EM-BASED SEMI-BLIND MIMO-OFDM CHANNEL ESTIMATION

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ABSTRACT

This paper deals with semi-blind (SB) channel estimation of Multiple-Input Multiple-Output Orthogonal Frequency-Division Multiplexing (MIMO-OFDM) wireless communications system in the uplink transmission. Herein, we propose a new channel estimation approach using the well known EM technique. More precisely, we derive first the SB EM algorithm in the MIMO case. Then, a parallelizable version of this algorithm is introduced relying on the decomposition of the MIMO system into several MISO systems. Finally, we propose a reduced cost EM version where only the lattice points in the neighboring of the pilot-based detected symbols are considered.

Index Terms— MIMO-OFDM, MISO-OFDM, EM algorithm, Channel estimation, Cramèr Rao bound.

1. INTRODUCTION

With advances in technology, demand for high-speed Internet and mobile communications is becoming increasingly important. A Multiple-Input Multiple-Output Orthogonal Frequency-Division Multiplexing (MIMO-OFDM) wireless communications system provides many advantages as the channel capacity enhancement and the improvement of the communication reliability. However to achieve good performance, the receiver should have an accurate channel state information (CSI).

Many channel estimation approaches have been developed and can be divided into three main classes: blind [1],[2], pilot-based [3] and semi-blind methods [4]. For the latter class, it is shown in [5] that the SB approach improves the throughput by reducing the training sequences up to 95%. Furthermore, in [6], semi-blind approaches are used to reduce the transmitted power ('green communications').

The objective of this paper is to propose an efficient semi-blind channel estimation for MIMO-OFDM system based on the EM algorithm. Indeed, the latter is a widely used technique that has been already considered in the literature for the MIMO blind channel estimation. In particular, Our EM-based algorithm is distinct from the previous ones ([7], [8]) in term of the channel parameters to be estimated. Instead of estimating the subcarrier channel coefficients, we estimate directly the channel taps so one can obtain a significant gain as analyzed in [9].

Furthermore, we propose two EM-based algorithms. The first one considers the MIMO-OFDM block as one system to estimate one channel vector through the iterative process. The second one, important for the case of parallel processing machine, decomposes the MIMO-OFDM system into parallel MISO-OFDM systems to estimate the different vector channel taps independently for each receiver. In the case of underdetermined system (i.e. number of transmitters is greater than the receivers one), where the traditional methods could not estimate the transmit data, we succeed through the two proposed EM-based algorithms to estimate the channel taps and data properly.

Another originality of this work, consists of a simplified EM-based, denoted S-EM, method that allows to reduce the computational heaviness based on an initial estimation of the channel and the data using the pilots.

2. DATA MODEL

We consider a MIMO system composed of N_t transmit antennas and N_r receive antennas. The transmitted signal is assumed to be an OFDM one, composed of K samples (subcarriers) and L Cyclic Prefix (CP) samples. The CP length is assumed to be greater or equal to the maximum multipath channel delay denoted N (i.e. $N \leq L$). After removing the CP and taking the K-point DFT, the received signal at the k-th sub-carrier by the r-th receive antenna, denoted $y_r(k)$, is given by:

$$y_r\left(k\right) = \sum_{i=1}^{N_t} \sum_{n=0}^{N-1} h_{ri}(n) w_K^{nk} d_i(k) + v_r(k) \quad 0 \leq k \leq K-1, \tag{1}$$

where $d_i(k)$ is the transmitted data at subcarrier k by the i-th transmitter. The noise v is assumed to be additive independent white Circular Complex Gaussian satisfying $E\left[\mathbf{v}(k)\mathbf{v}(i)^H\right] = \sigma_{\mathbf{v}}^2 \mathbf{I}_K \delta_{ki}$; $(.)^H$ being the Hermitian operator; $\sigma_{\mathbf{v}}^2$ the noise variance; $\mathbf{I}_K$ the identity matrix of size $K \times K$ and δ_{ki} the Kronecker symbol. $h_{ri}(n)$ represents the n-th channel coefficient between the i-th transmitter and the r-th receiver. w_K^{nk} represents the (n,k)-th coefficient of the K-DFT point matrix.

