UNDERWATER OPTICAL SENSOR NETWORKS LOCALIZATION WITH LIMITED CONNECTIVITY

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ABSTRACT

In this paper, a received signal strength (RSS) based localization technique is investigated for underwater optical wireless sensor networks (UOWSNs) where optical noise sources (e.g., sunlight, background, thermal, and dark current) and channel impairments of seawater (e.g., absorption, scattering, and turbulence) pose significant challenges. Hence, we propose a localization technique that works on the noisy ranging measurements embedded in a higher dimensional space and localize the sensor network in a low dimensional space. Once the neighborhood information is measured, a weighted network graph is constructed, which contains the one-hop neighbor distance estimations. A novel approach is developed to complete the missing distances in the kernel matrix. The output of the proposed technique is fused with Helmert transformation to refine the final location estimation with the help of anchors. The simulation results show that the root means square positioning error (RMSPE) of the proposed technique is more robust and accurate compared to baseline and manifold regularization.

***Index Terms*—** Underwater, Sensor Networks, Optical Communication, Localization

1. INTRODUCTION

The recent urge on the high quality of service communication for commercial, scientific and military applications of underwater exploration necessitates high data rate, low latency, and long-range networking solutions [1–3]. Achieving such goals is a daunting challenge because of the highly attenuating medium of seawater for most electromagnetic frequencies. In the past decades, acoustic systems have therefore received a considerable attention. However, underwater acoustic communication has low achievable rates (10-100 kbps) due to the limited bandwidth. Moreover, the low propagation speed of acoustic waves (1500 m/s) induces a high latency, especially for long-range applications [4].

On the other hand, underwater optical wireless communication (UOWC) has the advantages of higher bandwidth, lower latency, and enhanced security [5]. Nevertheless, UOWC has a very limited range attainability (10-100 m) due to absorption, scattering, and turbulence impairments of seawater. It is also susceptible to many noise sources such as sunlight, background, thermal, and dark current noises [6].

In particular, localization of nodes in underwater optical wirless sensor network (UOWSN) is of utmost importance since the gathered data from the sensor nodes must be associated with the location information as it is useful only if it refers to a particular location within the network [7]. Localization is especially useful for a number of applications such as target detection, intruder detection, routing protocols, and data tagging. Hence, a number of acoustic underwater sensor network localization techniques have been proposed in the past. These localization techniques consider different parameters such as signal propagation model, network topology, environmental factors, localization accuracy, the number of anchor nodes, the geometry of the anchor nodes, and the relative location of the sensor node to the anchors [8–14]. Anchor-based localization techniques can be broadly categorized into range-free and range-based techniques. The performance of every localization technique mainly relies on the initial reference position, number of sensor nodes, ranging technique, number of anchors, and the position of the anchors in the network [15].

However, aforementioned optical communication challenges do not allow the use of existing acoustic localization techniques for underwater sensor nodes. To the best of our knowledge, UOWSN localization is only addressed in [16] where RSS and time of arrival methods are investigated for an optical code-division multiple access networks. In [16], fully connected network is considered where the source is able to communicate with all the anchors directly. In UOWSNs, the transmission range is limited and muti-hop communication is more practical where the sensor nodes may not be directly connected to the anchors [17, 18]. Accordingly, in this paper, we investigate an RSS based UOWSN localization with the following contributions: We propose a localization technique for UOWSNs with limited connectivity that operates on the noisy ranging measurements embedded in a higher dimensional space and localizes the sensor network in a low dimensional space. A novel approach is developed to complete the missing distances in the kernel matrix which reduces the shortest path distance estimation error. To further reduce

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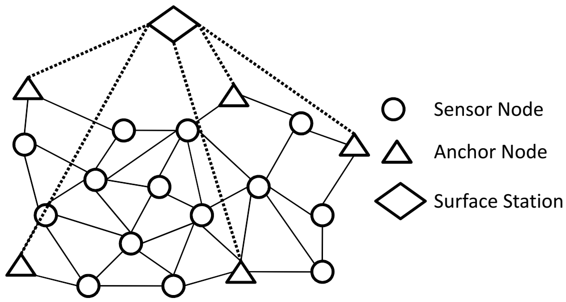

Fig. 1. Underwater optical wireless sensor network setup.

the localization error, the output of the proposed technique is fused with the Helmert transformation. Simulation results show that the root mean square positioning error (RMSPE) of the proposed technique is more robust and accurate compared to the baseline [19] and manifold regularization [20].

