# LOW-COMPLEXITY WEIGHTED MRT MULTICAST BEAMFORMING IN MASSIVE MIMO CELLULAR NETWORKS

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**ABSTRACT**

We consider downlink multicast beamforming in a massive MIMO multi-cell network. Aiming at maximizing the minimum SINR among users, for both non-cooperative and cooperative multicasting, we propose a multicast beamforming scheme based on weighted maximum ratio transmission (MRT), and transform the beamforming optimization problem into a weight optimization problem that is solved via the semi-definite relaxation (SDR) approach. The proposed method has a low computational complexity which does not grow with the number of antennas, and thus is suitable for massive MIMO systems. Simulation shows that our proposed multicast beamforming solution yields comparable or better performance than existing approaches but with significantly lower complexity for practical systems with a large but finite number of antennas.

***Index Terms*—** Multicast beamforming, massive MIMO, multi-cell coordination, base station cooperation

## 1. INTRODUCTION

With the fast development of wireless applications and networks, wireless services that delivers common contents to multiple users have becoming increasingly popular. Examples include video streaming, information or media sharing, and mobile application downloads. Wireless multicasting is the underlying technology to enable these services. With massive MIMO emerged as a promising key technology for the 5th generation wireless systems [1], [2], multicast beamforming for downlink content delivery in a massive MIMO cellular system is expected to a promising transmission techniques.

Multicast beamforming design was initially considered for a single cell serving a single user group [3]–[5] and multiple user groups [6], [7]. The resulting beamforming optimization problems are NP-hard in general. The focus in the literature has been on developing signal processing techniques or numerical algorithms to obtain suboptimal beamforming solutions. Among existing methods, semi-definite relaxation (SDR) is a prevailing approach to find a good suboptimal solution [3]–[8]. However, the computational complexity of the SDR approach becomes very high as the problem size grows, making the approach inefficient for large-scale antenna systems. For multi-group multicast beamforming, several recent works have proposed low-complexity or fast algorithms to obtain beamforming vectors [9]–[11].

In a multi-cell network, multicast beamforming needs to consider inter-cell interference. For non-cooperative multicasting, coordinated multicast beamforming, where beamforming vectors among multiple cells were designed jointly to reduce inter-cell interference, was considered in [12], [13] for maximizing the minimum signal-to-interference-and-noise ratio (SINR) among users. Clustering base stations (BSs) for cooperative multicast beamforming was recently considered jointly with caching to minimize network cost in [14]. In these works, conventional finite number of BS antennas was assumed, and the SDR approach was adopted to find the beamforming vectors. Very few works have studied multicast beamforming design for a massive MIMO multi-cell network. In [15], coordinated multicast beamforming was studied, where it is shown that, the inter-cell interference vanishes as the number of BS antennas goes to infinity, and the asymptotically optimal beamforming solution is obtained in closed-form as a linear combination of the channel vectors. However, both existing studies [16] and our study show that the multicast inter-cell interference vanishes at a rather slow rate as the number of BS antennas increases, and the asymptotically optimal beamformer is rather suboptimal and may not be a good choice for practical systems with a large but finite number of antennas. On the other hand, the direct SDR approach suffers from high computational complexity when the number of antennas is large, making it impractical to obtain a beamforming solution. Thus, our goal is to design a low-complexity multicast beamforming solution for a massive MIMO system.

In this work, we design downlink multicast beamforming in a massive MIMO multi-cell network, aiming at maximizing the minimum SINR among users. Both non-cooperative and cooperative multicast beamforming scenarios are considered. We propose a multicast beamforming structure using the weighted maximum ratio transmission (MRT) beamforming, and transform the multicast beamforming optimization problem into a weight optimization problem which is solved via the SDR approach. The complexity of our proposed solution does not grow with the number of BS antennas and is suitable for massive MIMO systems. Simulation shows that our proposed solution delivers comparable performance to the traditional direct SDR approach but with significantly lower complexity for the large-scale antenna systems, and substantially outperforms the asymptotically optimal beamforming solution.

