

SMALL-SAMPLE-SUPPORT CHANNEL ESTIMATION FOR MASSIVE MIMO SYSTEMS

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ABSTRACT

We consider the problem of blind channel estimation with minimal pilot signaling in multi-cell multi-user MIMO systems with very large antenna arrays at the base station. We develop a least-squares (LS)-type algorithm that iteratively extracts channel and data estimates in short-data record multi-cell massive MIMO environments with no prior channel state information. The proposed algorithm utilizes a novel initialization step that is based on auxiliary-vector (AV) subspace decomposition. Simulation studies show that for pilot signaling of about 4%, information data extraction can be achieved with lower probability of error than eigendecomposition-based initialization techniques, while for observation records of sufficient length it nearly attains the error rate performance achieved with complete knowledge of the channels.

Index Terms— Massive MIMO, channel estimation, small-sample support

1. INTRODUCTION

Massive multiple-input multiple-output (MIMO) has emerged as a potential technology candidate for the development and deployment of next-generation spectrally and energy efficient wireless networks [1]. Under “favorable” propagation conditions, the channel vectors for different (intra-/inter-cell) users are considered to become asymptotically orthogonal as the number of antennas at the base station (BS) grows infinitely large. As a result, simple signal filtering and detection techniques such as matched filtering (MF) can be utilized to suppress multi-user interference under the assumption of accurate knowledge of the channels. In practice, however, perfect channel-state information (CSI) is not available.

In uplink time-division duplex (TDD) massive MIMO multi-cell systems, pilot signaling in the form of training sequences is used to acquire CSI within the channel coherence time, therefore, the number of training sequences is equal to the number of users. Assuming that the number of orthogonal pilot sequences is limited by the channel coherence

time interval, the same training sequences must be re-used in neighboring cells. Interference between pilot sequences of neighboring cells, also known as the pilot contamination effect [2], may lead to poor channel estimates, and thus affect significantly the achievable rate of massive MIMO systems that rely on linear signal processing [3].

Both blind and supervised channel estimation techniques [4], [5] have been considered in the literature as potential means to address pilot contamination and increase spectral efficiency. Under the assumption of sufficiently large sample support, blind channel estimation methods [6] that are based on subspace partitioning of the received samples can achieve near maximum-likelihood (ML) performance. Blind channel estimation schemes proposed in [7] and [8] utilize prior channel information and consider separable uplink signal and interference subspaces to apply principal component analysis (PCA) to the uplink data and estimate the channel vectors up to a scalar ambiguity. A semi-blind technique that offers unique channel estimates up to a unitary rotation in the interference-free case is proposed in [9]. The unitary ambiguity is then resolved with minimal pilot signaling. Finally, [10] offers a maximum a-posteriori (MAP) problem formulation and proposes a heuristic semi-blind estimation technique that is not robust in the case of inaccurate subspace information.

In this paper, we develop a light-weight least-squares (LS)-type algorithm that iteratively extracts channel and data estimates in multi-cell multi-user massive MIMO systems. We observe that arbitrary initialization of iterative least-squares (ILS) techniques may result in poor channel and data estimates in possibly dense multi-user massive MIMO systems. We propose to utilize a novel initialization scheme based on auxiliary-vector (AV) subspace decomposition [11–13]. The proposed algorithm does not require prior channel information and offers fast convergence under small records of data observations. Simulation studies show that the proposed channel estimation method demonstrates superior bit-error rate (BER) performance than pilot-based techniques and ILS methods based on eigendecomposition.

2. SYSTEM MODEL

We consider a cellular network with L cells, each containing an M -antenna base-station and K single-antenna users. We consider equal-power, simultaneous uplink symbol transmissions (drawn from a complex constellation \mathcal{A}) from all K users to their respective base-stations in each cell. The down-converted and pulse-matched filtered received signal vector at the l -th base-station is given by:

$$\mathbf{y}_l[n] = \sqrt{p_u} \sum_{i=1}^L \mathbf{H}_i \mathbf{x}_i[n] + \mathbf{n}_l[n] \in \mathbb{C}^{M \times 1} \quad (1)$$

