JOINT PROBABILISTIC FORECASTS OF TEMPERATURE AND SOLAR IRRADIANCE

Raksha Ramakrishna[†], *Andrey Bernstein*[‡], *Emiliano Dall'Anese*[‡], *Anna Scaglione*[†]

[†]School of ECEE, Arizona State University, Tempe, AZ, USA.
[‡]National Renewable Energy Laboratory, Golden, CO,USA.

ABSTRACT

In this paper, a mathematical relationship between temperature and solar irradiance is established in order to reduce the sample space and provide joint probabilistic forecasts. These forecasts can then be used for the purpose of stochastic optimization in power systems. A Volterra system type of model is derived to characterize the dependence of temperature on solar irradiance. A dataset from NOAA weather station in California is used to validate the fit of the model. Using the model, probabilistic forecasts of both temperature and irradiance are provided and the performance of the forecasting technique highlights the efficacy of the proposed approach. Results are indicative of the fact that the underlying correlation between temperature and irradiance is well captured and will therefore be useful to produce future scenarios of temperature and irradiance while approximating the underlying sample space appropriately.

Index Terms— Volterra system, solar irradiance, temperature, probabilistic forecasts, stochastic optimization

1. INTRODUCTION

It is well understood that temperatures affect energy consumption, with regions utilizing electricity for heating and/or cooling observing considerable seasonalities in load, largely attributed to thermal load demand fluctuating with changes in outdoor temperature. Naturally, solar irradiation affects both temperature as well as solar power production, making the two statistically dependent. These underlying dependencies are not completely ignored in traditional power systems operations, as most load forecasts use temperature forecasts as an input. However, the literature on probabilistic forecasts that explicitly deals with these issues is scarce. What motivates this paper are two basic questions: *"Does it make sense to ignore these dependencies in a stochastic optimization framework?"* and *"How can one account for them in the decision model?"*. Many problems use stochastic optimization when they encounter uncertainty in load, renewable generation and electricity prices. Basically this involves constructing scenario trees for multidimensional random processes (random variables being load, renewable generation, electricity prices). Certainly many of these are correlated. By treating them as independent random variables, the size and scale of scenario trees increases making computation intractable.

In this work, we consider the dependency of temperature on solar irradiance. Since outdoor temperature affects thermal loads and solar irradiance is responsible for solar photo-voltaic (PV) power, by capturing the relationship between temperature and solar irradiance, we can relate renewable infeed (solar PV power) with thermostatically controlled loads (TCL). Such modeling leads to reduction in the size of the sample space when solar PV power and TCLs are described together.

Related work Empirical studies such as [1, 2] establish relationship between temperature and solar PV power. In [3], the authors take a data driven approach to relate solar PV, indoor and outdoor temperature and power consumption of an AC using an autoregressive (AR) model. In [4], ARMA models are learned for PV power, load and wind separately and their profiles are used to establish reliability in the hybrid system. In the realm of stochastic optimization, clustering methods like k-Nearest Neighbors (k-NN) along with auxiliary variables like temperature are used in [5] to create k scenarios for PV power and TCL load. It is only this extent that they acknowledge the correlation. Furthermore, they assign equal probabilities to all k scenarios which is not entirely accurate.

We take a model-based approach by firstly deriving a Volterra type system model to establish the relationship between temperature and irradiance. Then we write the conditional distribution of temperature given the irradiance using the proposed Volterra model. In our prior work in [6] we proposed a stochastic model for probabilistic forecasts of solar PV power. We assume the same model for irradiance since the underlying stochasticity is the same. Afterwards, we proceed to jointly forecast temperature and irradiance in the probabilistic sense using the reduced uncertainty from joint description. Our results highlight that the joint probabilistic forecasts are reliable [7] and have a low continuous rank probability score (CRPS).