## 2. SYSTEM MODEL

We consider downlink multicasting in a cellular network consisting of $N$ cells and $K$ users per cell, as shown in Fig. 1. The base station (BS) in each cell is equipped with $M$ antennas, where $M \gg 1$ for a massive MIMO system. Each user is equipped with single antenna. We assume that all BSs and users are perfectly synchronized in time and use the same spectrum for transmission. With multiple BSs, we design multicast beamforming for two types of multicast transmissions: non-cooperative multicasting and cooperative multicasting.

### 2.1. Non-cooperative Multicasting

We consider coordinated multicasting in a multi-cell scenario, each BS multicasts information to $K$ users in its own cell, and the

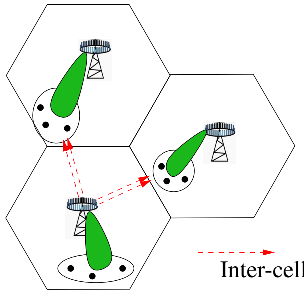


**Fig. 1**. A multi-cellular downlink non-cooperative multicast beamforming scenario.

beamforming vectors among BSs are jointly determined. Define $\mathcal{N} \triangleq \{1, \cdots, N\}$, $\mathcal{K} \triangleq \{1, \cdots, K\}$. Let $\mathbf{h}_{njk}$ denote the $M \times 1$ channel vector from BS $n$ to user $k$ in cell $j$, for $n, j \in \mathcal{N}$ and $k \in \mathcal{K}$. Let $s_n$ denote the multicast information symbol from BS $n$ with $\mathbb{E}[|s_n|^2] = 1$. Let $\mathbf{w}_n$ denote the $M \times 1$ multicast beamforming vector at BS $n$. The received signal at user $k$ in cell $n$ is given by

$$y_{nk} = \mathbf{w}_n^H \mathbf{h}_{nnk} s_n + \sum_{i \neq n}^{N} \mathbf{w}_i^H \mathbf{h}_{ink} s_i + n_{nk}, \ \ k \in \mathcal{K}, n \in \mathcal{N} \quad (1)$$

where $n_{nk}$ is the received additive Gaussian noise at user $k$ in cell $n$ with zero mean and variance $\sigma^2$. The first term in (1) is the desired signal to user $k$ and the second term is the interference from BSs of neighboring cells. The transmit power at BS $n$ is limited by its maximum power $P_{\text{tot}}$, and we have $\|\mathbf{w}_n\|^2 \leq P_{\text{tot}}$, for $n \in \mathcal{N}$. We assume that all the channel state information (CSI) are perfectly known at BSs.

From (1), the received SINR at user $k$ in cell $n$ is given by

$$\text{SINR}_{nk} = \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{\sum_{i \neq n}^{N} |\mathbf{h}_{ink}^H \mathbf{w}_i|^2 + \sigma^2}. \quad (2)$$

For multicasting, the performance at each cell is characterized by the minimum SINR among all users in the cell. Our objective is to design the beamforming vectors $\{\mathbf{w}_n\}$ of all BSs to maximize the minimum SINR of the network, under the transmit power constraint. The optimization problem is formulated by

$$\mathcal{P}_{\text{NC}}: \quad \max_{\{\mathbf{w}_n\}} \min_{k \in \mathcal{K}, n \in \mathcal{N}} \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_n|^2}{\sum_{i \neq n}^{N} |\mathbf{h}_{ink}^H \mathbf{w}_i|^2 + \sigma^2}$$
$$\text{s.t. } \|\mathbf{w}_n\|^2 \leq P_{\text{tot}}, \ n \in \mathcal{N}. \quad (3)$$

### 2.2. Cooperative Multicasting

In the cooperative multicasting case, multiple BSs form a cluster to cooperatively multicast data to a user group. We consider the general case where users are divided into groups, with $J$ users per group. Each user group is served by its serving BS cluster. This setup includes the scenario where users in multiple cells request the same content, and they can be formed into the same group and served by the BS cluster. Cooperative multicasting may be especially beneficial in this scenario.

