PARALLEL BEAMFORMING DESIGN IN FULL DUPLEX SYSTEMS WITH PER-ANTENNA POWER CONSTRAINTS

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ABSTRACT

We investigate the max-min weighted downlink signal-to-interference ratio (SINR) problem under uplink SINR constraints and practical per-antenna constraints in full-duplex systems. The successive convex approximation (SCA) method is adopted to iteratively deal with this non-convex problem. Within each SCA iteration, to lower the complexity, a parallel beamforming algorithm based on alternating direction method of multipliers (ADMM) is proposed. Specifically, local variables are introduced to decompose the problem to multiple independent subproblems with closed-form solutions. Numerical results show that our proposed algorithm can achieve the similar performance with existing algorithms, but runs much faster especially in large-scale systems.

Index Terms— Full-duplex, ADMM, parallel algorithm

1. INTRODUCTION

With the widespread popularity of smart mobile phones and the emerging special-purpose sensors, the wireless data traffic is increasing unprecedentedly. Full-duplex (FD) has been regarded as a potential technology to double the network throughput, which is brought by the simultaneous information transmission and reception [1]. However, operating in FD mode also leads to severe self-interference (SI) from the transmit antennas to the receive antennas. Fortunately, recent advances in hardware and algorithm design for SI cancellation (SIC) have been able to make the residual SI to the background noise level [2]. Due to the practicality of FD, many research works have been done in FD networks.

In [3], the transmission power minimization problem is considered with downlink and uplink signal-to-interference ratio (SINR) constraints. Then, the transceiver design is optimized to maximize the sum of downlink rate and uplink rate [4, 5]. In these works, second-order algorithms (i.e., interior point method) are adopted and beamforming designs are all implemented in a centralized way. This would result in an unacceptable high complexity when the number of antennas or associated users becomes large. To tackle this issue, some distributed low-complexity algorithms need to be designed for large-scale networks [6, 7, 8, 9].

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In this paper, we propose a low-complexity parallel beamforming algorithm in FD systems, which is based on alternating direction method of multipliers (ADMM). To achieve a better trade-off between user requirement and user fairness, the minimum weighted downlink SINR is maximized subject to uplink SINR constraints and per-antenna constraints, which is more practical than the sum power constraint in [10]. We adopt the successive convex approximation (SCA) method to iteratively deal with this non-convex problem. Within each SCA iteration, a low-complexity ADMM-based parallel algorithm is proposed. In specific, some local variables are introduced as copies of coupled beamforming vectors, which makes the original coupled problem decomposable. Each of the decomposed subproblem has a closed-form solution and can be updated concurrently. Numerical results show that our proposed algorithm can scale well to large-size problem and runs much faster than state-of-the-art algorithms.

2. SYSTEM MODEL

We consider a FD wireless communication network, where a FD base station (FD-BS) simultaneously serves K_d downlink users and K_u uplink users. FD-BS is equipped with $N = N_t + N_r$ antennas with N_t antennas for downlink transmission and N_r antennas for uplink reception. Each user has a single antenna and works in the half-duplex mode. Besides, we assume that $N_t \geq K_u + K_d$, which is practical when massive antennas are employed at FD-BS. All channels are assumed to be frequency flat slow fading and remain constant within a time slot but vary from one to another.

Suppose that, $s_{d_m} \in \mathbb{C}$ with $E[|s_{d_m}|^2] = 1$ denotes the desired message for the m-th downlink user, and then the downlink signal transmitted by FD-BS is expressed as $\mathbf{x}_d = \sum_{m=1}^{K_d} \mathbf{w}_{d_m} s_{d_m}$, where $\mathbf{w}_{d_m} \in \mathbb{C}^{N_t \times 1}$ is the beamforming vector for downlink user U_{d_m}. Let $s_{u_n} \in \mathbb{C}$ with $E[|s_{u_n}|^2] =$ 1 represent the data symbol sent by the n-th uplink user and p_{u_n} denote its corresponding transmission power, the uplink signal transmitted by the n-th uplink user is $x_{u_n} = \sqrt{p_{u_n}} s_{u_n}$.