The rest of the paper is organized as follows: Section 2 discusses the problem formulation and proposed technique. In Section 3 performance of the proposed technique is analyzed. In Section 4, simulations are conducted for the performance evaluation of the proposed technique and Section 5 concludes the paper with a few remarks.

2. PROBLEM FORMULATION AND PROPOSED TECHNIQUE

We consider an UOWSN consisting of m sensor nodes and n anchor nodes, which are all capable of optical communication. It is assumed that the optical sensor nodes and anchor nodes are capable of sweeping the circular region around itself. Therefore the network is represented as a undirected graph $H = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ are the vertices (nodes) of the graph and $\mathcal{E}$ are the associated links. The link e_{ij} exists between any two nodes i and j if they are in communication range of each other. The proposed system configuration is shown in Fig. 1. Given the location of anchor nodes and single hop neighborhood distances, the problem of UOWSN localization is to find the location of m sensor nodes. Modeling of RSS based ranging in aquatic medium is different and more challenging than the free space. Optical light passing through the aquatic medium suffers from widening and attenuation in angular, temporal and spatial domains [16]. The widening and attenuation of the underwater optical signals are dependent on the wavelength. Combining the absorption coefficient $a(\lambda)$ and scattering coefficient $s(\lambda)$ results in the extinction coefficient [21], i.e.,

$$e(\lambda) = a(\lambda) + s(\lambda). \tag{1}$$

The propagation loss as a function of distance d_{ij} and wavelength λ between any two sensor nodes i and j is given by

$$L_{ij} = \exp^{-e(\lambda)d_{ij}}, \tag{2}$$

where $d_{ij} = \|\boldsymbol{c}_i - \boldsymbol{c}_j\|$, $\boldsymbol{c}_i$ and $\boldsymbol{c}_j$ represents the 2-dimensional locations of node i and j, respectively. In this paper we consider line of sight communication, where the sensor node i directs the trajectory of its transmitter toward the receiver node j. Then, the received power at sensor node j is given as [22],

$$P_{r_j} = P_{t_i}\eta_i\eta_j L_{ij}\frac{A_j \cos\theta}{2\pi d_{ij}^2(1-\cos\theta_0)}, \tag{3}$$

where P_{t_i} is the optical power transmitted by node i, η_i and η_j are the optical efficiencies of node i and j, respectively, A_j is the aperture area of node j, θ is the angle between node i and j trajectory, and θ_0 is the divergence angle of transmitted signal.

The problem of UOWSN localization is to find the estimated locations for the m nodes given that the noisy range measurements $\boldsymbol{\Pi} = [\gamma_{ij}]_{m+n\times m+n}$ between all the nodes in the network are available. If nodes i and j are not in the range of each other, then the geodesic distances between them is considered using the intermediate nodes. The missing elements in the kernal matrix $\boldsymbol{\Pi}$ are approximated by