Assume there are $C$ clusters, where $C \leq N$. Assuming each BS cluster serves one user group, we use the same index for the user group and its serving BS cluster. Denote $\mathcal{C} = \{1, \cdots, C\}$ and $\mathcal{J} = \{1, \cdots, J\}$. Let $\mathcal{Q}_c$ denote the set of BS indices for BS cluster $c$, where $\mathcal{Q}_c \subseteq \mathcal{N}$, for $c \in \mathcal{C}$. Note that a BS may be in multiple BS clusters to serve multiple user groups simultaneously. Thus, sets $\{\mathcal{Q}_c\}_{c=1}^C$ may overlap with each other. Let $\mathcal{B}_n$ denote the set of cluster indices that BS $n$ belongs to. Let $\tilde{\mathbf{w}}_{nc}$ denote the beamforming vector at BS $n$ for cluster $c$. Let $\tilde{\mathbf{h}}_{ncj}$ denote the channel vector from BS $n$ to the user $j$ in group $c$. The received signal at user $j$ in group $c$, for $j \in \mathcal{J}$, $c \in \mathcal{C}$, is given by

$$y_{cj} = \sum_{n \in \mathcal{Q}_c} \tilde{\mathbf{w}}_{nc}^H \tilde{\mathbf{h}}_{ncj} s_c + \sum_{i \neq c}^{C} \sum_{n \in \mathcal{Q}_i} \tilde{\mathbf{w}}_{ni}^H \tilde{\mathbf{h}}_{ncj} s_i + n_{cj} \quad (4)$$

where $s_c$ is the common multicast symbol from BS cluster $c$, and $n_{cj}$ is the received Gaussian additive noise at user $j$ in group $c$. The transmit power constraint at BS $n$ is expressed by $\sum_{c \in \mathcal{B}_n} \|\tilde{\mathbf{w}}_{nc}\|^2 \leq P_{\text{tot}}$.

Based on (4), the SINR at user $j$ in group $c$ under the cooperative multicasting is given by

$$\text{SINR}_{cj} = \frac{|\sum_{n \in \mathcal{Q}_c} \tilde{\mathbf{h}}_{ncj}^H \tilde{\mathbf{w}}_{nc}|^2}{\sum_{i \neq c}^{C} |\sum_{n \in \mathcal{Q}_i} \tilde{\mathbf{h}}_{ncj}^H \tilde{\mathbf{w}}_{ni}|^2 + \sigma^2}. \quad (5)$$

To maximize the minimum SINR among all users, the problem is give by

$$\mathcal{P}_{\text{CP}}: \quad \max_{\{\tilde{\mathbf{w}}_{nc}\}} \min_{j \in \mathcal{J}, c \in \mathcal{C}} \frac{|\sum_{n \in \mathcal{Q}_c} \tilde{\mathbf{h}}_{ncj}^H \tilde{\mathbf{w}}_{nc}|^2}{\sum_{i \neq c}^{C} |\sum_{n \in \mathcal{Q}_i} \tilde{\mathbf{h}}_{ncj}^H \tilde{\mathbf{w}}_{ni}|^2 + \sigma^2}$$
$$\text{s.t. } \sum_{c \in \mathcal{B}_n} \|\tilde{\mathbf{w}}_{nc}\|^2 \leq P_{\text{tot}}, \ \ n \in \mathcal{N}. \quad (6)$$

## 3. WEIGHTED MRT MUTICAST BEAMFORMING

Both the optimization problems $\mathcal{P}_{\text{NC}}$ and $\mathcal{P}_{\text{C}}$ are non-convex and NP-hard problems, and the optimal solutions typically cannot be obtained. To find a good sub-optimal solution, a typical approach is to apply the SDR approach to find sub-optimal $\mathbf{w}_n$. However, the complexity of the SDR approach grows with the size of the problem which is determined by $M$. For massive MIMO, as $M \gg 1$, the SDR approach incurs very high computational complexity, thus directly obtaining $\mathbf{w}_n$ through SDR is not suitable for the large-scale antenna systems. Below, we propose a low-complexity algorithm via a special mutlicast beamforming structure to find a sub-optimal solution $\{\mathbf{w}_n\}$ whose complexity does not grow with the number of antennas.

