}^{K}$ are set to be equal to zero for simplicity. To guarantee a non-empty intersection in (P), a point is chosen randomly from a ball of radius $\Delta \in \mathbb{R}_{>0}$, centered at the origin of $\mathbb{R}^D$, and scalars $\{c_k\}_{k=1}^{K}$ are defined s.t. the previously selected point satisfies all constraints $\{C_k\}_{k=1}^{K}$. Per such scenario, all methods are run for 10 times, each time with a randomly generated initial point, which is drawn randomly from a ball of radius $2\Delta$ centered at the origin of $\mathbb{R}^D$. A number of 10 scenarios are generated, and the average computation time of all realizations is recorded. A maximum number of iterations is also set for each method: If feasibility is not achieved for a method during the prescribed number of iterations, then the method is stopped and a "failure flag" is raised. For RSPM, SAPM and PPM, it is assumed that all eigen-decompositions of $\{\mathbf{Q}_k\}_{k=1}^{K}$ are cached in memory. This holds true also for CADMM.

Fig. 2 corresponds to the case where $D = 50$. The previous randomly generated scenarios are run under a variable number of inequality constraints: 20%, 50% and 90% of the dimensionality $D$ of the ambient space. Fig. 2 lists computation times and average feasibility-success rates (in %). As Fig. 2 demonstrates, RSPM scores the least computation time, for all possible configurations of the number of constraints, achieving at the same time perfect score in the average feasibility-success rate. The second best method is the classical GD, while SAPM follows. It is worth noticing that PPM fails to achieve feasibility within the prescribed maximum number of iterations. The same trend appears also in Fig. 3, where the dimensionality $D = 100$.

A number of 100 inequality constraints are considered for $D = 50$ in Fig. 4. Still, RSPM exhibits the best performance among competing methods. Once again, PPM fails to follow the tracks of the other two projection-based techniques and achieves zero average feasibility-success rate within the prescribed maximum number of iterations. It is worth noticing that in Figs. 2–4 the relaxation coefficient $\xi$ of Alg. 1 is set to be equal to 1.9, which outperformed the ($\xi = 1$)-value case in almost all tests (*cf.* Fig. 1a).




(a) Relaxed successive projections method (RSPM)




(b) Successive averaged projections method (SAPM)

**Fig. 1**. (a) Steps 4–9 of Alg. 1, under $\xi = 1$, in the case of three non-convex quadratic constraints, defined as the hypographs of three parabolas in the two-dimensional plane. Relaxation of the projection mapping is also demonstrated via the point $\tilde{\mathbf{z}}_1 := \xi\mathbf{y}_0^{(1)} + (1-\xi)\mathbf{z}_0$, where $\mathbf{y}_0^{(1)} := \mathbf{y}_{\mathbf{z}_0}^{(1)}$ and $\xi \in (0, 2)$. In this specific depiction, $\xi \in (1, 2)$, since values of $\xi$ larger than 1 promote "over-relaxation" and move the point farther than the original projection, having thus the potential to improve the convergence speed of the algorithm. Such an improvement was verified by the numerical tests in Sec. 3. (b) Steps 4–9 of Alg. 2.

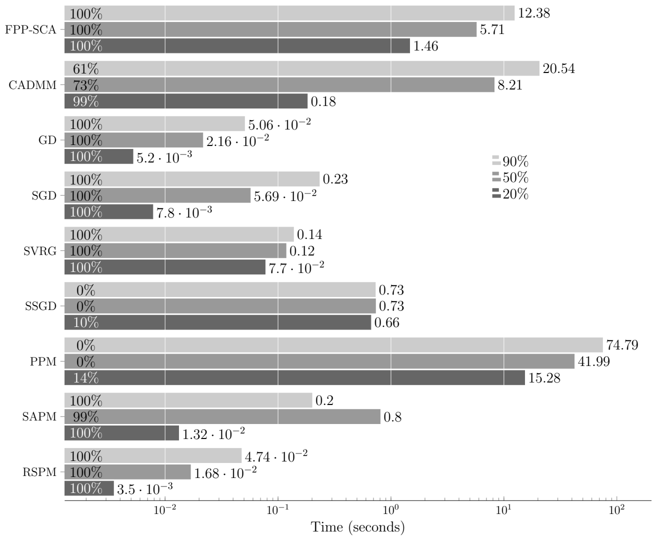



**Fig. 2**. Computation times and average feasibility-success rate for the case where the dimensionality $D$ of the ambient space is 50. The number of inequality constraints in (P) varies: 20%, 50% and 90% of the dimensionality $D$.