where p_u denotes the average transmit power of each user at the l -th base-station in the i -th cell, $\mathbf{x}_i[n] \in \mathcal{A}^{K \times 1}$, $n = 1, \dots, N$ contains the n -th transmitted symbol by all K users in the i -th cell, and $\mathbf{H}_i = [\mathbf{h}_{i,1}, \dots, \mathbf{h}_{i,K}] \in \mathbb{C}^{M \times K}$ models the channel matrix between the l -th base-station and the K users in the i -th cell. Finally, $\mathbf{n}_l \in \mathbb{C}^{M \times 1}$ accounts for additive zero-mean and unit-variance white noise. The channel vector from the k -th user to the l -th base station at the i -th cell is given by:

$$\mathbf{h}_{i,k} = \mathbf{a}_{i,k} \sqrt{\beta_{i,k}} \in \mathbb{C}^{M \times 1} \quad (2)$$

where $\mathbf{a}_{i,k}$ are i.i.d. complex Gaussian, i.e. $\mathbf{a}_{i,k} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ describe the fast fading channel coefficients, and the scaling factor $\beta_{i,k}$ describes the quasi-static shadow fading and path-loss (slow fading). Consequently, the channel matrix \mathbf{H}_i can be written as:

$$\mathbf{H}_i = \mathbf{A}_i \mathbf{B}_i^{\frac{1}{2}} \quad (3)$$

where $\mathbf{A}_i = [\mathbf{a}_{i,1}, \dots, \mathbf{a}_{i,K}]$ is the $M \times K$ matrix of fast-fading coefficients, and $\mathbf{B}_i = \text{diag}(\beta_{i,1}, \dots, \beta_{i,K}) \in \mathbb{R}^{K \times K}$ is a diagonal matrix that contains the slow-fading coefficients.

We assume block flat-fading channels, where channel statistics remain constant over a certain coherence time interval T that is divided into T_{ul} time slots for up-link data transmissions, and $T_{tr} \geq K$ slots for transmission of up-link pilot/training sequences. The remaining time slots are used for down-link data transmission. Assuming sample support of $N \leq T_{ul}$ symbols, the received up-link signal is written as:

$$\mathbf{Y}_l = \sqrt{p_u} \sum_{i=1}^L \mathbf{H}_i \mathbf{X}_i + \mathbf{N}_l \in \mathbb{C}^{M \times N} \quad (4)$$

where $\mathbf{X}_i \in \mathcal{A}^{K \times N}$ is the up-link data transmitted to the l -th base station in the i -th cell, and \mathbf{N}_l is additive noise with zero-mean and unit-variance entries. For notational convenience we rewrite the received signal as:

$$\mathbf{Y}_l = \sqrt{p_u} \mathbf{H}_l \mathbf{X}_l + \sqrt{p_u} \tilde{\mathbf{H}} \tilde{\mathbf{X}} + \mathbf{N}_l \in \mathbb{C}^{M \times N} \quad (5)$$

where $\tilde{\mathbf{H}} = [\mathbf{H}_2, \dots, \mathbf{H}_L]$ and $\tilde{\mathbf{X}} = [\mathbf{X}_2, \dots, \mathbf{X}_L]$, denote the channel and uplink data matrices of the $L - 1$ interfering cells.

The autocorrelation matrix of \mathbf{Y}_l is given by:

$$\mathbf{R}_Y \triangleq \mathbb{E} \{ \mathbf{Y}_l \mathbf{Y}_l^H \} = p_u \sum_{i=1}^L \mathbf{A}_i \mathbf{B}_i \mathbf{A}_i^H + \mathbf{I}_M \quad (6)$$

where channel statistics are assumed to remain constant over a certain coherence time interval $T = T_{ul} + T_{tr}$. In practice, however, the autocorrelation matrix is unavailable. Instead \mathbf{R}_Y is sample-average estimated over $N \leq T_{ul}$ snapshots, say $\mathbf{y}_l[1], \dots, \mathbf{y}_l[N]$ by:

$$\hat{\mathbf{R}}_Y \triangleq \frac{1}{N} \sum_{n=1}^N \mathbf{y}_l[n] \mathbf{y}_l^H[n]. \quad (7)$$