The received signal at downlink user U_{d_m} is given by

$$\begin{aligned} y_{d_m} =& \mathbf{h}_{d_m}^H \mathbf{w}_{d_m} s_{d_m} + \sum\nolimits_{i=1, i\neq m}^{K_d} \mathbf{h}_{d_m}^H \mathbf{w}_{d_i} s_{d_i} \\ &+ \sum\nolimits_{n=1}^{K_u} \sqrt{p_{u_n}} g_{d_{mn}} s_{u_n} + n_{d_m}, \end{aligned} \quad (1)$$

where $\mathbf{h}_{d_m} \in \mathbb{C}^{N_t \times 1}$ and $g_{d_{mn}} \in \mathbb{C}$ are channel coefficients from FD-BS to downlink user U_{d_m} and from uplink user U_{u_n} to downlink user U_{d_m}, respectively; n_{d_m} is the additive white Gaussian noise (AWGN) at user U_{d_m} with $n_{d_m} \sim \mathcal{CN}(0, \sigma_{d_m}^2)$, where $\sigma_{d_m}^2$ is the noise power. Note that the third term in this expression is the co-channel interference (C-CI). Hence, the received SINR at m-th downlink user is

$$\gamma_{d_m} = \frac{\left|\mathbf{h}_{d_m}^H \mathbf{w}_{d_m}\right|^2}{\sum_{i=1, i\neq m}^{K_d} \left|\mathbf{h}_{d_m}^H \mathbf{w}_{d_i}\right|^2 + \sum_{n=1}^{K_u} p_{u_n} \left|g_{d_{mn}}\right|^2 + \sigma_{d_m}^2}. \quad (2)$$

For the uplink, the received signal at FD-BS is

$$\mathbf{y}_u = \sum_{n=1}^{K_u} \sqrt{p_{u_n}} \mathbf{g}_{u_n} s_{u_n} + \sum_{m=1}^{K_d} \mathbf{H}_{SI} \mathbf{w}_{d_m} s_{d_m} + \mathbf{n}_u, \quad (3)$$

where $\mathbf{g}_{u_n} \in \mathbb{C}^{N_r \times 1}$ is channel vector from uplink user U_{u_n} to FD-BS, $\mathbf{n}_u \sim \mathcal{CN}(\mathbf{0}, \sigma_u^2 \mathbf{I}_{N_r})$ is the AWGN noise at user U_{u_n}. $\mathbf{H}_{SI} \in \mathbb{C}^{N_r \times N_t}$ is the residual SI channel from the transmit antennas to receive antennas at FD-BS, whose value depends on the capability of the SIC technique, and the second term in (3) is the residual SI after SIC.

The zero-forcing (ZF) receiver vector is adopted in this paper, since when the system is interference-limited [11] or the number of antennas is large [12], the ZF receiver can cancel the inter-user interference and approximately obtain the performance of optimal minimum square error (MMSE) receiver. That is, $\mathbf{w}_{u_n} = \left(\mathbf{v}_{u_n} \mathbf{G}_u^\dagger\right)^H$, where $\mathbf{v}_{u_n} \in \{0,1\}^{1 \times K_u}$ is a zero vector except that the n-th element is 1, $\mathbf{G}_u^\dagger = \left(\mathbf{G}_u^H \mathbf{G}_u\right)^{-1} \mathbf{G}_u^H$ and $\mathbf{G}_u = [\mathbf{g}_{u_1}, ..., \mathbf{g}_{u_{K_u}}]$. By applying ZF receiver, $\mathbf{w}_{u_n}$, the received SINR for uplink user U_{u_n} can be expressed as

$$\gamma_{u_n} = \frac{p_{u_n} \left|\mathbf{w}_{u_n}^H \mathbf{g}_{u_n}\right|^2}{\sum_{m=1}^{K_d} \left|\mathbf{h}_{SI_n} \mathbf{w}_{d_m}\right|^2 + \sigma_u^2 \left\|\mathbf{w}_{u_n}\right\|^2}, \quad (4)$$

where $\mathbf{h}_{SI_n} = \mathbf{H}_{SI}^H \mathbf{w}_{u_n}$. Clearly, the inter-user interference, $\sum_{i=1, i\neq n}^{K_u} \sqrt{p_{u_i}} \mathbf{g}_{u_i} s_{u_i}$, can be cancelled, and thus the residual SI becomes the dominant interference.