2. RELATING TEMPERATURE TO SOLAR IRRADIANCE

There is an obvious relationship between solar irradiance and temperature which is that of cause and effect. Solar irradiance is the source of heat during the day when the surrounding land and water absorb this heat, causing the temperature in that area to rise. Once the heat energy in surroundings exceeds the heat from sun, the heat dissipation is now from the land and water leading to higher temperatures on days with higher irradiance. Analogously, we can express the phenomenon of solar irradiance, $w[n]$ at time instant n, heating the landmass using charging and discharging of a capacitor in an RC circuit. The voltage across the capacitor is analogous to temperature $\tau[n]$ and $P_{in}^w[n]$, a function of solar irradiance, serves as a current source. The resistor $R[n]$ and capacitor $C[n]$ themselves are quanti-
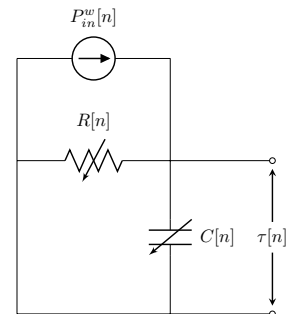

Fig. 1: Equivalent thermal circuit to describe the effect of irradiance $w[n]$ on temperature $\tau[n]$

ties dependent on temperature since they relate to thermal resistance of air and heat capacity of the landmass respectively. The temperature is also affected by the wind and other weather related phenomenon which can be accounted for in the model as noise. Fig.1 depicts the equivalent circuit describing the physics of the system.

We hypothesize that thermal resistivity $R[n]$ is linearly related to temperature [8] ,

$$R[n] \approx \sum_{q=1}^{M_1} a[q]\tau[n-q] \tag{1}$$

In [9], heat capacity is defined as "*the proportionality constant between the heat that the object absorbs or loses and the resulting temperature change of the object*". The heat absorbed by the surroundings to change the temperature from $\tau[n-1]$ to $\tau[n]$ can also be modeled as a linear relationship with temperature. Therefore, heat capacity $C[n]$ is postulated as

$$C[n] = \frac{C_0 + \sum_{q=1}^{M_2} b[q]\tau[n-q]}{(\tau[n] - \tau[n-1])}. \tag{2}$$

where C_0 is some internal heat energy and the term $\sum_{q=1}^{M_2} b[q]\tau[n-q]$ defines the amount of heat energy at time n. Now, we consider the input $P_{in}^w[n]$ to be the weighted sum of irradiance accumulated with a certain memory,

$$P_{in}^w[n] \approx \sum_{q=0}^{M_2-1} g[q]w[n-q] \tag{3}$$

In discrete time, the dynamics of the circuit in Fig.1 are described as

$$\tau[n] - \tau[n-1] = -\frac{\tau[n]}{R[n]C[n]} + \frac{P_{in}^w[n]}{C[n]} \tag{4}$$

$$R[n]C[n]\left(\tau[n]-\tau[n-1]\right) = -\tau[n] + R[n]P_{in}^w[n] \tag{5}$$

From (3) and (5),

$$R[n]P_{in}^w[n] \approx \sum_{q=0}^{M_2-1} \sum_{p=1}^{M_1} a[p]g[q]\tau[n-p]w[n-q] \tag{6}$$

Define $a[p]g[q] \triangleq \beta[k]$, $k = p - q$ by assuming that the coefficient $\beta[k]$ only depends on the time-lag between temperature and irradiance. Also, let the maximum lag be $M_2(M_2 \leq M_1)$. Then (6) can be written as,

$$R[n]P_{in}^w[n] \approx \sum_{k=0}^{M_2-1} \sum_{p=1}^{M_2-k} \beta[k]\tau[n-p]w[n-p+k+1] \tag{7}$$

We can also write the L.H.S of (5) as

$$R[n]C[n](\tau[n]-\tau[n-1]) \approx R[n]C_0 + \sum_{q=1}^{M_2} R[n]b[q]\tau[n-q]$$

$$R[n]C_0 \approx \sum_{q=1}^{M_1} C_0 a[q]\tau[n-q] \triangleq \sum_{q=1}^{M_1} \gamma[q]\tau[n-q] \text{ and} \tag{8}$$

$$R[n]\sum_{q=1}^{M_2} b[q]\tau[n-q] = \sum_{q=1}^{M_1}\sum_{p=1}^{M_2} a[q]b[p]\tau[n-q]\tau[n-p]$$

from (2). With the same assumption that coefficients only depend on time-lag and that significant coefficients have a maximum lag of M_2, we rewrite the term,

$$R[n]\sum_{q=1}^{M_2} b[q]\tau[n-q] \approx \sum_{k=0}^{M_2-1}\sum_{p=1}^{M_2-k} \alpha[k]\tau[n-p]\tau[n-p-k] \tag{9}$$
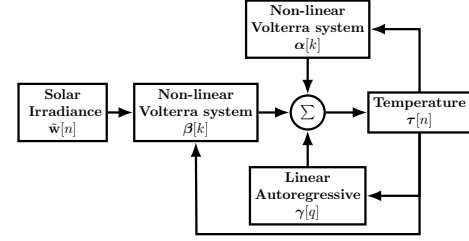