3. CHANNEL ESTIMATION

We first consider the signal model in (5) where the channel matrix \mathbf{H}_1 for the cell of interest is unknown. Clearly, \mathbf{H}_1 can be estimated in a supervised fashion by utilizing a unit-norm training sequence of length N_{tr} for uplink pilot signaling in each cell. Let $\Psi_i \in \mathcal{A}^{K \times N_{tr}}$, $i = 1, \dots, L$ denote the training data matrix of the l -th base-station in the i -th cell. The received training signal is written as:

$$\mathbf{Y}_l^{tr} = \sqrt{p_{tr}} \mathbf{H}_1 \Psi_1 + \sqrt{p_{tr}} \tilde{\mathbf{H}} \tilde{\Psi} + \mathbf{N}_{tr} \in \mathbb{C}^{M \times N_{tr}} \quad (8)$$

where $p_{tr} = p_u N_{tr}$, and $\tilde{\Psi} = [\Psi_2, \dots, \Psi_L] \in \mathcal{A}^{K(L-1) \times N_{tr}}$ contains the training sequences that are used in the interfering cells. The least squares (LS) channel estimate for the cell of interest is given by:

$$\hat{\mathbf{H}}_1 = \arg \min_{\mathbf{H}_1} \left\| \frac{1}{\sqrt{p_{tr}}} \mathbf{Y}_l^{tr} - \mathbf{H}_1 \Psi_1 \right\|_{\mathbb{F}}^2 \quad (9)$$

where $\|\cdot\|_{\mathbb{F}}$ denotes the Frobenius norm. If we consider that the disturbance $\mathbf{I}_l^{tr} \triangleq \sqrt{p_{tr}} \tilde{\mathbf{H}} \tilde{\Psi} + \mathbf{N}_{tr}$ is complex Gaussian with i.i.d. zero-mean and unit-variance entries, then $\hat{\mathbf{H}}_1$ is the maximum-likelihood (ML) channel estimate.

The solution to (9) is given by:

$$\hat{\mathbf{H}}_1 = \frac{1}{\sqrt{p_{tr}}} \mathbf{Y}_l^{tr} \Psi_1^\dagger \quad (10)$$

where the superscript $(\cdot)^\dagger$ denotes the pseudo-inverse. Treating the above as the true channel estimate, the minimum-mean-squared error (MMSE) filter for the cell of interest takes the following form:

$$\mathbf{w}_{\text{MMSE},1} = \mathbf{R}_Y^{-1} \hat{\mathbf{H}}_1 \quad (11)$$

where \mathbf{R}_Y is assumed to be the true autocorrelation matrix. However, the columns of Ψ_1 are not necessarily orthogonal due to interference from other cells during the pilot/training phase. Therefore, channel estimates may be dominated by estimation errors (i.e. pilot contamination effect) [2, 14].

Iterative LS procedure

- 1) $d := 0$; initialize $\widehat{\mathbf{H}}_1^{(0)}$
 - 2) $d := d + 1$;

$$\widehat{\mathbf{X}}_1^{(d)} = \arg \min_{\mathbf{X}_1 \in \mathcal{A}^{K \times N}} \left\| \frac{1}{\sqrt{p_u}} \mathbf{Y}_l - \widehat{\mathbf{H}}_1^{(d-1)} \mathbf{X}_1 \right\|_F^2$$

$$\widehat{\mathbf{H}}_1^{(d)} = \frac{1}{\sqrt{p_u}} \mathbf{Y}_l \left(\widehat{\mathbf{X}}_1^{(d)} \right)^H \left[\left(\widehat{\mathbf{X}}_1^{(d)} \right) \left(\widehat{\mathbf{X}}_1^{(d)} \right)^H \right]^{-1}$$
 - 3) Repeat Step 2 until $\widehat{\mathbf{X}}_1^{(d)} = \widehat{\mathbf{X}}_1^{(d-1)}$
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Fig. 1. Pseudocode for the iterative least-squares algorithm.

