MULTIPLE PEER-TO-PEER BIDIRECTIONAL COOPERATIVE COMMUNICATIONS USING MASSIVE MIMO RELAYS

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ABSTRACT

We study a two-way relay network where multiple multi-antenna relays facilitate two-way communications between multiple pairs of transceivers. Each relay is equipped with a massive number of antennas. As a result, we can assume that the transceiver-relay channel vectors are approximately orthogonal, and thus, intra- and inter-pair interference will be negligible. Aiming to maintain the signal-to–noise ratio (SNR) at receiver front-end of each transceiver above a certain threshold, we obtain the relay beamforming matrices and the transceiver powers such that the total transmit power consumed in the entire network is minimized. To do so, we assume that the channel vectors between each relay and different transceivers are asymptotically orthogonal. For such power minimization problem, we derive computationally efficient solutions.

Index Terms— Two-way relay networks, massive MIMO, total power minimization, multi-pair, multiple relay, MRC/MRT.

1. INTRODUCTION

In the past decade, cooperative wireless networks have attracted much attention from the research community. Due to the spectral efficiency improvements, two-way relay networks have been of particular interest. Majority of studies on two-way relay networks consider a simple configuration comprising a pair of single-antenna transceivers and single-antenna relays. Aiming to exploit the benefits of local beamforming at the relays, the authors of [1–4] study bi-directional networks where two transceivers and a relay node are equipped with multiple antennas. Assuming multiple single-antenna relays, the studies in [5–7] investigate the problem of bi-directional collaborative distributed beamforming. To exploit the advantages offered by joint local and distributed beamforming the investigations in [8, 9] consider cooperative networks, where multiple multi-antenna relays are employed to facilitate communication between a pair of transceivers. The problem of multiple peer-to-peer communications using two-way relay networks is studied in [10] and [11], where a two-way relay network enables peer-to-peer communication among multiple pairs of transceivers. In such a network, inter- and intra-pair interference must be suppressed. One way to suppress interference at the relays is to equip the relays with a very large number of antennas, thereby materializing a massive MIMO relay network. Equipping network nodes with a massive number of antennas (often referred to as massive multiple input multiple output (MIMO) scheme) has been the center focus of a significant volume of studies. However, published results on two-way network with massive MIMO relays are still scarce.

We herein consider a two-way relay network with multiple massive MIMO relays which aim to enable bidirectional multiple peer-to-peer communications. Assuming maximum ratio combining/transmitting (MRC/MRT) schemes at the relays and exploiting the approximate orthogonality among relay-transceiver channel vectors, we provide a computationally efficient solution to the problem of minimizing the total transmit when the transceivers' signal-to-noise ratios (SNRs) are to be above given thresholds.

2. SYSTEM MODEL

We study a two-way relay network consisting of K pairs of single-antenna transceivers which wish to establish pairwise communications with the help of n_r relays. The relays are equipped with M antennas, and M is considered to be very large. Each relay uses an MRC/MRT method to obtain the relay's vector of transmitted signals from the vector of signals received by that relay. Here, the two time-slot multiple access broadcast (MABC) relaying scheme is considered. As such, in the first time-slot the transceivers simultaneously transmit their signals toward the relays. In the second time-slot, each relay forwards a linearly transformed version of the relay's received signal vector toward the transceivers. Denoting $\mathbf{x}_i$ as the $M \times 1$ vector of the signals received at the i-th relay in the first time-slot, we can write

$$\mathbf{x}_i = \mathbf{H}_i \mathbf{P}^{1/2} \mathbf{s} + \mathbf{n}_i, \quad \text{for} \quad i \in \{1, 2, \ldots, n_r\} \tag{1}$$