In this paper, we focus on the downlink beamforming design at FD-BS to maximize the minimum weighted downlink SINR subject to the uplink SINR constraints and the downlink per-antenna power constraints. The formulated problem can be casted as

$$\max_{\{\mathbf{w}_{d_m}\}} \quad \min_{m=1,...,K_d} \frac{\gamma_{d_m}}{\Gamma_{d_m}} \quad (5a)$$

$$\text{s. t.} \quad \gamma_{u_n} \geq \Gamma_{u_n}, \forall n, \quad (5b)$$

$$\sum_{m=1}^{K_d} \mathbf{w}_{d_m}^H \mathbf{R}_i \mathbf{w}_{d_m} \leq P_i^{BS}, i = 1, ..., N_t, \quad (5c)$$

where Γ_{d_m} and Γ_{u_n} are the desired SINRs of downlink user U_{d_m} and uplink user U_{u_n}, respectively; $\mathbf{R}_i \in \{0,1\}^{N_t \times N_t}$ is a zero matrix except that the i-th diagonal element is 1; P_i^{BS} is the allowed maximum per-antenna transmission power for the downlink. Different from the commonly seen sum power constraint of BS, we consider the per-antenna constraint (5c) here, owing to the fact that the per-antenna power is always limited in practical systems. With the reciprocal of each downlink user's desired SINR, $\frac{1}{\Gamma_{d_m}}$, as weighting factor, our considered max-min problem can achieve a better trade-off between the user requirement and the fairness.

3. PARALLEL BEAMFORMING DESIGN

Obviously, the formulated max-min fairness (MMF) problem is non-convex. An non-negative parameter, t, can be introduced to equivalently rewrite problem (5) as

$$\mathcal{T}: \quad \max_{\{\mathbf{w}_{d_m}\}, t} \quad t \quad (6a)$$

$$\text{s. t. } \gamma_{d_m} \geq t\Gamma_{d_m}, \forall m, \text{and (5b), (5c).} \quad (6b)$$

Define $\boldsymbol{\gamma} = [\Gamma_{d_1}, ..., \Gamma_{d_{K_d}}]$ as the desired downlink SINR vector and $\mathbf{p} = [P_1^{BS}, ..., P_{N_t}^{BS}]$ as the maximum power vector of downlink transmission, the optimal value of problem $\mathcal{T}$ can be regarded as $t^* = \mathcal{T}(\boldsymbol{\gamma}, \mathbf{p})$.

To effectively handle this kind of MMF problems, quality of service (QoS) dual problem is exploited in downlink scenario with per-antenna power constraints in [13]. In this paper, we further extend this dual theory to our FD scenario. The QoS dual problem here can be formulated as

$$\mathcal{R}: \quad \min_{\{\mathbf{w}_{d_m}\}, r} \quad r \quad (7a)$$

$$\text{s. t.} \quad \gamma_{d_m} \geq \Gamma_{d_m}, \forall m, \; \gamma_{u_n} \geq \Gamma_{u_n}, \forall n, \quad (7b)$$

$$\frac{1}{P_i^{BS}} \sum_{m=1}^{K_d} \mathbf{w}_{d_m}^H \mathbf{R}_i \mathbf{w}_{d_m} \leq r, i = 1, ..., N_t, \quad (7c)$$

where r is an introduced variable. Similar to problem $\mathcal{T}$, The value of problem $\mathcal{R}$ can also be seen as $r^* = \mathcal{R}(\boldsymbol{\gamma}, \mathbf{p})$.

Proposition 1: With $N_t \geq K_u + K_d$, the relations between problem $\mathcal{T}$ and problem $\mathcal{R}$ are

$$1 = \mathcal{R}(\mathcal{T}(\boldsymbol{\gamma}, \mathbf{p}) \cdot \boldsymbol{\gamma}, \mathbf{p}), \text{ and } t = \mathcal{T}(\boldsymbol{\gamma}, \mathcal{R}(t \cdot \boldsymbol{\gamma}, \mathbf{p}) \cdot \mathbf{p}). \quad (8)$$

The proof is similar to [13] and thus omits here. According to **proposition** 1, we can deal with problem $\mathcal{T}$ by iteratively solving problem $\mathcal{R}$. In specific, with given t, problem $\mathcal{R}(t \cdot \boldsymbol{\gamma}, \mathbf{p})$ is first solved to obtain the achieved objective value r^*. Then, the optimal t can be found by the bisection method. If $r^* < 1$, t should be increased and otherwise be decreased.