Equation (1) can be rewritten in the following matrix form:

$$y_r(k) = \mathbf{w}^T(k)\mathbf{H}_r\mathbf{d}(k) + v_r(k), \tag{2}$$

where $\mathbf{d}(k) = [d_1(k), \cdots, d_{N_t}(k)]^T$,

and $\mathbf{w}(k) = \begin{bmatrix} 1 & w_K^k, \cdots, w_K^{(N-1)k}\end{bmatrix}^T$. $\mathbf{H}_r$ is given by

$$\mathbf{H}_r = \begin{bmatrix} h_{r1}(0) & \cdots & h_{rN_t}(0) \\ \vdots & \ddots & \vdots \\ h_{r1}(N-1) & \cdots & h_{rN_t}(N-1)\end{bmatrix}. \tag{3}$$

When considering all the outputs in a single vector $\mathbf{y}(k) = [y_1(k), \cdots, y_{N_r}(k)]^T$, the previous model can be written in the following compact form:

$$\mathbf{y}(k) = \boldsymbol{\mathcal{W}}(k)\boldsymbol{\mathcal{H}}\mathbf{d}(k) + \mathbf{v}(k), \tag{4}$$

where $\boldsymbol{\mathcal{W}}(k) = \mathbf{I}_{N_r} \otimes \mathbf{w}^T(k)$ ($\otimes$ being the Kronecker product) and $\boldsymbol{\mathcal{H}} = [\mathbf{H}_1^T, \cdots, \mathbf{H}_{N_r}^T]^T$

In the sequel the received OFDM symbols are assumed to be i.i.d and arranged according to the comb-type scheme with K_p sub-carriers dedicated to pilots corresponding (upon appropriate permutation) to $k = 0, \cdots, K_p - 1$ and K_d data sub-carriers. Also, we denote by D (respectively $|D|$) the finite set of all possible realizations of the data vector $\mathbf{d}$ (respectively its cardinal).

3. ML CHANNEL ESTIMATION

In the sequel, the unknown parameters are grouped in $\boldsymbol{\theta}$ containing the channel taps ($vec(\boldsymbol{\mathcal{H}})$ or $vec(\mathbf{H}_r)$) and the noise power ${\sigma_{\mathbf{v}}}^2$ (for simplicity, the signal power is assumed to be known).

3.1. EM algorithm

The EM algorithm looks for finding the ML estimate of the unknown parameters using the marginal likelihood of the observed data $\mathbf{y}$ by an iterative optimization process.

Considering $\mathbf{y}$ as the incomplete data and $\mathbf{d}$ as the missing data, the EM-algorithm is based on the two following steps:

- Expectation step (E-step):

$$Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{[i]}\right) = E_{\mathbf{d}|\mathbf{y},\boldsymbol{\theta}^{[i]}}\left[\log p\left(\mathbf{y}|\mathbf{d},\boldsymbol{\theta}\right)\right] \tag{5}$$

- Maximization step (M-step):

$$\boldsymbol{\theta}^{[i+1]} = \arg\max_{\boldsymbol{\theta}} Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{[i]}\right) \tag{6}$$

This process is proven in [10] to increase the ML value, i.e $p(\mathbf{y}|\mathbf{d},\boldsymbol{\theta})$, and hence leads to the algorithm's convergence to a local maximum point.

3.2. MIMO channel estimation

Under the data model assumption, the likelihood function is expressed by:

$$p(\mathbf{y};\boldsymbol{\theta}) = \prod_{k=0}^{K_p-1} p(\mathbf{y}(k);\boldsymbol{\theta}) \prod_{k=K_p}^{K-1} p(\mathbf{y}(k);\boldsymbol{\theta}), \tag{7}$$