where $\mathbf{H}_i \triangleq [\mathbf{h}_{1i} \ \mathbf{h}_{2i} \ \cdots \ \mathbf{h}_{2K,i}]$ is the $M \times 2K$ matrix of the channel vectors between the i-th relay and the $2K$ transceivers. Indeed, the l-th column of $\mathbf{H}_i$, denoted as $\mathbf{h}_{li}$, is the $M \times 1$ channel vector between the M antennas of the i-th relay and the l-th transceiver, for $l \in \{1, 2, \ldots, 2K\}$ and $i \in \{1, 2, \ldots, n_r\}$. The $2K \times 2K$ matrix $\mathbf{P} \triangleq \mathrm{diag}\{p_1, p_2, \ldots, p_{2K}\}$ is a diagonal matrix whose k-th diagonal entry, denoted as p_l, represents the transmit power of the l-th transceiver, the vector $\mathbf{s} \triangleq [s_1 \ s_2 \ \cdots \ s_{2K}]^T$ denotes the $2K \times 1$ vector of the signals transmitted by all transceivers, and s_l represents the symbol transmitted by the l-th transceiver. Note that $\mathbf{h}_{(2k-1),i}$ and $\mathbf{h}_{2k,i}$ are the channel vectors between the two transceivers in the k-th pair, for $k \in \{1, 2, \ldots, K\}$. Similarly, s_{2k-1} and s_{2k} are the symbols transmitted by the two transceivers in the k-th pair with p_{2k-1} and p_{2k} as the corresponding transmit powers. That is, s_{2k-1} (s_{2k}) is transmitted by Transceiver $2k-1$ ($2k$) and is meant to be received by Transceiver $2k$ ($2k-1$). The $M \times 1$ vector $\mathbf{n}_i$ denotes the vector of the noises received at the M antennas of the i-th relay. Here, the elements of $\mathbf{n}_i$ are assumed to be zero-mean spatially and temporally white Gaussian noise with variance σ^2. The vector $\mathbf{x}_i$ received at the i-th relay is multiplied by a beamforming matrix $\mathbf{A}_i$. Let the $M \times 1$ vector $\mathbf{t}_i$ represent the vector of the signals transmitted by the i-th relay. We can then write $\mathbf{t}_i = \mathbf{A}_i \mathbf{x}_i$. The received signal at Transceiver l, denoted as y_l, can be written as

$$y_l = \sum_{i=1}^{n_r} \mathbf{h}_{li}^T \mathbf{t}_i + \eta_l = \sum_{i=1}^{n_r} \mathbf{h}_{li}^T \mathbf{A}_i \mathbf{H}_i \mathbf{P}^{1/2} \mathbf{s} + \sum_{i=1}^{n_r} \mathbf{h}_{li}^T \mathbf{A}_i \mathbf{n}_i + \eta_l. \tag{2}$$

where η_l denotes the noise at the receiver front-end of the l-th transceiver. Using the definitions of $\mathbf{x}_i$ in (1) along with that of $\mathbf{t}_i$, the total relay transmit power, denoted as P_r, can be written as

$$P_{\mathrm{r}} \triangleq \sum_{i=1}^{n_r} \|\mathbf{A}_i \mathbf{H}_i \mathbf{P}^{1/2}\|^2 + \sigma^2 \sum_{i=1}^{n_r} tr(\mathbf{A}_i \mathbf{A}_i^H). \tag{3}$$

The total transmit power consumed in the entire network, denoted as P_T, is given by

$$P_{\mathrm{T}} = \sum_{l=1}^{2K} p_l + P_{\mathrm{r}} \tag{4}$$

which is defined as the sum of the transceivers' transmit powers and the total relay transmit power. The channel coefficient from the k-th transceiver to the m-th antenna of the i-th relay is herein modeled as the product of three terms: the complex small-scale fading coefficient, an amplitude factor that accounts for the path-loss (attenuation), and the shadowing factor. That is, we can write the channel coefficient matrix $\mathbf{H}_i$ as

$$\mathbf{H}_i = \mathbf{G}_i \mathbf{D}_i^{1/2}. \tag{5}$$

Here, $\mathbf{G}_i$ denotes the matrix of small-scale fading coefficients and $\mathbf{D}_i$ is a diagonal matrix with positive real-valued diagonal entries representing path-loss and shadowing (i.e., the large-scale fading effect). Note that contrary to the large-scale effect, the small-scale fading occurs over distances on the order of the signal wavelength. As a result, the small-scale fading coefficients for different transceivers can be independent. As such, channel vectors from different transceivers become asymptotically orthogonal when M, the number of relay antennas, is large [12, 13]. This asymptotic orthogonality enables us to write $\mathbf{H}_i^H \mathbf{H}_i$ approximately as a diagonal matrix, that is

$$\mathbf{H}_i^H \mathbf{H}_i = \mathbf{D}_i^{1/2} \mathbf{G}_i^H \mathbf{G}_i \mathbf{D}_i^{1/2} \approx M \mathbf{D}_i^{1/2} \mathbf{I}_{2K} \mathbf{D}_i^{1/2} = M \mathbf{D}_i. \tag{6}$$