Taking the above considerations into account, we begin by formulating the following joint data detection and channel estimation problem with the following LS-type solution:

$$\widehat{\mathbf{H}}_1, \widehat{\mathbf{X}}_1 = \arg \min_{\substack{\mathbf{X}_1 \in \mathcal{A}^{K \times N} \\ \mathbf{H}_1 \in \mathbb{C}^{M \times K}}} \left\| \frac{1}{\sqrt{p_u}} \mathbf{Y}_l - \mathbf{H}_1 \mathbf{X}_1 \right\|_F^2 \quad (12)$$

where the last two terms in (5) are considered as disturbance. Regretfully, the cost for calculating the optimal solution to (12) is unacceptable, therefore we attempt to reach to a quality approximation of the optimal solution by alternating LS-type estimates of $\widehat{\mathbf{H}}_1$ and $\widehat{\mathbf{X}}_1$ in an iterative fashion. The iterative procedure is depicted in Fig. 1.

Proper initialization affects greatly the convergence of the LS algorithm, while convergence to the optimal solution is not, in general, guaranteed. Although arbitrary initialization proves to work well, mainly in single user systems, it results in poor channel and data estimates in possibly dense multi-user massive MIMO systems. A common initialization strategy in non-linear optimization with mixed variables is to utilize the solution of the continuous problem as an estimate for the mixed problem. Therefore, a possible initialization point could be to select the K columns of $\widehat{\mathbf{H}}_1^{(0)}$ as the K eigenvectors corresponding to the K largest eigenvalues of the estimated autocorrelation matrix $\widehat{\mathbf{R}}_{\mathbf{Y}}$. However, in practice, autocorrelation matrix estimates are imperfect, since the matrix dimensions grow with the number of antennas and channel statistics change over time. Sample-average estimates of $\widehat{\mathbf{R}}_{\mathbf{Y}}$ are acquired over a finite number of data observations that should be at least the same order as the number of antennas at the BS. Consequently, eigendecomposition-based approaches may result in high-variance eigenvector estimates for cases that $\widehat{\mathbf{R}}_{\mathbf{Y}}$ is estimated from a short data record.

4. SMALL-SAMPLE SUPPORT ALGORITHM

We propose to use a non-eigenvector basis for the design of an iterative LS-type algorithm that exhibits lower computational complexity and superior BER performance in small-sample support environments from its eigendecomposition-

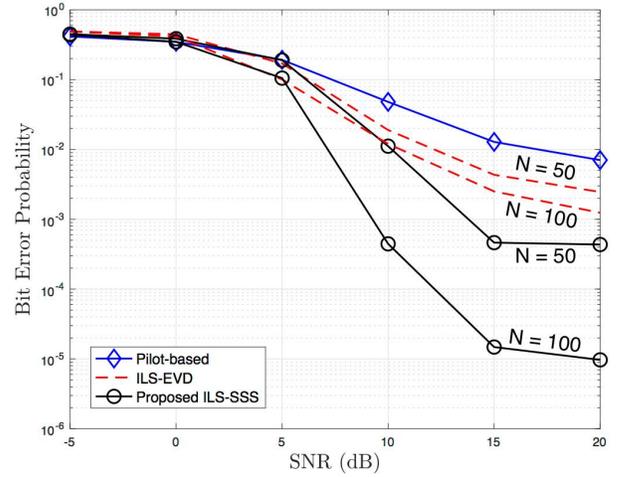


Fig. 2. Bit error probability versus SNR for BPSK modulation, $M = 100$ BS antennas, and $p_u = \text{SNR}/M$.

based counterpart. The proposed algorithm is motivated by current state-of-the-art in adaptive linear filtering theory [11, 12] and avoids any form of explicit or implicit autocorrelation matrix inversion, decomposition, or diagonalization. More specifically, we utilize the AV-type subspace decomposition to determine an effective initialization point for the iterative LS-type algorithm presented in Fig. 1. The AV algorithm utilizes sample-average estimated input data statistics, and provides a sequence of estimates of the ideal MMSE or minimum-variance-distortionless-response (MVDR) filter for the given signal processing design application [11, 12].