### 3.1. Non-cooperative Multicasting

Instead of directly finding $\{\mathbf{w}_n\}$ for problem $\mathcal{P}_{\text{NC}}$, we propose the structure of $\mathbf{w}_n$ at each BS $n$ as a weighted sum of the channel vectors between BS $n$ and its each serving users, given by

$$\mathbf{w}_n \triangleq \sum_{k=1}^{K} a_{nk} \mathbf{h}_{nnk}, \ \ n \in \mathcal{N}. \quad (7)$$

where $a_{nk}$ is the complex weight for the channel between BS $n$ and user $k$. We name this as the weighted MRT multicast beamforming.

Define $\mathbf{H}_n \triangleq [\mathbf{h}_{nn1}, \cdots, \mathbf{h}_{nnK}]$ as the $M \times K$ channel matrix between BS $n$ and its serving user group. Define $\mathbf{a}_n \triangleq [a_{n1}, \cdots, a_{nK}]^T$ as the $K \times 1$ weight vector associated with beamforming vector $\mathbf{w}_n$ at BS $n$. Based on $\mathbf{w}_n$ in (7), SINR expression in (2) can now be rewritten as

$$\text{SINR}_{nk} = \frac{|\mathbf{h}_{nnk}^H \mathbf{H}_n \mathbf{a}_n|^2}{\sum_{i \neq n}^{N} |\mathbf{h}_{ink}^H \mathbf{H}_i \mathbf{a}_i|^2 + \sigma^2} = \frac{\mathbf{a}_n^H \mathbf{A}_{nnk} \mathbf{a}_n}{\sum_{i \neq n}^{N} (\mathbf{a}_i^H \mathbf{A}_{ink} \mathbf{a}_i) + \sigma^2}$$

where $\mathbf{A}_{ink} \triangleq \mathbf{H}_i^H \mathbf{h}_{ink} \mathbf{h}_{ink}^H \mathbf{H}_i, i, n \in \mathcal{N}$. The transmitting power of BS $n$ can be written as

$$\|\mathbf{w}_n\|^2 = \|\mathbf{H}_n \mathbf{a}_n\|^2 = \mathbf{a}_n^H \mathbf{B}_n \mathbf{a}_n \tag{8}$$

where $\mathbf{B}_n \triangleq \mathbf{H}_n^H \mathbf{H}_n$.

The optimization problem $\mathcal{P}_{\text{NC}}$ is now transformed into the optimization of weight vectors $\{\mathbf{a}_n\}$ for the same objective. The max-min SINR optimization problem $\mathcal{P}_{\text{NC}}$ can now be rewritten as

$$\begin{aligned} \mathcal{P}_{\text{NC2}}: \max_{\{\mathbf{a}_n\}} \min_{k \in \mathcal{K}, n \in \mathcal{N}} & \frac{\mathbf{a}_n^H \mathbf{A}_{nnk} \mathbf{a}_n}{\sum_{i \neq n}^N (\mathbf{a}_i^H \mathbf{A}_{ink} \mathbf{a}_i) + \sigma^2} \\ \text{s.t.} \ & \mathbf{a}_n^H \mathbf{B}_n \mathbf{a}_n \leq P_{\text{tot}}, \quad n \in \mathcal{N}. \end{aligned}$$

Note that the problem size of $\mathcal{P}_{\text{NC2}}$ is $NK$ based on the optimization variables in $\{\mathbf{a}_n\}$, as opposed to $NM$ for $\mathcal{P}_{\text{NC}}$, and has the same number of constraints. The size of the new problem is independent of the number of BS antennas $M$, making the approach especially attractive for BSs with massive MIMO. Problem $\mathcal{P}_{\text{NC2}}$ can be transformed to the following

$$\begin{aligned} \mathcal{P}_{\text{NC3}}: \min_{\{\mathbf{a}_n\}} \ & t \\ \text{s.t.} \ & \frac{\mathbf{a}_n^H \mathbf{A}_{nnk} \mathbf{a}_n}{\sum_{i \neq n}^N (\mathbf{a}_i^H \mathbf{A}_{ink} \mathbf{a}_i) + \sigma^2} \geq \frac{1}{t}, \ k \in \mathcal{K}, n \in \mathcal{N} \\ & \mathbf{a}_n^H \mathbf{B}_n \mathbf{a}_n \leq P_{\text{tot}}, \ n \in \mathcal{N}, \quad t > 0. \end{aligned}$$