where $p(\mathbf{y}(k);\boldsymbol{\theta}) \sim \mathcal{N}\left(\boldsymbol{\mathcal{W}}(k)\boldsymbol{\mathcal{H}}\mathbf{d}_p(k), \sigma_{\mathbf{v}}^2\mathbf{I}\right)$, for $k = 0, \cdots, K_p - 1$, $\mathbf{d}_p(k)$ being the pilot vector at the k-th sub-carrier, and

$$p(\mathbf{y}(k);\boldsymbol{\theta}) = \sum_{\xi=1}^{|D|} p(\mathbf{y}(k)|\mathbf{d}_\xi;\boldsymbol{\theta})\, p(\mathbf{d}_\xi), \tag{8}$$

with $p(\mathbf{y}(k)|\mathbf{d}_\xi;\boldsymbol{\theta}) \sim \mathcal{N}\left(\boldsymbol{\mathcal{W}}(k)\boldsymbol{\mathcal{H}}\mathbf{d}_\xi, \sigma_{\mathbf{v}}^2\mathbf{I}\right)$

3.2.1. E-step

After some straightforward derivations, $Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{[i]}\right)$ can be written as:

$$\begin{aligned} Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{[i]}\right) &= \sum_{k=0}^{K_p-1} \log p(\mathbf{y}(k)|\mathbf{d}_p(k);\boldsymbol{\theta}) \\ &+ \sum_{k=K_p}^{K-1}\sum_{\xi=1}^{|D|} \alpha_{k,\xi}\left(\boldsymbol{\theta}^{[i]}\right) \log p(\mathbf{y}(k)|\mathbf{d}_\xi;\boldsymbol{\theta}), \end{aligned} \tag{9}$$

where

$$\alpha_{k,\xi}\left(\boldsymbol{\theta}^{[i]}\right) = \frac{p\left(\mathbf{y}(k)|\mathbf{d}_\xi;\boldsymbol{\theta}^{[i]}\right) p(\mathbf{d}_\xi)}{\sum_{\xi'=1}^{|D|} p\left(\mathbf{y}(k)|\mathbf{d}_{\xi'};\boldsymbol{\theta}^{[i]}\right) p(\mathbf{d}_{\xi'})}. \tag{10}$$

In this paper, all the realizations $\mathbf{d}_\xi$ are equi-probable and hence the term $p(\mathbf{d}_\xi)$ can be ignored in equation (10).

3.2.2. M-step

The goal of the M-step is to find $\boldsymbol{\theta}$, i.e. the channel matrix $\boldsymbol{\mathcal{H}}$ and the noise power $\sigma_{\mathbf{v}}^2$ that maximizes the auxiliary function:

$$\boldsymbol{\theta}^{[i+1]} = \arg\max_{\boldsymbol{\theta}} Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{[i]}\right). \tag{11}$$

By setting to zero the derivative of $Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{[i]}\right)$ in (9) w.r.t. $\boldsymbol{\mathcal{H}}$, one obtains:

$$\begin{aligned} vec\left(\boldsymbol{\mathcal{H}}^{[i+1]}\right) &= \left[\sum_{k=0}^{K_p-1}\left(\mathbf{d}_p(k)^*\mathbf{d}_p(k)^T \otimes \boldsymbol{\mathcal{W}}(k)^H\boldsymbol{\mathcal{W}}(k)\right)\right. \\ &\left. + \sum_{k=K_p}^{K-1}\sum_{\xi=1}^{|D|}\alpha_{k,\xi}\left(\boldsymbol{\theta}^{[i]}\right)\left({\mathbf{d}_\xi}^*{\mathbf{d}_\xi}^T \otimes \boldsymbol{\mathcal{W}}(k)^H\boldsymbol{\mathcal{W}}(k)\right)\right]^{-1} \\ &\times \left[\sum_{k=0}^{K_p-1} vec\left(\boldsymbol{\mathcal{W}}(k)^H\mathbf{y}_p(k)\,\mathbf{d}_p(k)^H\right)\right. \\ &\left. + \sum_{k=K_p}^{K-1}\sum_{\xi=1}^{|D|}\alpha_{k,\xi}\left(\boldsymbol{\theta}^{[i]}\right) vec\left(\boldsymbol{\mathcal{W}}(k)^H\mathbf{y}(k)\,{\mathbf{d}_\xi}^H\right)\right]. \end{aligned} \tag{12}$$