$$\gamma_{ij} = \frac{\acute{R}_{min} + \acute{R}_{max}}{2}, \tag{4}$$

where $\acute{R}_{min}$ and $\acute{R}_{max}$ are the minimum and maximum achievable distances in the network respectively. $\acute{R}_{min}$ is the length of the shortest edge and $\acute{R}_{max}$ is the length of the longest edge in $H = (\mathcal{V}, \mathcal{E})$ respectively. Note that the approach in (4) is different than the shortest path distance estimation used in [19] and [20]. It is well known that noise is distance dependent factor, i.e., the noise increases with the distance. Moreover, the underwater optical communication signals are affected by different environmental parameters other than distance which makes it more unpredictable. Thus, the transformation from the noisy measurements in a high dimensional space to a low dimension space is a non-linear process. Therefore, UOWSN localization requires to develop a non-linear projection technique from a noisy high dimensional space to the actual low dimensional space. In the proposed non linear projection technique, if there are m anchor nodes and n optical sensor nodes, then by exploiting the hidden information in the network, all the nodes are localized. The nonlinear projection function $g : d_{ij} \longrightarrow \boldsymbol{C}$, where $\boldsymbol{C} = \{\boldsymbol{c}_1, ..., \boldsymbol{c}_{m+n}\}$ denotes the actual location of all the nodes in the network, is defined as

$$g(d_{ij}|\boldsymbol{C}) = \sqrt{\sum_{i\neq j=1\ldots m+n} (\gamma_{ij} - d_{ij})^2 \,\mathsf{W}_{ij}}, \tag{5}$$

where $\mathsf{W}_{ij} = \frac{1}{\gamma_{ij}^2}$ are the associated weights for each link. The above problem is a well known optimization problem which can be solved by spectral decomposition of the double centered matrix for the kernel matrix $\boldsymbol{\Pi}$. Squaring and centering $\boldsymbol{\Pi}$, we obtain

$$\boldsymbol{A} = -1/2(\boldsymbol{T}\boldsymbol{\Pi}\boldsymbol{T}), \tag{6}$$

where $\boldsymbol{T} = \boldsymbol{I}_{(m+n)} - (1/(m+n))\mathbf{1}_{(m+n)}\mathbf{1}^T_{(m+n)}$ is centering operator, $\boldsymbol{I}_{(m+n)}$ is $(m+n)\times(m+n)$ is identity matrix, and $\mathbf{1}_{(m+n)}$ is all ones vector. Decomposing $\boldsymbol{A}$ yields

$$\boldsymbol{A} = \boldsymbol{E}\boldsymbol{Q}\boldsymbol{E}^T, \tag{7}$$

where $\boldsymbol{E}$ and $\boldsymbol{Q}$ are the eigen-vectors and eigen-values of $\boldsymbol{A}$. The lower 2 dimensional graph for all the nodes in the network is drawn from the 2 largest eigen-vectors in $\boldsymbol{E}$ and the corresponding eigen-values in $\boldsymbol{Q}$ as

$$\hat{\boldsymbol{C}} = \boldsymbol{E}_{(m+n)\times 2}\sqrt{\boldsymbol{Q}_{2\times 2}}. \tag{8}$$

After getting the initial estimated locations, they are transformed into actual locations using the Helmert transformation [23]. Based on n anchor nodes, $1 \leq h \leq n$, their locations are known in the 2-dimensional space, having absolute locations $\boldsymbol{O}_1 = \{\boldsymbol{c}_1, ...\boldsymbol{c}_n\}$. The relative locations $\hat{\boldsymbol{O}}_1 = \{\hat{\boldsymbol{c}}_1, ..., \hat{\boldsymbol{c}}_n\}$ need to match the absolute locations $\boldsymbol{O}_1$ for n anchors. The mis-matching between the absolute and relative locations is calculated by

$$\nu = \sum_{h=1}^{n} (\hat{\boldsymbol{c}}_h - \boldsymbol{c}_h)^T (\hat{\boldsymbol{c}}_h - \boldsymbol{c}_h). \tag{9}$$

The actual function for Helmert transformation is defined as

$$\begin{aligned} t(\Phi, \Psi, \Omega) \;&=\; \sum_{h=1}^{n} (\hat{\boldsymbol{c}}_h - \Phi\Omega^T\boldsymbol{c}_h - \Psi)^T \\ &\quad (\hat{\boldsymbol{c}}_h - \Phi\Omega^T\boldsymbol{c}_h - \Psi), \end{aligned} \tag{10}$$

where Φ, Ψ and Ω represents the scaling, shifting, and rotation, respectively. The Helmert function is minimized by using optimum values of Φ, Ψ and Ω

$$\{\hat{\Phi}, \hat{\Psi}, \hat{\Omega}\} = arg \min_{\Phi,\Psi,\Omega} t(\Phi, \Psi, \Omega). \tag{11}$$