In the following, we put our effort on solving problem $\mathcal{R}$. To tackle this non-convex problem, the SCA method is adopted [14]. Toward this end, we first reformulate downlink SINR constraints (7b) as

$$\Gamma_{d_m}\left(\sum_{i=1, i\neq m}^{K_d} \left|\mathbf{h}_{d_m}^H \mathbf{w}_{d_i}\right|^2 + \sum_{n=1}^{K_u} p_{u_n} |g_{d_{mn}}|^2 + \sigma_{d_m}^2\right) \leq \left|\mathbf{h}_{d_m}^H \mathbf{w}_{d_m}\right|^2, \forall m, \quad (9)$$

which is non-convex due to the right side. Fortunately, $\left|\mathbf{h}_{d_m}^H \mathbf{w}_{d_m}\right|^2$ is convex and can be approximated by its first-order Tayor expansion iteratively. Suppose that, at the $j+1$-th iteration, $\mathbf{w}_{d_m}^{(j)}$ is given and thus the approximated problem can be casted as

$$\min_{\{\mathbf{w}_{d_m}\}, r} \quad r \quad \text{s. t.} \quad (10a)$$

$$\Gamma_{d_m}\left(\sum_{i=1, i\neq m}^{K_d} \left|\mathbf{h}_{d_m}^H \mathbf{w}_{d_i}\right|^2 + \sum_{n=1}^{K_u} p_{u_n} \left|g_{d_{mn}}\right|^2 + \sigma_{d_m}^2\right)$$

$$\leq 2\mathrm{Re}\{(\mathbf{w}_{d_m}^{(j)})^H \mathbf{h}_{d_m}\mathbf{h}_{d_m}^H\mathbf{w}_{d_m}\} - |\mathbf{h}_{d_m}^H\mathbf{w}_{d_m}^{(j)}|^2, \forall m, \quad (10b)$$

$$\Gamma_{u_n}\Big(\sum\nolimits_{m=1}^{K_d}|\mathbf{h}_{SI_n}\mathbf{w}_{d_m}|^2 + \sigma_u^2\|\mathbf{w}_{u_n}\|^2\Big)$$
$$\leq p_{u_n}|\mathbf{w}_{u_n}^H\mathbf{g}_{u_n}|^2, \forall n, \text{ and (7c)}, \quad (10c)$$

which is convex [15], and can be solved by CVX solver, such as SDPT3 [16]. However, the complexity of interior point method is very high when the number of antennas or users is large. To lower the complexity, inspired by [17], we propose a ADMM-based parallel algorithm to solve problem (10).

Observed that variable r is coupled in per-antenna power constraints (7c) and variables $\{\mathbf{w}_{d_m}\}$ are coupled in all constraints. To decouple these variables and parallelize problem (10), a set of local variables need to be introduced. That is,

$$a_{m,i} = \mathbf{h}_{d_m}^H\mathbf{w}_{d_i}, \forall m, i \in \{1, ..., K_d\}, \quad (11)$$

$$b_{n,m} = \mathbf{h}_{SI_n}^H\mathbf{w}_{d_m}, \forall n \in \{1, ..., K_u\}, \forall m \in \{1, ..., K_d\}, \quad (12)$$

$$\mathbf{v}_{d_m} = \mathbf{w}_{d_m}, \forall m \in \{1,...,K_d\}, \alpha_i^d = r, \forall i \in \{1, ..., N_T\}. \quad (13)$$