For any given autocorrelation matrix $\mathbf{R}_{\mathbf{Y}}$, the initial vector in our basis calculations, \mathbf{v}_0 is defined as:

$$\mathbf{v}_0 \triangleq \frac{\mathbf{R}_{\mathbf{Y}} \mathbf{h}_0}{\|\mathbf{R}_{\mathbf{Y}} \mathbf{h}_0\|} \quad (13)$$

where $\mathbf{h}_0 \in \mathbb{C}^{M \times 1}$ can be either chosen arbitrarily or obtained from minimal pilot signaling from (10).

After forming the first vector of the basis, we search for the “auxiliary” vector \mathbf{g}_1 that maximizes the magnitude of the statistical cross-correlation between $\mathbf{y}_l^H \mathbf{v}_0$ and $\mathbf{y}_l^H \mathbf{g}_1$ subject to an orthonormality constraint with respect to \mathbf{v}_0 :

$$\mathbf{g}_1 = \frac{(\mathbf{I}_M - \mathbf{v}_0 \mathbf{v}_0^H) \mathbf{R}_{\mathbf{Y}} \mathbf{v}_0}{\|(\mathbf{I}_M - \mathbf{v}_0 \mathbf{v}_0^H) \mathbf{R}_{\mathbf{Y}} \mathbf{v}_0\|} \quad (14)$$

The proposed basis set is then filled with the intermediate vector that is defined as:

$$\mathbf{w}_1 \triangleq \mathbf{v}_0 - \mu_1 \mathbf{g}_1 \quad (15)$$

where the scalar μ_1 minimizes the output variance of \mathbf{w}_1 pro-

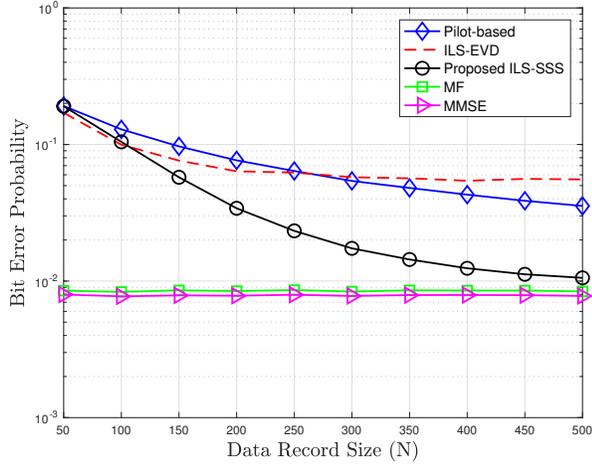


Fig. 3. Bit error probability versus data record size N for BPSK modulation, $M = 100$ BS antennas, and $\text{SNR} = 5$ dB.

cessed data $\mathbb{E} \left[\|\mathbf{w}_1^H \mathbf{Y}_l\|^2 \right]$, and is given by:

$$\mu_1 = \frac{\mathbf{g}_1^H \mathbf{R}_Y \mathbf{v}_0}{\mathbf{g}_1^H \mathbf{R}_Y \mathbf{g}_1} \quad (16)$$

For the rest of $K - 1$ users at the l -th base station, the basis is optimized recursively, for $n = 2, \dots, K$ as:

$$\mathbf{g}_n = \frac{\left(\mathbf{I}_M - \mathbf{v}_0 \mathbf{v}_0^H - \sum_{i=1}^{n-1} \mathbf{g}_i \mathbf{g}_i^H \right) \mathbf{R}_Y \mathbf{w}_{n-1}}{\left\| \left(\mathbf{I}_M - \mathbf{v}_0 \mathbf{v}_0^H - \sum_{i=1}^{n-1} \mathbf{g}_i \mathbf{g}_i^H \right) \mathbf{R}_Y \mathbf{w}_{n-1} \right\|}, \quad (17)$$

$$\mu_n = \frac{\mathbf{g}_n^H \mathbf{R}_Y \mathbf{w}_{n-1}}{\mathbf{g}_n^H \mathbf{R}_Y \mathbf{g}_n}, \quad (18)$$

$$\mathbf{w}_n = \mathbf{w}_{n-1} - \mu_n \mathbf{g}_n. \quad (19)$$

We note that:

$$\text{span} \{ \mathbf{v}_0, \mathbf{g}_1, \dots, \mathbf{g}_{K-1} \} = \text{span} \{ \mathbf{h}_{1,1}, \dots, \mathbf{h}_{1,K} \} \quad (20)$$

and the vectors $\{ \mathbf{v}_0, \mathbf{g}_1, \dots, \mathbf{g}_{K-1} \}$ form an orthonormal basis of the true signal subspace. Therefore, the iterative LS-type procedure in Fig. 1 can be initialized to:

$$\hat{\mathbf{H}}_1^{(0)} = [\mathbf{v}_0, \mathbf{g}_1, \dots, \mathbf{g}_{K-1}]. \quad (21)$$

5. SIMULATION STUDIES AND CONCLUSIONS

We consider a massive MIMO system with $L = 3$ cells and $K = 3$ single-antenna users per cell. We simulate the BER performance of BPSK transmissions for user 1 in cell of interest 1. The slow-fading coefficients for the cell of interest in our simulations are selected as $\mathbf{B}_1 = \text{diag}(0.98, 0.36, 0.47)$, $\mathbf{B}_2 = \text{diag}(0.36, 0.29, 0.05)$, and $\mathbf{B}_3 = \text{diag}(0.32, 0.14, 0.11)$. BER performance of the

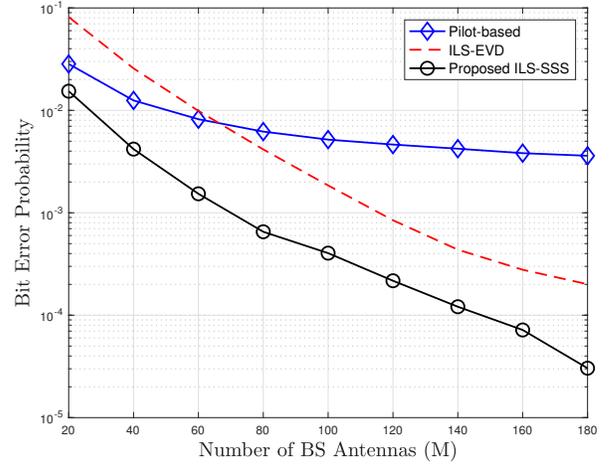


Fig. 4. Bit error probability versus number of BS antennas for $\text{SNR} = 20$ dB, data record $N = 50$, and $d_{\max} = 5$ iterations for the ILS procedure.

proposed iterative LS-type small-sample support (ILS-SSS) algorithm, that uses 4% of pilot signaling for initialization of \mathbf{v}_0 in (13), is compared to (i) an iterative LS-type method, that is initialized using eigendecomposition and uses a single training symbol for sign/phase ambiguity resolution, and (ii) a pilot-based method that uses 10% of pilot signaling.

Figure 2 evaluates the BER achieved by each channel estimation method as a function of the common signal-to-noise ratio (SNR) of all users in all cells. We consider $M = 100$ BS antennas and the average transmit power of each user is proportional to $1/M$. BER is evaluated for two data records of $N = 50$ and $N = 150$. We observe that the proposed algorithm outperforms both the iterative LS scheme based on eigendecomposition and the pilot-based method. As expected, the effectiveness of all methods increases when data record N increases. Figure 3 depicts the dependence of error probability on the size of data observations. The user's SNR is fixed at 5 dB and the number of BS antennas is $M = 100$. Interestingly, the proposed algorithm is the only one approaching the performance of MF and MMSE (that use the true \mathbf{H}_1 and \mathbf{R}_Y) for large sample support. Figure 4 demonstrates the BER performance of the proposed channel estimation algorithm for varying number of BS antennas. The SNR of all users is set at 20 dB and the maximum number for iterations in the ILS algorithm is fixed at $d_{\max} = 5$. The proposed algorithm significantly outperforms both eigendecomposition-based ILS and pilot-based methods as M increases.

In conclusion, in this work, we propose an iterative LS-type channel estimation algorithm for massive MIMO systems that utilizes a novel initialization step based on a non-eigenvector basis. The proposed algorithm demonstrates superior BER performance in small-sample support environments with about 4% of pilot signaling.

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