Let a_o and r_o be the centroids of the absolute locations and relative locations of the anchors, respectively, and given as $a_o = \frac{1}{n}\sum_{h=1}^{n}\boldsymbol{c}_h$ and $r_o = \frac{1}{n}\sum_{h=1}^{n}\hat{\boldsymbol{c}}_h$. The optimal shifting is then

$$\begin{aligned} t(\Phi, \Psi, \Omega) \;&=\; \sum_{h=1}^{n}((\hat{\boldsymbol{c}}_h - r_o) - \Phi\Omega^T(\boldsymbol{c}_h - a_o) \\ &\quad +\hat{\boldsymbol{c}}_h - \Phi\Omega^T\boldsymbol{c}_h - \Psi)^T \\ &\quad \times((\hat{\boldsymbol{c}}_h - r_o) - \Phi\Omega^T(\boldsymbol{c}_h - a_o) \\ &\quad +\hat{\boldsymbol{c}}_h - \Phi\Omega^T\boldsymbol{c}_h - \Psi), \end{aligned} \tag{12}$$

The optimal shifting element Ψ that minimizes the Helmert objective function is $\Psi = r_o - \Phi^T a_o$. With this transformation a_o and r_o becomes zero, thus

$$t(\Phi, \Psi, \Omega) = \sum_{h=1}^{n} (\hat{\boldsymbol{c}}_h - \Phi\Omega^T\boldsymbol{c}_h)^T (\hat{\boldsymbol{c}}_h - \Phi\Omega^T\boldsymbol{c}_h). \tag{13}$$

Differentiating (13) with respect to Φ, $t(\Phi, \Psi, \Omega)$ is minimized at the optimal Φ, i.e.,

$$\Phi = \frac{\mathrm{Tr}(\boldsymbol{O}_1\Omega\hat{\boldsymbol{O}}_1^T)}{\mathrm{Tr}(\boldsymbol{O}_1\boldsymbol{O}_1^T)}. \tag{14}$$

The rotation Ω is then given as $\Omega = \boldsymbol{v}\boldsymbol{u}$, where $\boldsymbol{v}$ are the eigen-vectors and $\boldsymbol{u}$ are the eigen-values of $\boldsymbol{O}_1\hat{\boldsymbol{O}}_1^T$ respectively. The estimated global locations of all the nodes in the network are then given as

$$\tilde{\boldsymbol{C}} = \Phi\Omega^T(\hat{\boldsymbol{C}}) + \Psi. \tag{15}$$

3. PERFORMANCE ANALYSIS

The UOWSN localization problem is similar to the parameter estimation. Therefore, to evaluate any parameter estimation problem, the minimum unbiased variance estimation is taken as the evaluation criteria of any localization algorithm. Thus, the Cramer-Rao lower bound (CRLB) is commonly used as an unbiased parameter estimator to evaluate the performance of parameter estimation. Since the range measurements γ_{ij} between nodes are affected by Gaussian noise, the probability density function (PDF) for the range measurements γ_{ij} is given by

$$f(\gamma_{ij}|\boldsymbol{c}_i, \boldsymbol{c}_j) = \frac{1}{\eta_r\sqrt{2\pi}}\exp^{\left(-\frac{(\gamma_{ij}-d_{ij})^2}{2\eta_r^2}\right)}, \tag{16}$$

where η_r is the noise variance. To construct the corresponding fisher information matrix (FIM), log-likelihood ratio for the given PDF in (16) can be written as