Instead of copying $\{\mathbf{w}_d\}$ for each SINR constraint, the introduced $\{a_{m,i}\}$ and $\{b_{n,m}\}$ can reduce the dimension of local variables. Then, problem (10) can be reformulated as

$$\min_{\mathbf{w}_d, r, \mathbf{a}, \mathbf{b}, \mathbf{v}_d, \boldsymbol{\alpha}^d} \quad r \quad \text{s. t.} \quad (14a)$$

$$\Gamma_{d_m}\Big(\sum\nolimits_{i=1, i\neq m}^{K_d}|a_{m,i}|^2 + \sum\nolimits_{n=1}^{K_u}p_{u_n}|g_{d_{mn}}|^2 + \sigma_{d_m}^2\Big)$$
$$\leq 2\mathrm{Re}\{(\mathbf{w}_{d_m}^{(j)})^H\mathbf{h}_{d_m}a_{m,m}\} - |\mathbf{h}_{d_m}^H\mathbf{w}_{d_m}^{(j)}|^2, \forall m, \quad (14b)$$

$$\Gamma_{u_n}\Big(\sum_{m=1}^{K_d}|b_{n,m}|^2 + \sigma_u^2\|\mathbf{w}_{u_n}\|^2\Big) \leq p_{u_n}|\mathbf{w}_{u_n}^H\mathbf{g}_{u_n}|^2, \forall n, \quad (14c)$$

$$\frac{1}{P_i^{BS}}\sum\nolimits_{m=1}^{K_d}\mathbf{v}_{d_m}^H\mathbf{R}_i\mathbf{v}_{d_m} \leq \alpha_i^d, i = 1, ..., N_t, \quad (14d)$$

$$(11) - (13), \quad (14e)$$

where $\mathbf{w}_d$ is the aggregated beamforming vector $\mathbf{w}_{d_m}$, other variables are defined in the same manner. By making (14b)-(14d) implicit in the objective, problem (14) is equivalent to

$$\min_{\mathbf{w}_d, r, \mathbf{a}, \mathbf{b}, \mathbf{v}_d, \boldsymbol{\alpha}^d} r + g_{c_1}(\mathbf{a}) + g_{c_2}(\mathbf{b}) + g_{c_3}(\mathbf{v}_d, \boldsymbol{\alpha}^d) \text{ s. t. (14e)} \quad (15)$$

where g_{c_i} is the indicator function of the feasible region for i-th constraint c_i in problem (14). For instance, if $\mathbf{a}$ is within the feasible region of (14b), i.e., c_1, $g_{c_1} = 0$, otherwise $g_{c_1} = \infty$. Note that the objective is separable across the global variable set $\{\mathbf{w}_d, r\}$ and the local variable set $\{\mathbf{a}, \mathbf{b}, \mathbf{v}_d, \boldsymbol{\alpha}^d\}$, and the constraints are linear equations. Thus, ADMM can be applied to update these two variable sets alternatively. Since problem (15) is convex and only two variable sets are optimized alternatively, the ADMM can converge to the optimal point [17].

Define $\boldsymbol{\lambda}^d, \boldsymbol{\lambda}^u, \boldsymbol{\mu}, \boldsymbol{\eta}^d$ respectively are the multipliers of constraints (11)-(13) and ρ is the penalty parameter. According to [17], the scaled form of augmented Lagrangian for problem (15) can be expressed as

$$\mathcal{L}_\rho(\{\mathbf{w}_d, r\}, \{\mathbf{a}, \mathbf{b}, \mathbf{v}_d, \boldsymbol{\alpha}^d\}; \boldsymbol{\lambda}^d, \boldsymbol{\lambda}^u, \boldsymbol{\mu}, \boldsymbol{\eta}^d) = r + g_{c1}(\mathbf{a}) +$$
$$g_{c2}(\mathbf{b}) + g_{c3}(\mathbf{v}, \boldsymbol{\alpha}^d) + \frac{\rho}{2}\sum_{m=1}^{K_d}\sum_{i=1}^{K_d}|a_{m,i} - \mathbf{h}_{d_m}^H\mathbf{w}_{d_i} + \lambda_{m,i}^d|^2$$
$$+\frac{\rho}{2}\sum_{m=1}^{K_d}\sum_{n=1}^{K_u}|b_{n,m} - \mathbf{h}_{SI_n}^H\mathbf{w}_{d_m} + \lambda_{m,n}^u|^2 + \frac{\rho}{2}\sum_{i=1}^{N_t}|\alpha_i^d - r + \eta_i^d|^2$$
$$+\frac{\rho}{2}\sum\nolimits_{m=1}^{K_d}\|\mathbf{v}_{d_m} - \mathbf{w}_{d_m} + \mu_m\|^2. \quad (16)$$