Fig. 2: Block diagram showing the proposed relationship between temperature and solar irradiance

From (7), (8) and (9) the model for temperature can be written as,

$$\tau[n] \approx \sum_{q=1}^{M_1}\gamma[q]\tau[n-q] + \sum_{k=0}^{M_2-1}\sum_{p=1}^{M_2-k}\alpha[k]\tau[n-p]\tau[n-p-k]$$

$$+ \sum_{k=0}^{M_2-1}\sum_{p=1}^{M_2-k}\beta[k]w[n-p+1]\tau[n-p-k] + \epsilon[n] \tag{10}$$

where $\epsilon[n]$ is the modeling error. This model for temperature can be interpreted as a Volterra series expansion [10, 11] up to a second order with solar irradiance as an input. In this way, the non-linearity in the system which is a result of temperature dependent thermal resistivity and heat capacity is captured. The parameters $\alpha[k], \beta[k], \gamma[q]$ can be estimated by solving a least-squares problem using training data of temperature and irradiance. Let us define a vector of parameters $\mathbf{x} \in \mathbb{R}^{M_1+2M_2}$ as $\mathbf{x}^T = \begin{bmatrix} \boldsymbol{\alpha} & \boldsymbol{\beta} & \boldsymbol{\gamma} \end{bmatrix}$ where $[\boldsymbol{\alpha}]_k = \alpha[k]$, $[\boldsymbol{\beta}]_k = \beta[k], [\boldsymbol{\gamma}]_q = \gamma[q]$. Define

$$\mathbf{Q}(n, 1:2M_1+M_2) = \begin{bmatrix} \boldsymbol{r}_{\tau\tau}^T & \boldsymbol{r}_{w\tau}^T & \boldsymbol{r}_{\tau}^T \end{bmatrix}, \mathbf{Q} \in \mathbb{R}^{N \times 2M_1+M_2}$$

$$[\boldsymbol{r}_{\tau\tau}]_k = \sum_{p=1}^{M_2-k}\tau[n-p]\tau[n-p-k], \ [\boldsymbol{r}_{\tau}]_q = \tau[n-q]$$

$$[\boldsymbol{r}_{w\tau}]_k = \sum_{p=1}^{M_2-k}w[n-p+1]\tau[n-p-k], k = 0, \dots M_2-1,$$

$$q = 1, \dots M_1, \boldsymbol{r}_{\tau} \in \mathbb{R}^{M_1}, \boldsymbol{r}_{\tau\tau} \in \mathbb{R}^{M_2}, \boldsymbol{r}_{w\tau} \in \mathbb{R}^{M_2}$$

Then, the estimate of $\hat{\mathbf{x}}$ is given by solving the least-squares problem

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{Q}\mathbf{x} - \boldsymbol{\tau}\|_2^2, \ \text{ where } \boldsymbol{\tau} \in \mathbb{R}^N, [\boldsymbol{\tau}]_n = \tau[n] \tag{11}$$

3. JOINT PROBABILISTIC FORECASTS

After establishing a relationship between temperature and solar irradiance, we provide the joint description of future values i.e. probabilistic forecasts of temperature and solar irradiance.

3.1. Stochastic model for solar irradiance

In our prior work in [6], a stochastic model for solar PV power was described using a regime-switching process as shown in Fig.3 by assuming that solar PV power comes from one of the three regimes: *sunny*, *partly cloudy* and *overcast*. These models were then used to provide probabilistic forecasts of solar PV power. We can extend the same method for solar irradiance by recognizing that the stochasticity in power comes from irradiance.