$$\begin{aligned} \vartheta_{ij}[\mathrm{dB}] \;&=\; -\log\sqrt{2\pi\mu} - \frac{\beta_{ij}}{4}\log(\|\boldsymbol{c}_i - \boldsymbol{c}_j\|^2) \\ &\quad -\frac{1}{2\mu}\frac{(\gamma_{ij} - \|\boldsymbol{c}_i - \boldsymbol{c}_j\|)^2}{(\|\boldsymbol{c}_i - \boldsymbol{c}_j\|^2)^{\beta_{ij}/2}}, \end{aligned} \tag{17}$$

where μ is scalar constant and β_{ij} is the path loss exponent, then the joint log-likelihood ratio for all the nodes in the network is

$$\boldsymbol{\Theta} = \sum_{i=1}^{m+n}\sum_{j=i+1}^{m+n}\log\left(f(\gamma_{ij}|\boldsymbol{c}_i, \boldsymbol{c}_j)\right). \tag{18}$$

From (18), a sub-matrix $\mathbf{F}_i$ of FIM is defined as

$$\mathbf{F}_i = \begin{bmatrix} \mathbf{F}_{2i-1,2i-1} & \mathbf{F}_{2i-1,2i} \\ \mathbf{F}_{2i,2i-1} & \mathbf{F}_{2i,2i} \end{bmatrix}, \; i = 1, 2, 3......K, \tag{19}$$

where each element in (19) is defined as

$$\begin{aligned} &\mathbf{F}_{2i-1,2j-1} = \mathbf{F}_{2j-1,2i-1} = \\ &\begin{cases} \sum_{j\in H(i)} \frac{1}{\eta_r^2}\frac{(x_i - x_j)^2}{\|\boldsymbol{C}_i - \boldsymbol{C}_j\|^2} & i = j, \\ \frac{-1}{\eta_r^2}\frac{w_{ij}(x_i - x_j)^2}{\|\boldsymbol{C}_i - \boldsymbol{C}_j\|^2} & j \in H(i) \text{ and } j \neq i, \\ 0 & j \notin H(i) \end{cases} \end{aligned} \tag{20}$$

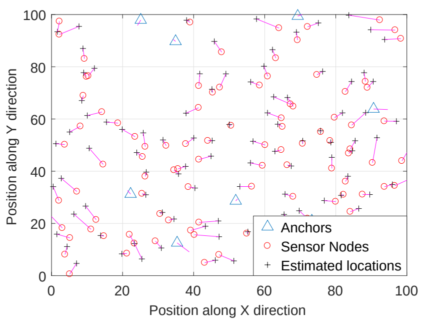

Fig. 2. Estimated Locations.

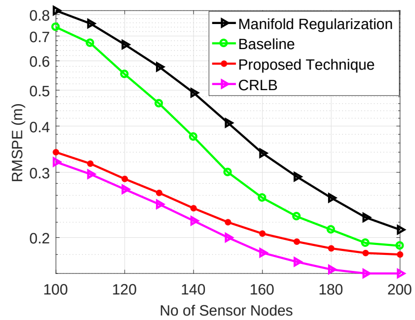

Fig. 3. RMSPE vs. No. of sensor nodes.

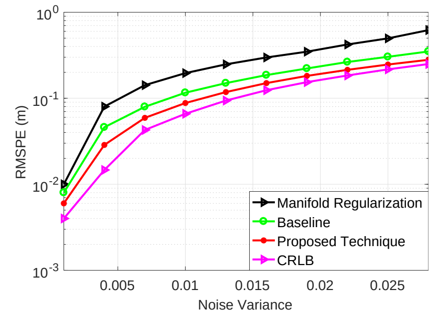

Fig. 4. RMSPE vs. Noise variance.