Note that the approximation $\mathbf{G}_i^H \mathbf{G}_i \approx M \mathbf{I}_{2K}$ holds true for large values of M that are no less than $2K$ (i.e., $M \geq 2K$). Employing the MRC/MRT-based scheme for signal processing at relays, the relay beamforming matrix at relay i, denoted as $\mathbf{A}_i$, can be written as

$$\mathbf{A}_i = \mathbf{H}_i^* \mathbf{C}_i \mathbf{H}_i^H \tag{7}$$

where $\mathbf{C}_i$ is a $2K \times 2K$ block diagonal matrix, that is

$$\mathbf{C}_i \triangleq \begin{bmatrix} \mathbf{B}_{1i} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{B}_{Ki} \end{bmatrix}. \tag{8}$$

Here, each block is a 2×2 anti-diagonal matrix, is associated with one of the transceiver pairs, and is given as

$$\mathbf{B}_{ki} \triangleq \begin{bmatrix} 0 & \beta_{(2k-1),i} \\ \beta_{2k,i} & 0 \end{bmatrix}. \tag{9}$$

Using the definitions in (7)-(9), we can simplify the problem of finding optimal beamforming matrices as finding the optimal values of parameters $\beta_{(2k-1),i}$ and $\beta_{2k,i}$ for $k \in \{1, \ldots, K\}$, and $i \in \{1, \ldots, n_r\}$. The role of the anti-diagonal matrix $\mathbf{B}_{ki}$ on the k-th diagonal block of $\mathbf{C}_i$ is to swap the linear estimates of the signals transmitted in the first phase so that the linear estimate of the symbol transmitted by Transceiver $2k$ can be forwarded to Transceiver $(2k-1)$ and the linear estimate of the symbol transmitted by Transceiver $(2k-1)$ can be forwarded to Transceiver $2k$. This goal is achieved by multiplying $\hat{\mathbf{s}}_i$ with $\mathbf{C}_i$, as (7) implies. As $\mathbf{H}_i^H \mathbf{H}_i \approx M \mathbf{D}_i$ holds for very large M (see (6)), the use of $\mathbf{H}_i^*$ as the left-most component in $\mathbf{A}_i$ guarantees that the transmitted symbol estimates will not interfere for $M \to \infty$, as these estimate will be transmitted over approximately orthogonal columns of $\mathbf{H}_i^*$. However, when M is a finite number, the MRC/MRT relaying scheme suffers from inter- and intra-interferences.

Using (6)-(9) in (3), the total relay transmit power for the MRC/MRC-based scheme can be expressed as

$$\begin{aligned} P_{\mathrm{r}} &= \sum_{i=1}^{n_r} \Big\{ tr(\mathbf{A}_i \mathbf{H}_i \mathbf{P} \mathbf{H}_i^H \mathbf{A}_i^H) + \sigma^2 tr(\mathbf{A}_i \mathbf{A}_i^H) \Big\} \\ &= \sum_{i=1}^{n_r} tr \Big\{ M^3 \mathbf{P} \mathbf{D}_i \mathbf{C}_i^H \mathbf{D}_i \mathbf{C}_i \mathbf{D}_i + \sigma^2 M^2 \mathbf{C}_i^H \mathbf{D}_i \mathbf{C}_i \mathbf{D}_i \Big\}. \end{aligned} \tag{10}$$