To solve $\mathcal{P}_{\text{NC3}}$, we apply the SDR approach. Define $\mathbf{X}_n \triangleq \mathbf{a}_n \mathbf{a}_n^H$, $n \in \mathcal{N}$, and drop the rank constrain $\text{Rank}(\mathbf{X}_n) = 1$, we transform $\mathcal{P}_{\text{NC3}}$ to the following problem

$$\begin{aligned} \mathcal{P}_{\text{NC4}}: \min_{\{\mathbf{X}_n\}, t} \ & t \\ \text{s.t.} \ & \text{tr}\Big[t\mathbf{A}_{nnk}\mathbf{X}_n - \sum_{i \neq n}^N \mathbf{A}_{ink}\mathbf{X}_i\Big] \geq \sigma^2, \ k \in \mathcal{K}, n \in \mathcal{N} \\ & \text{tr}\Big[\mathbf{B}_n \mathbf{X}_n\Big] \leq P_{\text{tot}}, \ n \in \mathcal{N}, \quad \mathbf{X}_n \succeq 0, \quad t > 0. \end{aligned}$$

Although $\mathcal{P}_{\text{NC4}}$ is not jointly convex w.r.t. $\mathbf{X}_n$ and $t$, when $t$ is fixed, it is convex w.r.t. $\mathbf{X}_n$. Thus, we are able to find $\mathbf{X}_n$ by applying the bi-section search over $t$, along with a feasibility test problem given by

$$\begin{aligned} \text{Find} \ & \{\mathbf{X}_n\} \\ \text{s.t.} \ & \text{tr}[t\mathbf{A}_{nnk}\mathbf{X}_n - \sum_{i \neq n}^N \mathbf{A}_{ink}\mathbf{X}_i] \geq \sigma^2, \ k \in \mathcal{K}, n \in \mathcal{N} \\ & \text{tr}[\mathbf{B}_n \mathbf{X}_n] \leq P_{\text{tot}}, \ n \in \mathcal{N}. \end{aligned}$$

The above problem is a semi-definite programming (SDP) which can be solved efficiently with standard SDP solvers by applying interior point methods. If the optimal solution $\mathbf{X}_n^o$ is rank one, the weight vector $\mathbf{a}_n$ for BS $n$ can be directly recovered from $\mathbf{X}_n^o = \mathbf{a}_n \mathbf{a}_n^H$. Otherwise, the Gaussian randomization method [5] can be applied to find a suboptimal rank-one solution.

### 3.2. Cooperative Multicasting

Similar to the non-cooperative case, we consider weighted MRT multicast beamforming, where we construct $\tilde{\mathbf{w}}_{nc}$ as a weighted sum of the channel vectors between BS $n$ and user group $c$, given by

$$\tilde{\mathbf{w}}_{nc} \triangleq \sum_{j=1}^J b_{ncj} \tilde{\mathbf{h}}_{ncj}, \ n \in \mathcal{Q}_c, c \in \mathcal{C} \tag{9}$$

where $b_{ncj}$ is the complex weight for the channel vector between BS $n$ and user $j$ in group $c$.

Again, define $\tilde{\mathbf{H}}_{nc} \triangleq [\tilde{\mathbf{h}}_{nc1}, \cdots, \tilde{\mathbf{h}}_{ncJ}]$ as the channel matrix between BS $n$ and user group $c$. Define $\mathbf{b}_{nc} \triangleq [b_{nc1}, \cdots, b_{ncJ}]^T$ as the weight vector for beamforming vector of BS $n$ to group $c$. The SINR of user $j$ in group $c$ in (5) can now be rewritten as

$$\text{SINR}_{cj} = \frac{|\sum_{n \in \mathcal{Q}_c} \tilde{\mathbf{h}}_{ncj}^H \tilde{\mathbf{H}}_{nc} \mathbf{b}_{nc}|^2}{\sum_{i \neq c}^C |\sum_{n \in \mathcal{Q}_i} \tilde{\mathbf{h}}_{ncj}^H \tilde{\mathbf{H}}_{ni} \mathbf{b}_{ni}|^2 + \sigma^2}. \tag{10}$$