Let $s_d[n]$ be the solar irradiance on a sunny day d. It can be calculated using formulas based on latitude, longitude and time [12]. Solar irradiance, $w[n]$, corresponding to the *sunny* regime is,

$$w[n] = s_d[n] + \eta[n], \ \eta[n] \sim \mathcal{N}(0, \sigma_s^2) \tag{12}$$

where σ_s^2 is the modeling error. At times with *overcast* regime, a scaling, ρ_d, of the sunny day (clear sky) irradiance is observed,

$$w[n] = \rho_d s_d[n] + \eta[n], \ \eta[n] \sim \mathcal{N}(0, \sigma_{oc}^2) \tag{13}$$
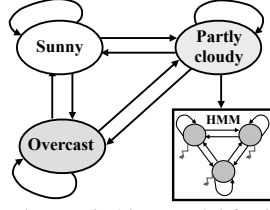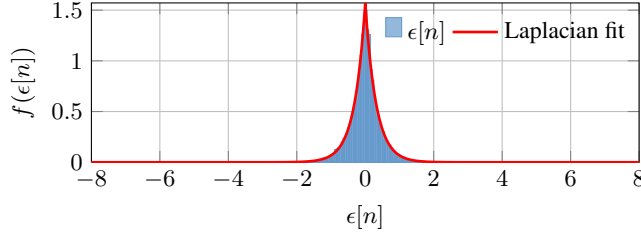
Fig. 3: Regime switching model for irradiance

Fig. 4: Laplacian fit for $\epsilon[n]$ with $\mu = 3.1422$

where σ_{oc}^2 is the variance of power in *overcast* regime. For the *partly cloudy* case, a hidden Markov model (HMM) is defined using parameters that pertain to attenuation of solar irradiance,

$$w[n] = s_d[n] - \mathbf{q}_n^T \mathbf{z}_n \tag{14}$$

$$\mathbb{E}\{\mathbf{q}_{n+1}|\mathbf{q}_n\} = \mathbf{A}^T \mathbf{q}_n, \quad \mathbf{A} \in \mathbb{R}^{N_s \times N_s} \tag{15}$$

$$\mathbf{z}_n = \begin{bmatrix} 0 & z_n(1) & \cdots & -z_n(N_s - 1) \end{bmatrix}^T \tag{16}$$

The matrix $\mathbf{A}$ is the state transition matrix where $\mathbf{A}(i,j)$ is the probability of going from state i to state j. The state vector $\mathbf{q}_n \in \mathbb{R}^{N_s \times 1}$ is a binary vector taking values from the set of coordinate vectors $\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_{N_s}\}$ where $\mathbf{e}_i$ has a 1 at position i and zero elsewhere. The coefficients in $\mathbf{z}_n$ are assumed to be drawn from exponential distributions with different parameters given the state i. Each of the states i correspond to attenuation of direct and diffuse beam irradiation components [12] of solar irradiance. Transition matrix $\mathbf{A}$ and the input $\mathbf{z}_n$ help to capture the memory in diffuse beam attenuation. More details can be found in [6]. In a simplified representation, the conditional probability distribution given the state i has the following form, $f_{pc}(w[n]|i) \triangleq f(w[n]|\mathbf{q}_n = \mathbf{e}_i)$ and is equal to,

$$f_{pc}(w[n]|i) = \begin{cases} \delta(s_d[n] - w[n]), & i = 1 \\ \lambda_i \exp\{-\lambda_i (s_d[n] - w[n])\}, & i = 2, \ldots, N_s \\ \lambda_i \exp\{-\lambda_i (w[n] - s_d[n])\}, & i = N_s \end{cases} \tag{17}$$

3.2. Conditional distribution of temperature given irradiance

After solving the least-squares problem in section 2, we analyzed the residual $\epsilon[n]$ from (10) and found that it can be approximated to be drawn from a Laplacian distribution with parameter μ. To write the conditional distribution of temperature at time n given all the past values of temperature and irradiance, we define the vectors

$$\mathbf{w}_n^{-M_2} = \begin{bmatrix} w[n] & w[n-1] & \cdots & w[n-M_2] \end{bmatrix}^T$$

$$\boldsymbol{\tau}_n^{-M_1} = \begin{bmatrix} \tau[n] & \tau[n-1] & \cdots & \tau[n-M_1] \end{bmatrix}^T$$

$$\mathbf{s}_n^{-M_2} = \begin{bmatrix} s_d[n] & s_d[n-1] & \cdots & s_d[n-M_2] \end{bmatrix}^T$$