Since $\mathbf{D}_i$ is a diagonal matrix and $\mathbf{C}_i$ is a block-diagonal matrix with blocks formed as anti-diagonal matrices, $\mathbf{C}_i^H \mathbf{D}_i \mathbf{C}_i \mathbf{D}_i$ becomes a diagonal matrix.[1] Hence, we can write

$$\begin{aligned} \mathbf{C}_i^H \mathbf{D}_i \mathbf{C}_i \mathbf{D}_i &= \mathrm{diag} \Big\{ |\tilde{\beta}_{2,i}|^2, |\tilde{\beta}_{1,i}|^2, \ldots, |\tilde{\beta}_{2K,i}|^2, |\tilde{\beta}_{2K-1,i}|^2 \Big\} \\ &= \mathrm{diag} \{ |\tilde{\beta}_{\bar{l},i}|^2 \}_{l=1}^{2K} \end{aligned} \tag{11}$$

where we define

$$\tilde{\beta}_{l,i} \triangleq \sqrt{d_{l,i} d_{\bar{l},i}} \beta_{\bar{l},i}, \quad \text{for } \ l \in \{1, 2, \ldots, 2K\} \tag{12}$$

and $\bar{l} \in \{1, 2, \ldots, 2K\}$ is defined as

$$\bar{l} \triangleq \begin{cases} l+1, & \text{if } l \in \{1, 3, \ldots, 2K-1\} \\ l-1, & \text{if } l \in \{2, 4, \ldots, 2K\} \end{cases}. \tag{13}$$

Using (10) and (11), P_{r} can be rewritten as

$$\begin{aligned} P_{\mathrm{r}} &= \sum_{i=1}^{n_r} tr \big(\underbrace{\left(M^3 \mathbf{P} \mathbf{D}_i + \sigma^2 M^2 \mathbf{I} \right)}_{\text{diagonal matrix}} \underbrace{\mathbf{C}_i^H \mathbf{D}_i \mathbf{C}_i \mathbf{D}_i}_{\text{diagonal matrix}} \big) \\ &= \sum_{i=1}^{n_r} \sum_{l=1}^{2K} \left(M^3 p_{\bar{l}} d_{\bar{l},i} + M^2 \sigma^2 \right) |\tilde{\beta}_{l,i}|^2. \end{aligned} \tag{14}$$

Defining $\mathbf{f}_l \triangleq [\sqrt{d_{l,1}} \ \sqrt{d_{l,2}} \cdots \sqrt{d_{l,n_r}}]^T$, $\mathbf{F}_l \triangleq \mathrm{diag}(\mathbf{f}_l \odot \mathbf{f}_l)$, and $\tilde{\boldsymbol{\beta}}_l \triangleq [\tilde{\beta}_{l,1} \ \tilde{\beta}_{l,2} \ \cdots \ \tilde{\beta}_{l,n_r}]^T$, and using (4) and (14), the total transmit power, P_{T} can be rewritten as

$$P_{\mathrm{T}} = \sum_{l=1}^{2K} \left(p_{\bar{l}} + M^2 \tilde{\boldsymbol{\beta}}_l^H (M p_{\bar{l}} \mathbf{F}_{\bar{l}} + \sigma^2 \mathbf{I}) \tilde{\boldsymbol{\beta}}_l \right). \tag{15}$$

We can use (6) and (7) to write y_l, the signal received at Transceiver l, for $l \in \{1, 2, \cdots, 2K\}$, as

$$\begin{aligned} y_l &= \sum_{i=1}^{n_r} \mathbf{h}_{l,i}^T \mathbf{H}_i^* \mathbf{C}_i \mathbf{H}_i^H \mathbf{H}_i \mathbf{P}^{1/2} \mathbf{s} + \sum_{i=1}^{n_r} \mathbf{h}_{l,i}^T \mathbf{H}_i^* \mathbf{C}_i \mathbf{H}_i^H \mathbf{n}_i + \eta_l \\ &= M^2 \sum_{i=1}^{n_r} d_{l,i} \beta_{l,i} d_{\bar{l},i} \sqrt{p_{\bar{l}}} s_{\bar{l}} + M \sum_{i=1}^{n_r} d_{l,i} \beta_{l,i} \mathbf{h}_{l,i}^H \mathbf{n}_i + \eta_l. \end{aligned} \tag{16}$$

Note that under the assumption of the orthogonality of the channel vectors, the received signal, y_l, contains only the signal from Transceiver $\bar{l}$. Using (16), the SNR at Transceiver l can be written as

$$\mathrm{SNR}_l = \frac{M^4 p_{\bar{l}} \left| \sum_{i=1}^{n_r} \sqrt{d_{l,i} d_{\bar{l},i}} \tilde{\beta}_{l,i} \right|^2}{M^3 \sigma^2 \sum_{i=1}^{n_r} \left| \sqrt{d_{l,i}} \tilde{\beta}_{l,i} \right|^2 + \sigma^2}, \quad \text{for } l \in \{1, 2, \ldots, 2K\} \tag{17}$$