By applying ADMM, with given dual variables $\{\boldsymbol{\lambda}^d, \boldsymbol{\lambda}^u, \boldsymbol{\mu}, \boldsymbol{\eta}^d\}$, primal variables $\{\mathbf{w}_d, r\}$ and $\{\mathbf{a}, \mathbf{b}, \mathbf{v}_d, \boldsymbol{\alpha}^d\}$ can be updated alternatively by minimizing lagrangian function (16). In the sequel, we will show that both of these two variable sets can be optimized in parallel.

For the local variable set update, Lagrangian function minimization problem can be decomposed into the following three independent problems with given global variable set:

$$\min_{\mathbf{a}} \sum_{m=1}^{K_d}\sum_{i=1}^{K_d}|a_{m,i} - \mathbf{h}_{d_m}^H\mathbf{w}_{d_i} + \lambda_{m,i}^d|^2 \quad \text{s. t. (14b)}, \quad (17)$$

$$\min_{\mathbf{b}} \sum_{m=1}^{K_d}\sum_{n=1}^{K_u}|b_{n,m} - \mathbf{h}_{SI_n}^H\mathbf{w}_{d_m} + \lambda_{m,n}^u|^2 \text{ s. t. (14c)}, \quad (18)$$

$$\min_{\boldsymbol{\alpha}^d, \mathbf{v}_d} \sum_{i=1}^{N_T}|\alpha_i^d - r + \eta_i^d|^2 + \sum_{m=1}^{K_d}\|\mathbf{v}_{d_m} - \mathbf{w}_{d_m} + \mu_m\|^2 \quad (19a)$$

$$\text{s. t.} \quad \text{(14d)}. \quad (19b)$$

Note that problem (17) can further be decomposed into K_d independent subproblems, one for each U_{d_m}:

$$\min_{\{a_{m,i}\}_{i=1}^{K_d}} \sum\nolimits_{i=1}^{K_d}|a_{m,i} - \mathbf{h}_{d_m}^H\mathbf{w}_{d_i} + \lambda_{m,i}^d|^2 \quad \text{s.t.} \quad (20a)$$

$$\Gamma_{d_m}\Big(\sum\nolimits_{i=1, i\neq m}^{K_d}|a_{m,i}|^2 + \sum\nolimits_{n=1}^{K_u}p_{u_n}|g_{d_{mn}}|^2 + \sigma_{d_m}^2\Big)$$
$$\leq 2\mathrm{Re}\{(\mathbf{w}_{d_m}^{(j)})^H\mathbf{h}_{d_m}a_{m,m}\} - |\mathbf{h}_{d_m}^H\mathbf{w}_{d_m}^{(j)}|^2, \quad (20b)$$

which is a convex quadratically constrained quadratic programming with one constraint (QCQP-1). The optimal closed-form solution can be achieved by the lagrangian dual decomposition method. Please refer to [6] for the details.

Similar to problem (17), problem (18) can also be decomposed into K_u independent subproblems, and one for each n-th uplink user. The details are omitted here.

Observe that problem (19) can be decomposed into N_t independent subproblems, one for each antenna i:

$$\min_{\tilde{\mathbf{v}}_{d_i}, \alpha_i^d} \quad |\alpha_i^d - r + \eta_i^d|^2 + \|\tilde{\mathbf{v}}_{d_i} - \tilde{\mathbf{w}}_{d_i} + \tilde{\boldsymbol{\mu}}_i\|^2 \quad (21a)$$

$$\text{s. t.} \quad \|\tilde{\mathbf{v}}_{d_i}\|^2 \leq \alpha_i^d P_i^{BS}, \quad (21b)$$

where $\tilde{\mathbf{w}}_{d_i} = [w_{d_{i,1}}, ..., w_{d_{i,K_d}}]^T \in \mathbb{C}^{K_d}$ is a vector including all beamformings transmitted from the i-th antenna, and $w_{d_{i,m}}$ is the i-th entry of $\mathbf{w}_{d_m}$. $\tilde{\mathbf{v}}_{d_i}$ and $\tilde{\boldsymbol{\mu}}_i$ are defined in the similar way. This is also a QCQP-1 problem, and can be optimally solved by Lagrangian dual decomposition method.