Then we can write the conditional distribution from (10) as

$$f\left(\tau[n]\bigg|\mathbf{w}_n^{-M_2}, \boldsymbol{\tau}_{n-1}^{-M_1}\right) = \frac{\mu}{2} \exp\left(-\mu\big|\tau[n] - g(\mathbf{w}_n^{-M_2}, \boldsymbol{\tau}_{n-1}^{-M_1})\big|\right), \tag{18}$$

$$g(\mathbf{w}_n^{-M_2}, \boldsymbol{\tau}_{n-1}^{-M_1}) = \sum_{q=1}^{M_1} \gamma[q]\tau[n-q] + \sum_{k=0}^{M_2-1}\sum_{p=1}^{M_2-k} \alpha[k]\tau[n-p]\tau[n-p-k]$$
$$+ \sum_{k=0}^{M_2-1}\sum_{p=1}^{M_2-k} \beta[k]w[n-p+1]\tau[n-p-k]$$

3.3. Joint distribution and probabilistic forecast

From law of total probability, we can write the joint distribution of $\tau[n], w[n]$ at time n given $\mathbf{w}_{n-1}^{-M_2}$ and $\boldsymbol{\tau}_{n-1}^{-M_1}$ as

$$f\left(\tau[n], w[n]\bigg|\mathbf{w}_{n-1}^{-M_2}, \boldsymbol{\tau}_{n-1}^{-M_1}\right) = f\left(\tau[n]\bigg|w[n], \mathbf{w}_{n-1}^{-M_2}, \boldsymbol{\tau}_{n-1}^{-M_1}\right) f(w[n]) \tag{19}$$

Define vectors of forecasts of $\tau[n]$ and $w[n]$ for m steps ahead as

$$\mathbf{w}_n^{+m} = \begin{bmatrix} w[n+m] & w[n+m-1] & \cdots & w[n+1] \end{bmatrix}^T, m = 1, 2, \ldots$$

$$\boldsymbol{\tau}_n^{+m} = \begin{bmatrix} \tau[n+m] & \tau[n+m-1] & \cdots & \tau[n+1] \end{bmatrix}^T, \boldsymbol{\tau}_n^{+0} = \boldsymbol{\tau}_n^{-M_1}$$

We are interested in probabilistic forecasts i.e. the joint distribution which following (19) and probability chain rule we write as,

$$f\left(\mathbf{w}_n^{+m}, \boldsymbol{\tau}_n^{+m}\bigg|\mathbf{w}_n^{-M_2}, \boldsymbol{\tau}_n^{-M_1}\right) = \tag{20}$$

$$\prod_{\ell=1}^{m} f\left(\tau[n+\ell]\bigg|\mathbf{w}_n^{+m}, \boldsymbol{\tau}_n^{+\ell-1}, \mathbf{w}_n^{-M_2}, \boldsymbol{\tau}_n^{-M_1}\right) f\left(\mathbf{w}_n^{+m}\bigg|\mathbf{w}_n^{-M_2}\right)$$

Each component in the product over ℓ is the conditional distribution of temperature $\tau[n+\ell]$ given current and all past values of irradiance, $\mathbf{w}_n^{+\ell}, \mathbf{w}_n^{-M_2}$ and past values of temperature $\boldsymbol{\tau}_n^{+\ell-1}, \boldsymbol{\tau}_n^{-M_1}$. This can be written using (18). For probabilistic forecast of solar irradiance, it is assumed that the current regime 'persists'. It is also assumed that $w[n]$ are temporally independent given the regime. Therefore,

$$f\left(\mathbf{w}_n^{+m}\bigg|\mathbf{w}_n^{-M_2}\right) = \begin{cases} f_s(\mathbf{w}_n^{+m}), & \|\mathbf{w}_n^{-M_2} - \mathbf{s}_n^{-M_2}\|_2 \leq \nu_s \\ f_{oc}(\mathbf{w}_n^{+m}), & \arg\min_{\rho} \|\mathbf{w}_n^{-M_2} - \rho\mathbf{s}_n^d\|_2 \leq \rho_{oc} \\ f_{pc}(\mathbf{w}_n^{+m}), & \text{otherwise} \end{cases}$$

where ν_s is the error threshold set to classify as *sunny*, ρ_{oc} is the threshold for parameter ρ_d to decide in favor of *overcast* regime. The probabilistic forecasts given the regime *sunny* from (12),

$$f_s(\mathbf{w}_n^{+m}) = (2\pi\sigma_s)^{-\frac{m}{2}} \exp\left(-\|\mathbf{w}_n^{+m} - \mathbf{s}_n^{+m}\|_2^2 / 2\sigma_s^2\right) \tag{21}$$