[1]The k-th block of the product $\mathbf{C}_i^H \mathbf{D}_i \mathbf{C}_i \mathbf{D}_i$ can be written as

$$\begin{bmatrix} 0 & \beta_{2k,i}^* \\ \beta_{2k-1,i}^* & 0 \end{bmatrix} \begin{bmatrix} d_{2k-1,i} & 0 \\ 0 & d_{2k,i} \end{bmatrix} \begin{bmatrix} 0 & \beta_{2k-1,i} \\ \beta_{2k,i} & 0 \end{bmatrix} \begin{bmatrix} d_{2k-1,i} & 0 \\ 0 & d_{2k,i} \end{bmatrix} = \begin{bmatrix} d_{2k-1,i} d_{2k,i} |\beta_{2k,i}|^2 & 0 \\ 0 & d_{2k-1,i} d_{2k,i} |\beta_{2k-1,i}|^2 \end{bmatrix}.$$

where we have used the approximation $E\{|\mathbf{h}_{\bar{l},i}^H \mathbf{h}_{\bar{l},i}|\} \approx M d_{\bar{l},i}$, and $E\{\mathbf{n}_i^H \mathbf{n}_i\} = \sigma^2 \mathbf{I}_M$, along with (6). Using the following definition:

$$\mathbf{g}_l \triangleq \mathbf{f}_l \odot \mathbf{f}_{\bar{l}}, \quad \text{for } l \in \{1, 2, \ldots, 2K\} \tag{18}$$

we can rewrite SNR_l in (17) as

$$\text{SNR}_l = \frac{M^4 p_{\bar{l}} |\mathbf{g}_l^T \tilde{\boldsymbol{\beta}}_l|^2}{\sigma^2 (1 + M^3 \tilde{\boldsymbol{\beta}}_l^H \mathbf{F}_l \tilde{\boldsymbol{\beta}}_l)}, \quad \text{for } l \in \{1, 2, \ldots, 2K\}. \tag{19}$$

It is worth emphasizing that (15) and (19) are obtained under the assumption that the channel vectors are asymptotically orthogonal as in (6). In reality, inter- and intra-pair interference will exist for finite values of M. It is however expected that as M is increased inter- and intra-pair interference vanish.

3. TOTAL POWER MINIMIZATION

We aim to find the beamforming matrices and the transceivers' transmit powers such that the total transmit power P_{T} is minimized, while the SNR at Transceiver l is maintained above given threshold γ_l, for $l \in \{1, 2, \ldots, 2K\}$. This power minimization problem can be expressed as

$$\min_{\mathcal{P}, \mathcal{A}} \; P_{\mathrm{T}} \quad \text{s.t.} \quad \text{SNR}_l \geq \gamma_l, \;\text{for } l \in \{1, 2, \ldots, 2K\} \tag{20}$$

where $\mathcal{P} \triangleq \{p_l\}_{l=1}^{2K}$ is the set of transceivers' transmit powers and $\mathcal{A} \triangleq \{\mathbf{A}_i\}_{i=1}^{n_r}$ is the set of relays' beamforming matrices. Using (15) and (19), the power minimization problem for the MRC/MRT-based scheme in (20) can be recast as

$$\begin{aligned} \min_{\mathcal{P}, \mathcal{B}} \; & \sum_{l=1}^{2K} \left(p_{\bar{l}} + M^2 \tilde{\boldsymbol{\beta}}_l^H (M p_{\bar{l}} \mathbf{F}_{\bar{l}} + \sigma^2 \mathbf{I}) \tilde{\boldsymbol{\beta}}_l \right) \\ \text{s.t.} \; & \frac{M^4 p_{\bar{l}} |\mathbf{g}_l^T \tilde{\boldsymbol{\beta}}_l|^2}{\sigma^2 (1 + M^3 \tilde{\boldsymbol{\beta}}_l^H \mathbf{F}_l \tilde{\boldsymbol{\beta}}_l)} \geq \gamma_l, \;\text{for } l \in \{1, 2, \ldots, 2K\} \end{aligned} \tag{21}$$