Next, we focus on the global variables update with given local variables, which can be decomposed into the following two independent problems:

$$\min_{\mathbf{w}_d} \sum\nolimits_{m=1}^{K_d}\sum\nolimits_{i=1}^{K_d}|a_{m,i} - \mathbf{h}_{d_m}^H\mathbf{w}_{d_i} + \lambda_{m,i}^d|^2 + \quad (22)$$

$$\sum_{m=1}^{K_d}\sum_{n=1}^{K_u}\left|b_{n,m}-\mathbf{h}_{SI_n}^H\mathbf{w}_{d_m}+\lambda_{m,n}^u\right|^2+\sum_{m=1}^{K_d}\|\mathbf{v}_{d_m}-\mathbf{w}_{d_m}+\mu_m\|^2,$$

$$\min_{r}\ r+\frac{\rho}{2}\sum_{i=1}^{N_t}\left|\alpha_i^d-r+\eta_i^d\right|^2. \tag{23}$$

For $\mathbf{w}_d$ update, problem (22) can be decomposed into K_d independent subproblems, one for each m-th downlink user:

$$\min_{\mathbf{w}_{d_m}}\ \sum_{i=1}^{K_d}\left|a_{m,i}-\mathbf{h}_{d_m}^H\mathbf{w}_{d_i}+\lambda_{m,i}^d\right|^2+ \tag{24}$$
$$\sum_{n=1}^{K_u}\left|b_{n,m}-\mathbf{h}_{SI_n}^H\mathbf{w}_{d_m}+\lambda_{m,n}^u\right|^2+\|\mathbf{v}_{d_m}-\mathbf{w}_{d_m}+\mu_m\|^2,$$

which is an unconstrained quadratic programming problem and the optimal closed-form solution is

$$\mathbf{w}_{d_m}=\left(\sum_{i=1}^{K_d}\mathbf{h}_{d_i}\mathbf{h}_{d_i}^H+\sum_{n=1}^{K_u}\mathbf{h}_{SI_n}\mathbf{h}_{SI_n}^H+\mathbf{I}\right)^{-1}\times \tag{25}$$
$$\left(\sum_{i=1}^{K_d}(a_{i,m}+\lambda_{i,m}^d)\mathbf{h}_{d_i}+\sum_{n=1}^{K_u}(b_{n,m}+\lambda_{m,n}^u)\mathbf{h}_{SI_n}^H+\mathbf{v}_{d_m}+\boldsymbol{\mu}_m\right).$$

To update r, the optimal solution to problem (23) is

$$r=\left(\rho\sum_{i=1}^{N_t}(\alpha_i^d+\eta_i^d)-1\right)\Big/(\rho N_t)\ . \tag{26}$$

To summarize, the proposed iterative algorithm consists of two steps. i), with given t, SCA method is adopted to tackle problem $\mathcal{R}$ iteratively; at each iteration, the ADMM-based parallel **Algorithm 1** is proposed to solve problem (10); ii) the bisection method is applied to obtain the optimal t.

Algorithm 1 ADMM-based beamforming for problem (10)

1: Initialize global variables $\{\mathbf{w}_d,\ r\}$ and dual variables $\{\boldsymbol{\lambda}^d,\ \boldsymbol{\lambda}^u,\ \boldsymbol{\mu},\ \boldsymbol{\eta}^d\}$; set penalty factor ρ;
2: **while** the convergence condition is not met **do**
3: Update local variables $\{\mathbf{a},\mathbf{b},\mathbf{v}_d,\boldsymbol{\alpha}^d\}$ by solving (17)–(19);
4: Update global variables $\{\mathbf{w}_d, r\}$ by solving (22), (23);
5: Update dual variables $\{\boldsymbol{\lambda}^d,\boldsymbol{\lambda}^u,\boldsymbol{\mu},\boldsymbol{\eta}^d\}$: $\lambda_{m,i}^d=\lambda_{m,i}^d+a_{m,i}-\mathbf{h}_{d_m}^H\mathbf{w}_{d_i}$, $\lambda_{n,m}^u=\lambda_{n,m}^u+b_{n,m}-\mathbf{h}_{SI_n}^H\mathbf{w}_{d_m}$, $\mu_m=\mu_m+\mathbf{v}_{d_m}-\mathbf{w}_{d_m}$, $\eta_i^d=\alpha_i^d-r+\eta_i^d$;
6: **end while**