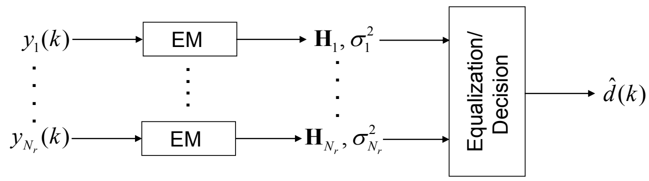

Fig. 1: MIMO-OFDM system model using N_r parallel MISO-OFDM systems.

Similarly, setting to zero the derivative of $Q\left(\boldsymbol{\theta}, \boldsymbol{\theta}^{[i]}\right)$ w.r.t. ${\sigma_{\mathbf{v}}}^2$ leads to:

$$\{{\sigma_{\mathbf{v}}}^2\}^{[i+1]} = \frac{1}{K}\left(\sum_{k=0}^{K_p-1}\left\|\mathbf{y}_p\left(k\right) - \boldsymbol{\mathcal{W}}(k)\boldsymbol{\mathcal{H}}^{[i+1]}\mathbf{d}_p(k)\right\|^2 + \sum_{k=K_p}^{K-1}\sum_{\xi=1}^{|D|}\alpha_{k,\xi}\left(\boldsymbol{\theta}^{[i]}\right)\left\|\mathbf{y}\left(k\right) - \boldsymbol{\mathcal{W}}(k)\boldsymbol{\mathcal{H}}^{[i+1]}\mathbf{d}_\xi\right\|^2\right). \tag{13}$$

The algorithm can be summarized as below:

Step 1 : Initialization $\boldsymbol{\theta}^{[0]} = \left[vec\left(\boldsymbol{\mathcal{H}}^{[0]}\right)^T, \{\sigma_{\mathbf{v}}^2\}^{[0]}\right]$ which represent the standard pilot-based channel and noise estimates;

Step 2 : Estimate $\boldsymbol{\mathcal{H}}^{[i+1]}$ using $\boldsymbol{\mathcal{H}}^{[i]}$ and $\{\sigma_{\mathbf{v}}^2\}^{[i]}$ according to equation (12);

Step 3 : Estimate $\{\sigma_{\mathbf{v}}^2\}^{[i+1]}$ using $\boldsymbol{\mathcal{H}}^{[i+1]}$, $\boldsymbol{\mathcal{H}}^{[i]}$ and $\{\sigma_{\mathbf{v}}^2\}^{[i]}$ according to equation (13);

Step 4 : repeat from step 1 using $\boldsymbol{\theta}^{[i+1]}$;

Remark: In the case of blind channel estimation, either in the MIMO or MISO approaches, one can ignore the pilot's terms in equations (12) and (13) and take into account only the data OFDM symbols.

In the sequel, the MIMO system is sub-divided into N_r parallel MISO systems, for which the EM is applied in a parallel scheme.

3.3. MISO channel estimation

Equations (2) and (3), allow the parallel decomposition of the MIMO system into N_r MISO-OFDM systems, as illustrated in Fig. 1. The estimation of the global parameters of the MIMO-OFDM system is done by concatenating the parameters of the N_r parallel MISO-OFDM system. The parameters of the r-th MISO-OFDM system are denoted as:

$$\boldsymbol{\theta}_r = \left[vec\left(\mathbf{H}_r\right)^T, \sigma_{\mathbf{v}_r}^2\right] \tag{14}$$

The computation of $\mathbf{H}_r$ and $\sigma_{\mathbf{v}_r}^2$, using the EM algorithm, leads to the same expressions as in the MIMO case given in the previous subsection where $\boldsymbol{\mathcal{H}}$ and $\boldsymbol{\mathcal{W}}$ are substituted by $\mathbf{H}_r$ and $\mathbf{w}$, respectively.