where $\mathcal{B} \triangleq \{\tilde{\boldsymbol{\beta}}_l\}_{l=1}^{2K}$ is the set of vectors $\tilde{\boldsymbol{\beta}}_l$, each with size $n_r \times 1$. A closer look at (21) shows that the total transmit power minimization problem can be decoupled into a set of $2K$ total power minimization problems each of which written as

$$\begin{aligned} \min_{\tilde{\boldsymbol{\beta}}_l, p_{\bar{l}}} \quad & p_{\bar{l}} + M^2 \tilde{\boldsymbol{\beta}}_l^H (M p_{\bar{l}} \mathbf{F}_{\bar{l}} + \sigma^2 \mathbf{I}_{n_r}) \tilde{\boldsymbol{\beta}}_l \\ \text{s.t.} \quad & \frac{M^4 p_{\bar{l}} |\mathbf{g}_l^T \tilde{\boldsymbol{\beta}}_l|^2}{\sigma^2 (1 + M^3 \tilde{\boldsymbol{\beta}}_l^H \mathbf{F}_l \tilde{\boldsymbol{\beta}}_l)} \geq \gamma_l. \end{aligned} \tag{22}$$

Indeed, the minimization problem (22) amounts to minimizing the total power consumed to guarantee a received SNR at Transceiver l. We can rewrite the optimization problem in (22) as

$$\begin{aligned} \min_{p_{\bar{l}}} \quad & p_{\bar{l}} + \min_{\tilde{\boldsymbol{\beta}}_l} M^2 \tilde{\boldsymbol{\beta}}_l^H (M p_{\bar{l}} \mathbf{F}_{\bar{l}} + \sigma^2 \mathbf{I}_{n_r}) \tilde{\boldsymbol{\beta}}_l \\ \text{s.t.} \quad & M^3 \tilde{\boldsymbol{\beta}}_l^H (M p_{\bar{l}} \mathbf{g}_l^* \mathbf{g}_l^T - \sigma^2 \gamma_l \mathbf{F}_l) \tilde{\boldsymbol{\beta}}_l \geq \sigma^2 \gamma_l. \end{aligned} \tag{23}$$

To solve (23), one can first fix $p_{\bar{l}}$ and solve the inner minimization problem over $\tilde{\boldsymbol{\beta}}_l$. It can be shown that the inner problem in (23) is feasible if and only if $p_{\bar{l}} > \sigma^2 \gamma_l / (M \|\mathbf{f}_l\|^2)$, and that the solution to the inner minimization problem can be written as

$$\tilde{\boldsymbol{\beta}}_l^{\text{opt}} = \mu_l \underbrace{M^2 (\sigma^2 \gamma_l M \mathbf{F}_l + \lambda_l (M p_{\bar{l}} \mathbf{F}_{\bar{l}} + \sigma^2 \mathbf{I}_{n_r}))^{-1} \mathbf{g}_l}_{\triangleq \mathbf{u}_l} \tag{24}$$

$$\mu_l = \sqrt{\frac{\sigma^2 \gamma_l}{\lambda_l M^2 \mathbf{u}_l^H (M p_{\bar{l}} \mathbf{F}_{\bar{l}} + \sigma^2 \mathbf{I}_{n_r}) \mathbf{u}_l}}. \tag{25}$$

Here, $\tilde{\boldsymbol{\beta}}_l^{\text{opt}}$ is the optimal value of $\tilde{\boldsymbol{\beta}}_l$ while $p_{\bar{l}}$ and λ_l, must satisfy the following two nonlinear equations:

$$\sigma^2 \gamma_l \frac{p_{\bar{l}}^{-2} - \lambda_l M^3 \mathbf{u}_l^H \mathbf{F}_{\bar{l}} \mathbf{u}_l}{\lambda_l^2 \mathbf{u}_l^H (M^3 p_{\bar{l}} \mathbf{F}_{\bar{l}} + M^2 \sigma^2 \mathbf{I}_{n_r}) \mathbf{u}_l} = 1 \tag{26}$$

$$p_{\bar{l}} M^2 \mathbf{g}_l^H (M \sigma^2 \gamma_l \mathbf{F}_l + \lambda_l (M p_{\bar{l}} \mathbf{F}_{\bar{l}} + \sigma^2 \mathbf{I}_{n_r}))^{-1} \mathbf{g}_l = 1 \tag{27}$$