regime *overcast* from (13),

$$f_s(\mathbf{w}_n^{+m}) = (2\pi\sigma_{oc})^{-\frac{m}{2}} \exp\left(-\|\mathbf{w}_n^{+m} - \hat{\rho}\mathbf{s}_n^{+m}\|_2^2 / 2\sigma_{oc}^2\right) \tag{22}$$

regime *partly cloudy* from (17) and (14),

$$f_{pc}(\mathbf{w}_n^{+m}) = \prod_{\ell=1}^{m}\sum_{j=1}^{N_s} \mathbf{A}^{\ell}(\hat{i}, j) f_{pc}(w[n]|j) \tag{23}$$

where $\hat{i}$ is the most probable state at time n. This is estimated by Viterbi algorithm [13, see section III.B] using current and past values of solar irradiance, $\mathbf{w}_n^{-M_2}$ and state transition matrix $\mathbf{A}$.
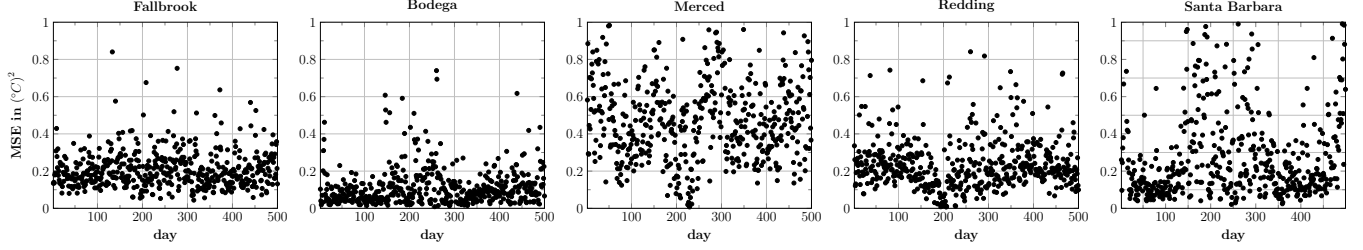
Fig. 5: Mean squared error (MSE) of the proposed model fit to temperature data from 5 NOAA weather stations in California, USA.

4. NUMERICAL RESULTS

Experiments were run on a dataset of solar irradiance (in W/m^2) and temperature (in $^\circ C$) obtained from 5 NOAA weather stations in California. Data is sampled at 15-minute intervals. For estimating the parameters of model for temperature from (10), $\alpha[k], \beta[k], \gamma[q]$, it was assumed that $M_1 = 12$ i.e. 3 hours and $M_2 = 4$ i.e. 1 hour. Temperature and irradiance data for 500 contiguous days were used for estimation. The mean squared error (MSE) after fitting the model for each of the 500 days is highlighted in Fig.5 for the 5 weather stations. The variance of MSE among different days is seasonal in nature and is higher for the stations of Merced and Santa Barbara. This can be attributed to erratic weather conditions in these regions as well as higher presence of wind and humidity (in Santa Barbara) and is considered as noise in the modeling process.

As an example, results from the Fallbrook station are shown in this paper. The fit of the model to temperature data along with the corresponding irriadiance is highlighted in Fig. 6 for a week in May 2014 In Fig.7, the estimated parameters $\hat{\alpha}[k], \hat{\beta}[k], \hat{\gamma}[q]$ are plotted.

Joint Probabilistic forecasts: To highlight the efficacy of the probabilistic forecasts, $m = 2$ step ahead forecast i.e. 30 minutes ahead was performed. The joint probability density takes the form of (20) with $m = 2$. This gives a 4-D probability distribution (2-D for temperature and 2-D for irradiance). The accuracy of the probabilistic forecast is evaluated in two ways: Continuous ranked probability score (CRPS),

$$\text{CRPS}(\boldsymbol{w}_n^{+m}, \boldsymbol{\tau}_n^{+m}) = \int\limits_{\tilde{\boldsymbol{w}}_n^{+m}, \tilde{\boldsymbol{\tau}}_n^{+m}} \Big[F\left(\tilde{\boldsymbol{w}}_n^{+m}, \tilde{\boldsymbol{\tau}}_n^{+m} \big| \boldsymbol{w}_n^{-M_2}, \boldsymbol{\tau}_n^{-M_1}\right) +$$