To facilitate the notations, let $Q_c = |\mathcal{Q}_c|$, and we describe the BS indices in BS cluster set $\mathcal{Q}_c = \{n_1, \cdots, n_{Q_c}\}$, where $n_k$ is the BS index for $k$th BS in BS cluster $c$, for $k = 1, \cdots, Q_c$. We further define $\mathbf{b}_c \triangleq \text{vec}([\mathbf{b}_{n_1 c}, \cdots, \mathbf{b}_{n_{Q_c} c}])$ as the weight vector associate with the beamforming vectors for BS cluster $c$. Define $\mathbf{g}_{icj} \triangleq \text{vec}([\tilde{\mathbf{H}}_{n_1 i}^H \tilde{\mathbf{h}}_{n_1 cj}, \cdots, \tilde{\mathbf{H}}_{n_{Q_c} i}^H \tilde{\mathbf{h}}_{n_{Q_c} cj}])$. Note that vector $\mathbf{g}_{ccj}$ contains the correlation of the channel vector from each BS in cluster $c$ to its user $j$ and channel vectors from that BS to all other users in group $c$. Using $\mathbf{b}_c$ and $\mathbf{g}_{icj}$, $\text{SINR}_{cj}$ expression in (10) can be further rewritten as

$$\text{SINR}_{cj} = \frac{\mathbf{b}_c^H \mathbf{G}_{ccj} \mathbf{b}_c}{\sum_{i \neq c}^C \mathbf{b}_i^H \mathbf{G}_{icj} \mathbf{b}_i + \sigma^2} \tag{11}$$

where $\mathbf{G}_{icj} \triangleq \mathbf{g}_{icj} \mathbf{g}_{icj}^H$. Similarly, the transmit power at BS $n$ can be rewritten as

$$\sum_{c \in \mathcal{B}_n} \|\tilde{\mathbf{w}}_{nc}\|^2 = \sum_{c \in \mathcal{B}_n} \mathbf{b}_{nc}^H \tilde{\mathbf{H}}_{nc}^H \tilde{\mathbf{H}}_{nc} \mathbf{b}_{nc} = \sum_{c \in \mathcal{B}_n} \mathbf{b}_c^H \mathbf{D}_{nc} \mathbf{b}_c \tag{12}$$

where $\mathbf{D}_{nc} \triangleq \text{bldg}(\mathbf{0}, \cdots, \mathbf{0}, \tilde{\mathbf{H}}_{nc}^H \tilde{\mathbf{H}}_{nc}, \mathbf{0}, \cdots \mathbf{0})$ is a block diagonal matrix consisting of $Q_c$ diagonal blocks of size $J \times J$ each; matrix $\tilde{\mathbf{H}}_{nc}^H \tilde{\mathbf{H}}_{nc}$ is located at the $k$th diagonal block, where $k$ is determined by the inverse mapping from BS index $n$ to the $k$th element in $\mathcal{Q}_c = \{n_1, \cdots, n_{Q_c}\}$, where $n_k = n$. The rest diagonal blocks are $J \times J$ zero matrices.

Using (11) and (12), optimization problem $\mathcal{P}_{\text{CP}}$ for the cooperative multicasting case is now transformed to

$$\begin{aligned} \mathcal{P}_{\text{CP2}}: \min_{\{\mathbf{b}_c\}} \ & t \\ \text{s.t.} \ & \frac{\mathbf{b}_c^H \mathbf{G}_{ccj} \mathbf{b}_c}{\sum_{i \neq c}^C (\mathbf{b}_i^H \mathbf{G}_{icj} \mathbf{b}_i) + \sigma^2} \geq \frac{1}{t}, \ j \in \mathcal{J}, c \in \mathcal{C} \\ & \sum_{c \in \mathcal{B}_n} \mathbf{b}_c^H \mathbf{D}_{nc} \mathbf{b}_c \leq P_{\text{tot}}, \ n \in \mathcal{N}, \quad t > 0. \end{aligned}$$

Comparing with the original problem $\mathcal{P}_{\text{CP}}$, the transformed problem $\mathcal{P}_{\text{CP2}}$ is of size $J \sum_{c=1}^C Q_c$ based on the optimization variables $\{\mathbf{b}_c\}$, which only depends on $N$ and $J$ and is independent of the number of BS antennas $M$, which makes the approach particularly suitable for a large-scale antenna systems.