Parameters	Specifications
Number of pilot sub-carriers	$K_p = 8$
Number of data OFDM symbols	$N_d = 16$
Number of data sub-carriers	$K_d = 56$
Pilot signal power	$\sigma_p^2 = 13$ dBm
Data signal power	$\boldsymbol{\sigma}_{\mathbf{d}}^2 = 10$ dBm
Number of sub-carriers	$K = 64$

Table 1: Simulation parameters.

3.4. Simplified EM algorithm

The computational heaviness in equations (12) and (13) is due to the summation taking all the possible realizations of the data vector $\mathbf{d}$ (i.e. $|D|$). In this subsection we propose a simplifying method to reduce the summation set from $|D|$ (which growth exponentially with the number N_t) to another reduced summation set of size $|D'|$ proportional to N_t.

The proposed approach is summarized in Fig. 2, where we use the Decision Feedback Equalizer technique (DFE) to re-estimate the channel using the EM-based algorithm. The first step consists of estimating the data $\hat{\mathbf{d}}_d$ using only pilots to estimate $\hat{\mathbf{h}}_{op}$ followed by Zero Forcing Equalizer (ZF) and hard decision. Using $\hat{\mathbf{d}}_d$, the summation in equations (12) and (13) is done on a reduced size set $|D'|$ corresponding to the neighborhood of $\hat{\mathbf{d}}_d$ defined here as the points differing from $\hat{\mathbf{d}}_d$ by at most one entry.

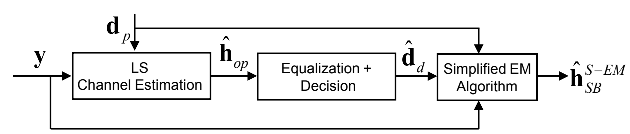

Fig. 2: Simplified EM algorithm.

4. SIMULATIONS RESULTS

Herein, we analyze the performance of the EM blind and semi-blind channel estimators in terms of the normalized Root Mean Square Error (NRMSE) for the two system configurations presented in this paper i.e MIMO system ($\mathbf{h}_{SB}^{EM}(MIMO)$ and $\mathbf{h}_B^{EM}(MIMO)$) and parallel MISO systems ($\mathbf{h}_{SB}^{EM}(MISO//)$ and $\mathbf{h}_B^{EM}(MISO//)$).

A (2×2) MIMO-OFDM communications system with BPSK data is considered. The IEEE 802.11n training sequence and channel model are used in this paper as pilots [11]. Simulation parameters are summarized in Table 1, where the used IEEE 802.11n channel model is of type B with path delay [0 10 20 30] μs and an average path gains of [0 -4 -8 -12] dB.

Fig. 3 provides a comparison between the two algorithms presented in this paper and the performance limits defined by the Cramèr Rao bound CRB, detailed in [5] versus the

SNR. The curves show clearly that both blind and semi-blind channel estimation algorithms give better results than the case where only pilots are used to estimate the channel, i.e the least square estimator ($\mathbf{h}_{OP}^{LS}$). Also, the semi-blind solutions show the best results (close to the CRB) with a slight advantage in favor of the MIMO version.

Fig. 4 represents the simulation results in the case of 4×2 underdetermined MIMO system, where we can see that, even in this configuration the EM-based algorithms perform well.

For a given $SNR = 10$ dB (around the operating mode of the IEEE 802.11n), Fig. 5 presents the behavior of EM algorithms when increasing the number of data OFDM symbols (N_d). The analysis of the curve confirms that when the number of data OFDM symbols increases, the performance of the EM algorithm in the blind and semi-blind approaches improves. One can see that in the MISO case one has taken advantage of the parallel computational architecture but at the cost of a reduced channel estimation quality as compared to the MIMO case.

In Fig. 6, we present the effect of the used approximation on the performance of the S-EM algorithm. We can see that the degradation is relatively minor for a computational gain that is significant.

5. CONCLUSION

This paper introduced the EM based blind and semi-blind channel identification in MIMO-OFDM wireless communications system. Through the simulation results, we have shown that the EM (blind or semi-blind) performs better than the pilot based channel estimation method. The decomposition of the MIMO system into N_r parallel MISO systems gives good results and is suitable for parallel machine processing allowing reduction of the EM execution time. The latter, is further reduced by a simplifying method employing a first estimation of the channel and transmitted data using the pilots.