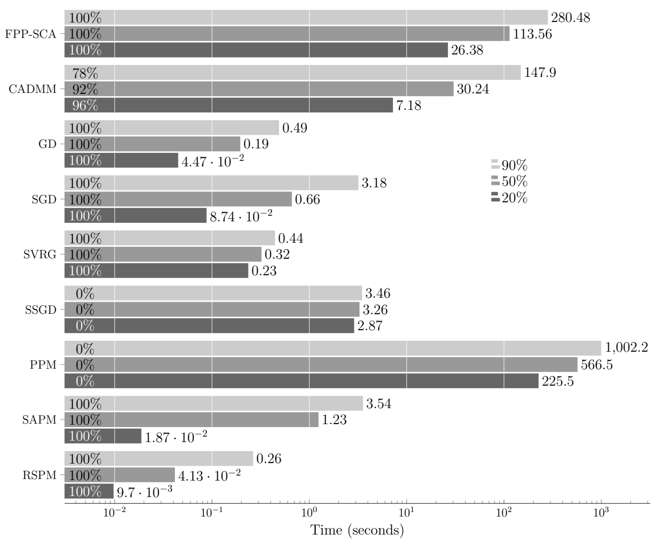



**Fig. 3**. Computation times and average feasibility-success rate for the case where the dimensionality $D$ of the ambient space is 100.

Finally, the dimensionality of the space is set again equal to 50, but a single equality constraint is also considered in Fig. 5. To avoid any numerical errors when checking for feasibility, the equality constraints are relaxed in the following sense: For a very small $\epsilon \in \mathbb{R}_{>0}$ (here, $\epsilon = 10^{-3}$), $\mathbf{x}^\top \mathbf{Q}_k \mathbf{x} - 2\mathbf{b}_k^\top \mathbf{x} - c_k \leq \epsilon$ and $-\mathbf{x}^\top \mathbf{Q}_k \mathbf{x} + 2\mathbf{b}_k^\top \mathbf{x} + c_k \leq \epsilon$ are considered. Moreover, as opposed to the previous tests, $\xi = 1$, *i.e.*, over-relaxation is not considered here. As Fig. 5 demonstrates, both RSPM and SAPM compare favorably against the rest of the competing methods.

## 4. CONCLUSIONS AND THE ROAD AHEAD

Following very recent advances on projection-based iterative techniques for non-convex feasibility problems, the present paper introduced two projection-based algorithms for computing feasible points of non-convex QCQPs. The first algorithm (RSPM; Alg. 1) builds on relaxed successive projection mappings, while the second one (SAPM; Alg. 2) combines successive projections with averaging. Both of the methods exhibit low computational complexity footprints, provided that eigen-decompositions of certain matrices are cached in memory. Extensive experiments on synthetically generated instances of non-convex quadratically constrained feasibility problems demonstrate that the simple successive-projection based technique compares favorably against state-of-the-art feasible point pursuit methods which capitalize on successive convex approximation, parallel projections and computationally demanding interior-point techniques.

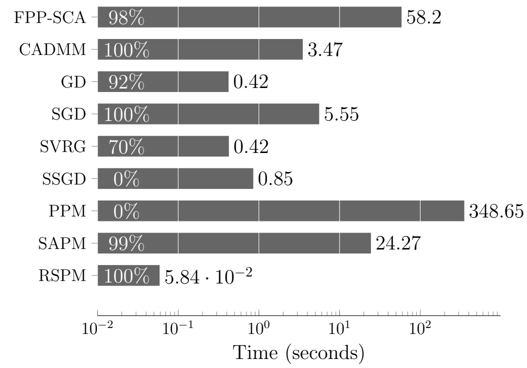



**Fig. 4**. $D = 50$ and the number of inequality constraints is set to $K = 100$ (200% of $D$).

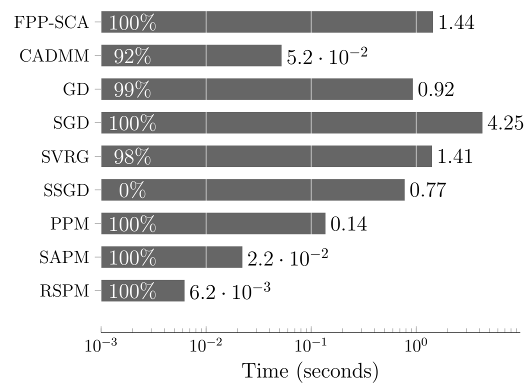



**Fig. 5**. $D = 50$. The number of inequality constraints is set to be equal to 8, while only one equality constraint is considered. In total, the number of constraints reaches 20% of $D$, since the equality constraint is recast as 2 inequality ones.

Open questions on RSPM and SAPM are abundant, revolving mainly around the performance analysis of the methods. In the case where $K = 2$, $\xi = 1$ and step 3 is not considered, the convergence analysis of RSPM boils down to that of MAP for semi-algebraic sets [3,4,7,8,14,15,19]. Partial theoretical results have been obtained for the general case of RSPM, based on the transversality arguments of [8, 14, 15, 19], but they are still premature to be included in this version of the manuscript.

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