$$- u\left(\tilde{\boldsymbol{w}}_n^{+m} - \boldsymbol{w}_n^{+m}\right) u\left(\tilde{\boldsymbol{\tau}}_n^{+m} - \boldsymbol{\tau}_n^{+m}\right) \Big]^2 d\tilde{\boldsymbol{w}}_n^{+m} d\tilde{\boldsymbol{\tau}}_n^{+m} \quad (24)$$

where $(\boldsymbol{w}_n^{+m}, \boldsymbol{\tau}_n^{+m})$ are the observed values and $F\left(\cdot|\cdot\right)$ is the joint CDF (from (20)) and $u\left(\boldsymbol{a}\right)$ is the Heaviside function acting on a
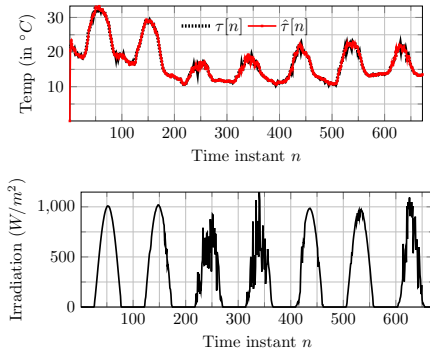

Fig. 6: Estimate of temperature, $\hat{\tau}[n]$ and corresponding irradiance $w[n]$ for May $2 - 12, 2014$ at NOAA weather station Fallbrook, CA.
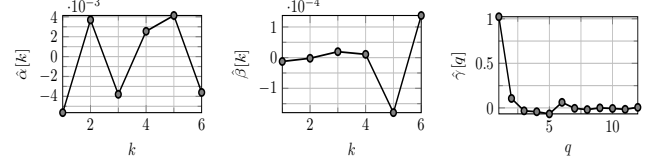

Fig. 7: Estimates, $\hat{\alpha}[k], \hat{\beta}[k], \hat{\gamma}[q]$ for Fallbrook.
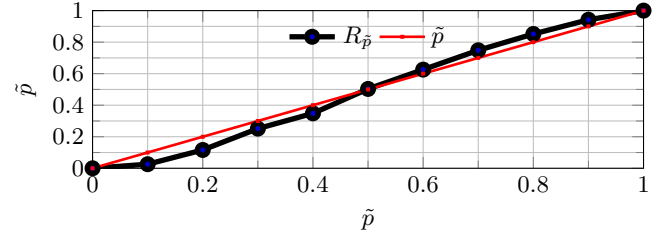

Fig. 8: Reliability $R_{\tilde{p}}$ of joint probabilistic forecasts for 2 steps ahead.

vector. CRPS in the point forecast (single value) case reduces to mean absolute error (MAE)[7] after averaging over all observed samples $\boldsymbol{w}_n^{+m}, \boldsymbol{\tau}_n^{+m}$. Thus, the units of CRPS in our case becomes $(W/m^2)^2 (^\circ C)^2$. We performed forecasting experiments for 30 days and for 15 time- intervals within each day and achieved a mean CRPS $= 489.5 \; (W/m^2)^2 (^\circ C)^2$.

Since it is difficult to evaluate the performance with CRPS without benchmarks, we use the metric of reliability[7]. Let a probability interval (PI), I, cover the observed values $\boldsymbol{w}_n^{+m}, \boldsymbol{\tau}_n^{+m}$ with probability $\tilde{p}$.

$$\text{Reliability: } R_{\tilde{p}} = \left(\textstyle\sum_n 1\right)^{-1} \textstyle\sum_n \mathbb{I}_{\left((\boldsymbol{w}_n^{+m}, \boldsymbol{\tau}_n^{+m}) \in I\right)} \quad (25)$$

where $\mathbb{I}$ is an indicator function with value 1 if the observed sample belongs to the probability interval obtained using the joint CDF. Fig.8 shows the reliability of prediction using our forecasting technique. Ideally, it should be close to the diagonal line $\tilde{p}$. The reliability of our method is good as can be seen from the figure. Broadly, it is demonstrative of the fact that the predictive distribution (CDF) we provide is close to the observed CDF of real data.

5. CONCLUSIONS

In this paper, we derived a Volterra-type system relating temperature and solar irradiance. Using this relationship, we provided joint probabilistic forecasts of solar irradiance and temperature for 30 minutes ahead. We evaluated the performance of our technique using a dataset from a weather station in California. The model fit very well with the temperature data. The joint probabilistic forecasts were reliable and provided a low (CRPS). Future work includes taking wind variability and spatial correlation into consideration.

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