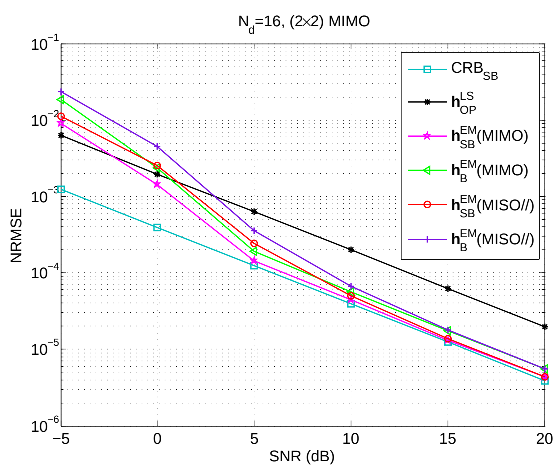

Fig. 3: NRMSE of the EM algorithms versus SNR .

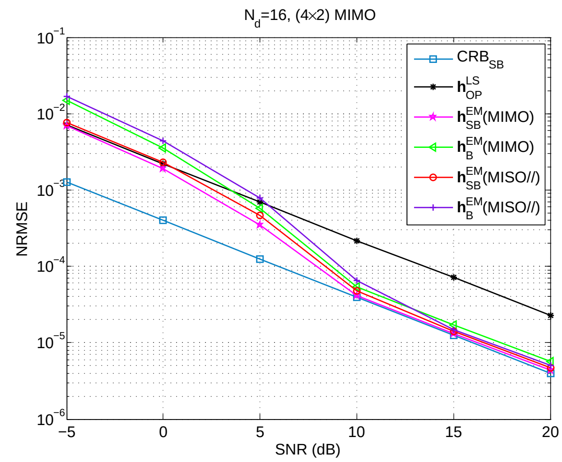

Fig. 4: NRMSE of the EM algorithms versus SNR in the underdetermined case ($N_t > N_r$).

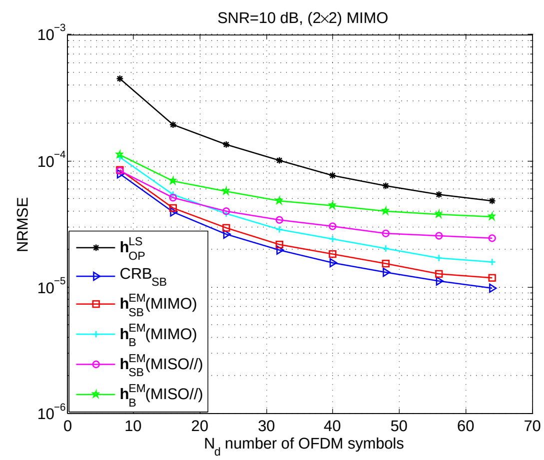

Fig. 5: NRMSE versus the number of OFDM symbols (N_d).

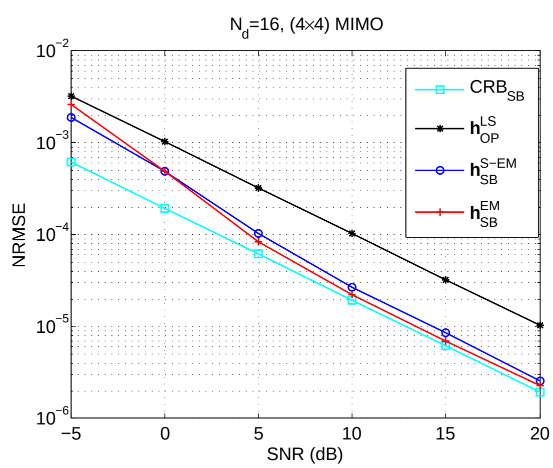

Fig. 6: S-EM-algorithm performance

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