Now $\mathcal{P}_{\text{CP2}}$ has a very similar structure as $\mathcal{P}_{\text{NCP3}}$ in the non-cooperative case. Likewise, we define $\mathbf{Y}_c \triangleq \mathbf{b}_c \mathbf{b}_c^H$, $c \in \mathcal{C}$, and use the SDR approach to find a solution for $\mathcal{P}_{\text{CP2}}$ as

$$\begin{aligned} \mathcal{P}_{\text{CP3}}: \min_{\{\mathbf{Y}_c\}, t} \ & t \\ \text{s.t.} \ & \text{tr}\Big[t\mathbf{G}_{ccj}\mathbf{Y}_c - \sum_{i \neq c}^C \mathbf{G}_{icj}\mathbf{Y}_i\Big] \geq \sigma^2, \ j \in \mathcal{J}, c \in \mathcal{C} \\ & \text{tr}\Big[\sum_{c \in \mathcal{B}_n} \mathbf{D}_{nc} \mathbf{Y}_c\Big] \leq P_{\text{tot}}, \ n \in \mathcal{N}, \quad \mathbf{Y}_n \succeq 0, \quad t > 0. \end{aligned}$$

The optimal solution $\mathbf{Y}_c^o$ can be obtained by solving the SDP feasibility problem along with 1-D bi-section search on $t$. Again,

| $M$ | Non-cooperative | | Cooperative | |
|---|---|---|---|---|
| | Weighted MRT (s) | Direct SDR (s) | Weighted MRT (s) | Direct SDR (s) |
| 5 | 12.68 | 12.49 | 12.38 | 22.48 |
| 10 | 13.42 | 16.31 | 12.73 | 105.78 |
| 20 | 14.61 | 42.52 | 13.46 | 799.51 |
| 40 | 15.84 | 266.0 | 14.08 | 6710 |
| 50 | 16.33 | 533.8 | 14.68 | 12435 |
| 100 | 15.79 | 4405 | 14.97 | N/A |
| 200 | 16.68 | 35042 | 15.84 | N/A |
| 500 | 17.86 | N/A | 16.82 | N/A |

**Table I**. Comparison of Average Computation Time ($N = 3$).

the solution $\mathbf{b}_c$ can be extracted from $\mathbf{Y}_c^o$, either directly as the optimal solution to $\mathcal{P}_{\text{CP2}}$ if $\mathbf{Y}_c^o$ is rank one, or through the Gaussian randomization approach as a sub-optimal solution to $\mathcal{P}_{\text{CP2}}$.

## 4. SIMULATION RESULTS

For simulation study, we generate i.i.d. channel vectors $\mathbf{h}_{nik} \sim \mathcal{CN}(\mathbf{0}, \beta_{nk}\mathbf{I})$ between each BS and each user, where channel variance $\beta_{nk}$ is using the large-scale pathloss model as $\beta_{nk} = K_o d_{nk}^{-\kappa}$, with $d_{nk}$ being the distances between BS $n$ and user $k$, and $\kappa$ being the path loss exponent set to $\kappa = 3.5$. We set $P_{\text{tot}}/\sigma^2 = 10$dB.

*1) Noncooperative Case*: We first consider a single cell setup without inter-cell interference with $K = 3$. For comparison, we consider i) directly obtaining $\{\mathbf{w}_n\}$ by solving the original problem $\mathcal{P}_{\text{NC}}$ using the SDR approach (direct SDR); ii) Asymptotically optimal solution for the non-cooperative case (asymptotic BF) [15] obtained by letting $M \to \infty$. In Fig. 2, we plot the minimum SINR performance versus $M$, where we see that all three methods provide very close performance to each other. However, their computational complexity are substantially different which is discussed next.