Table 1. Simulation Parameters

Parameter	Values
Area	$100 \times 100\ m^2$
Number of Sensor Nodes	100-200
Number of Anchors	10
Variance	0.001-0.029
Range	5-50 m

$$\mathbf{F}_{2i-1,2j} = \mathbf{F}_{2j,2i} = \begin{cases} \sum_{j\in H(i)} \frac{1}{\eta_r^2} \frac{(y_i-y_j)^2}{\|\boldsymbol{C}_i-\boldsymbol{C}_j\|^2} & i=j, \\ \frac{-1}{\eta_r^2} \frac{w_{ij}(y_i-y_j)^2}{\|\boldsymbol{C}_i-\boldsymbol{C}_j\|^2} & j\in H(i) \text{ and } j\neq i\ , \\ 0 & j\notin H(i) \end{cases} \tag{21}$$

$$\mathbf{F}_{2i-1,2j} = \mathbf{F}_{2j,2i-1} = \mathbf{F}_{2i,2j-1} = \mathbf{F}_{2j-1,2i} = \begin{cases} \sum_{j\in H(i)} \frac{1}{\eta_r^2} \frac{(x_i-x_j)(y_i-y_j)}{\|\boldsymbol{C}_i-\boldsymbol{C}_j\|^2} & i=j, \\ \frac{-1}{\eta_r^2} \frac{(x_i-x_j)(y_i-y_j)}{\|\boldsymbol{C}_i-\boldsymbol{C}_j\|^2} & j\in H(i) \text{ and } j\neq i\ . \\ 0 & j\notin H(i) \end{cases} \tag{22}$$

where ω_{ij} is the distance dependent scaling factor. Then the CRLB is obtained as

$$\mathrm{E}\left((\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2\right) \geq (\mathbf{F}_i^{-1})_{1,1} + (\mathbf{F}_i^{-1})_{2,2}, \tag{23}$$

where $(\mathbf{F}_i^{-1})_{1,1}$ and $(\mathbf{F}_i^{-1})_{2,2}$ are the diagonal elements of $(\mathrm{FIM})^{-1}$.

4. SIMULATION RESULTS

Unless it is explicitly stated otherwise, we evaluate the performance of the proposed localization method by employing the simulation parameters summarized in Table 1. Fig. 2 shows an UOWSN scenario with 100 sensor nodes and 10 anchors randomly distributed over $100 \times 100\ m^2$ area. The transmission ranges for this setup is set to $30m$ and the noise variance is distance dependent. The performance metric is considered to be root mean square positioning error which is defined as RMSPE $= \frac{\sqrt{\sum_{i=1}^{m+n}(\boldsymbol{C}_i - \tilde{\boldsymbol{C}}_i)^2}}{m+n}$.

Fig. 2 shows the actual and estimated locations of the nodes in the deployed area where $\bigcirc$ represents the sensor nodes, anchor nodes are represented by $\triangle$, the estimated locations for all the nodes in the network are represented by $+$, and the line connecting $\bigcirc$ to $\triangle$ shows the localization error of the proposed technique. Fig. 3 shows the performance of the proposed technique with respect to the network density. Increasing the number of nodes in the network reduces the RMSPE because it reduces the geodesic error in the network. To validate the performance of the proposed technique, results are compared with the CRLB, baseline [19] and manifold regularization [20]. Baseline and manifold regularization have higher RMSPE compare to the proposed technique reduces the shortest path estimation error for multi-hop networks. Fig. 4 presents the RMSPE performance with respect to the noise variance where the noise variance is increased from 0.001 to 0.029 m. The RMSPE increases with increase in the noise variance for the transmission range, it can be seen that the proposed technique absorbs the inaccuracies in the RSS measurements and is more robust compare to the baseline and manifold regularization.

5. CONCLUSIONS

In this paper, a novel matrix completion strategy is developed for UOWSNs localization. The RSS measurements for underwater optical communication are inaccurate and would introduce large localization error. However, the proposed technique is robust to absorb the inaccuracies in the RSS measurement and provide better results. The proposed technique first collects the noisy RSS measurements and estimate the initial location of sensor nodes and then the final coordinate transformation is achieved by using Helmert transformation. The CRLB have also been derived to analyze the performance of the proposed technique. Simulations show that the proposed technique provides enhanced localization performance to get more robust and accurate results and achieves the CRLB.

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