4. SIMULATION RESULTS

In this section, performance evaluation of our proposed parallel beamforming scheme is provided. All users are randomly distributed within a circle around the FD-BS, whose radius is 250 m. For simplicity, we assume $N_t=N_r$ and $K_d=K_u$. Besides, uplink SINR requirement of each uplink user, Γ_{u_n}, is set to be equal and the weighted factor for each downlink user, $\frac{1}{\Gamma_{d_m}}$, is assumed to be 1. Except for the residual SI channel, all remaining channels are modeled as the large-scale pathloss and small-scale Rayleigh fading. The large-scale pathloss is $PL=103.8+20.9\log(d)$ in dB, where d is in kilometer. The residual SI channel is modeled as $\mathbf{H}_{SI}\sim\mathcal{CN}(0,\sigma_{SI}^2)$, where $\sigma_{SI}^2=-60$ dB represents for the SIC capability. All noise power is assumed to be -74 dB.

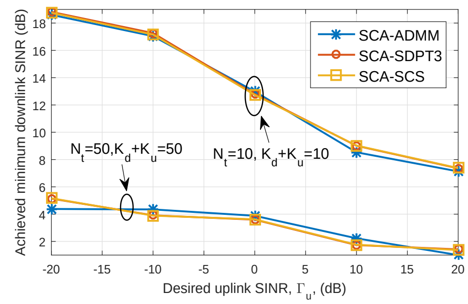

Fig. 1. Achievable downlink SINR vs uplink SINR requirement.

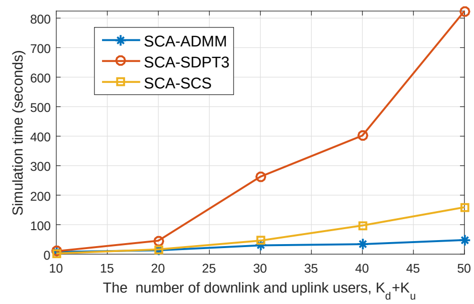

Fig. 2. Simulation time vs the number of users with $N_t=50, N_r=50$.

As mentioned in section 3, the convex QCQP problem (10) can be directly solved by the CVX solver. In the simulation, we compare our proposed parallel beamforming algorithm, labelled as 'SCA-ADMM', with two off-the-shelf CVX solvers, i.e., a second-order solver, SDPT3 and a first-order solver, SCS [16]. These two baseline schemes are respectively named as 'SCA-SDPT3' and 'SCA-SCS'.

In Fig. 1, the performance of our proposed scheme and two baseline schemes are shown. For the small-size system setup, we set $N_t=N_r=10$, $K_d=K_u=5$, $P_i^{BS}=1$ W and $p_{u_n}=0.5$ W. For the large-size system setup, we set $N_t=N_r=50$, $K_d=K_u=25$, $P_i^{BS}=2$ W and $p_{u_n}=1$ W. From Fig. 1, we can see that our proposed 'SCA-ADMM' scheme can obtain the similar performance with two baseline schemes for both small-size and large-size systems.

Fig. 2 gives the running time of different algorithms as the number of users increases. It is observed that proposed low-complexity parallel 'SCA-ADMM' scheme runs 4 times faster than 'SCA-SCS' scheme and 17 times faster than 'SCA-SDPT3' scheme when $N_t=50$. Thus, our proposed scheme can significantly reduce the computational complexity and is very suitable for large-scale systems.

5. CONCLUSIONS

This paper proposes a low-complexity parallel beamforming algorithm to maximize the minimum weighted downlink SINR with uplink SINR constraints and per-antenna constraints in FD systems. The SCA method and ADMM are utilized. Numerical results show that our proposed algorithm runs much faster than CVX solvers (SDPT3 and SCS) and can scale well to large-scale systems. For future work, we will design the distributed transceiver scheme in multiple FD-BSs networks.

6. REFERENCES

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