Fig. 3 shows the performance of different methods in a multi-cell scenario with $N = 3$ and $K = 3$. Besides the direct SDR method for coordinated beamforming $\mathcal{P}_{\text{NC}}$, we also consider the direct SDR method for non-coordinated beamforming (*i.e.,* single-cell) for comparison. The SDR upper bound is plotted as a benchmark. We see that the proposed weighted MRT method results in a small loss ($\sim$ 0.5 dB) as compared with the direct SDR method (coordinated) for $M \leq 200$, while using the latter for $M > 200$ becomes computationally prohibitive. The asymptotic BF and direct SDR (non-coordinated) methods have very similar performance, and our proposed method significantly outperforms the two methods by about 3 dB. The reason is that the inter-cell interference reduces at a very slow rate as $M$ increases, and the asymptotic solution (assuming inter-cell interference vanishes) is considerably sub-optimal for practical large value of $M$. The average computation time for the weighted MRT and the direct SDR methods are shown Table. I. The weighted MRT complexity is low and is almost unchanged as $M$ increases, while the direct SDR complexity increases significantly with $M$ and becomes impractical for finite but large $M$.

*2) Cooperative Case:* For the cooperative case, we consider $N = 3$ and set $C = 3$ clusters, where each cluster includes all 3 BSs, and each cluster is serving a different user group with $J = 3$ users per group. For comparison, we consider the direct SDR approach to obtain $\{\tilde{\mathbf{w}}_{nc}\}$ in $\mathcal{P}_{\text{CP}}$. In Fig.4, the gap between the weighted MRT and direct SDR is larger than that in the non-cooperative case. Similar to the non-cooperative case, we can derive the cooperative asymptotically optimal beamforming

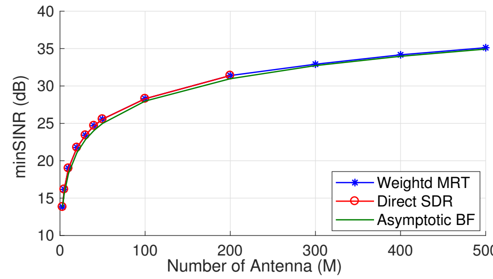


**Fig. 2**. Minimum SINR vs. $M$ for the single cell case.

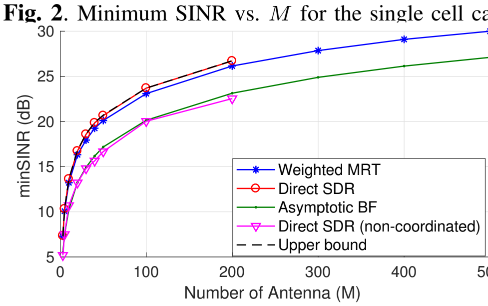


**Fig. 3**. Minimum SINR vs. $M$ for the non-cooperative case ($N = 3$, $K = 3$).

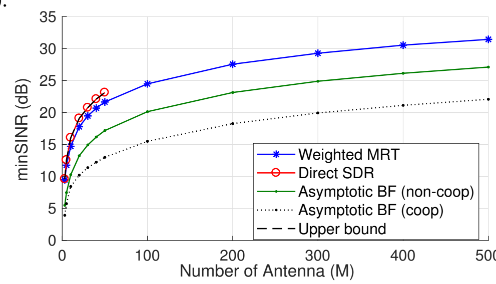


**Fig. 4**. Minimum SINR vs. $M$ for the cooperative case ($N = 3$, $J = 3$).

solution, whose performance shown in Fig.4 is significantly worse than the weighted MRT, as well as the asymptotical BF solution in the non-cooperative case. This is due to the increased interference when a BS participates multiple clusters, which decreases very slowly with $M$ and cannot be captured in the asymptotic solution for finite but large $M$. Comparing the performance of the weighted MRT in the non-cooperative and cooperative cases, we observe about 1.5dB gain due to cooperation among 3 BSs. From Table. I, we see that the processing time by weight MRT remains nearly unchanged across $M$ as well as compared with that in the non-cooperative case, while the processing time by the direct SDR method increases significantly in the cooperative case and with $M$.

## 5. CONCLUSION

In this work, we considered the non-cooperative and cooperative multicast beamforming designs in a massive MIMO multi-cell network. Aiming to maximize the minimum SINR among users, we proposed a weighted MRT beamforming structure which can be optimized through a weight optimization problem via the SDR approach with a low complexity independent of the number of BS antennas. Simulation shows the performance of our proposed solution is comparable to the solution using direct SDR approach but with much lower complexity. Our proposed solution is also significantly better than the asymptotic solution in a practical system with large $M$.

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