and $p_{\bar{l}} \in (\sigma^2 \gamma_l / (M \|\mathbf{f}_l\|^2), +\infty)$ must hold true. We now explain how $p_{\bar{l}}$ and λ_l can be obtained from (26) and (27). To do so, note that for any given value of $z \in (\frac{\sigma^2 \gamma_l}{M \|\mathbf{f}_l\|^2}, +\infty)$, one can prove that the following nonlinear equality

$$z M^2 \mathbf{g}_l^H (M \sigma^2 \gamma_l \mathbf{F}_l + \lambda (M z \mathbf{F}_{\bar{l}} + \sigma^2 \mathbf{I}_{n_r}))^{-1} \mathbf{g}_l = 1 \tag{28}$$

renders a unique positive solution for parameter λ. That is, in (28), the parameter λ can be viewed as a function of z. As such, the function

$$\sigma^2 \gamma_l \frac{z^{-2} - \lambda M^3 \mathbf{u}_l^H \mathbf{F}_{\bar{l}} \mathbf{u}_l}{\lambda^2 \mathbf{u}_l^H (z M^3 \mathbf{F}_{\bar{l}} + \sigma^2 M^2 \mathbf{I}_{n_r}) \mathbf{u}_l} - 1 \tag{29}$$

can be considered as a function of only z, where λ is obtained, for any value of $z \in (\frac{\sigma^2 \gamma_l}{M \|\mathbf{f}_l\|^2}, +\infty)$, from (28). Hence, the parameter $p_{\bar{l}}$ is the provably unique root of (29), and one can use a bisection method to find this root. Note that in this bisection method, the function in (29) has to be evaluated for intermediate values of z. As such, to obtain a value of λ corresponding to an intermediate value of z, one has to solve (28) using another bisection technique. Once $p_{\bar{l}}$, the root of (29), is obtained, the corresponding value of λ is indeed λ_l. Once $p_{\bar{l}}$ and λ_l are obtained, the value of the objective function in (23) is given by $(p_{\bar{l}} + \sigma^2 \gamma_l / \lambda_l)$. Based on (21)-(23), the minimum value of the total transmit power can be obtained as

$$P_{\mathrm{T}} = \sum_{l=1}^{2K} (p_{\bar{l}} + \frac{\sigma^2 \gamma_l}{\lambda_l}). \tag{30}$$

Using the values obtained for $p_{\bar{l}}$ and λ_l, the optimal vector $\tilde{\boldsymbol{\beta}}_l^{\text{opt}}$ is obtained from (24) and (25). Once $\tilde{\boldsymbol{\beta}}_l^{\text{opt}} \triangleq [\tilde{\beta}_{l,1}^{\text{opt}} \; \tilde{\beta}_{l,2}^{\text{opt}} \cdots \tilde{\beta}_{l,n_r}^{\text{opt}}]^T$ is obtained, the optimal values of $\beta_{l,i}^{\text{opt}}$ can be calculated from (12) as $\beta_{l,i}^{\text{opt}} = \tilde{\beta}_{\bar{l},i}^{\text{opt}} / \sqrt{d_{l,i} d_{\bar{l},i}}$, for $i \in \{1, 2, \ldots, n_r\}$. Replacing $\beta_{2k-1,i}$ and $\beta_{2k,i}$ in (9), respectively, with $\beta_{2k-1,i}^{\text{opt}}$ and $\beta_{2k,i}^{\text{opt}}$, the optimal value of $\mathbf{B}_{ki}$, denoted as $\mathbf{B}_{ki}^{\text{opt}}$, can be obtained. Replacing blocks $\mathbf{B}_{ki}$ for $k \in \{1, 2, \ldots, K\}$ in (8), with the so-obtained set of blocks $\{\mathbf{B}_{ki}^{\text{opt}}\}_{k=1}^K$, the effective beamforming matrix of i-th relay, denoted as $\mathbf{C}_i^{\text{opt}}$, can be formed. Finally, the optimal beamforming matrix of the i-th relay for MRC/MRT-based scheme, denoted as $\mathbf{A}_i^{\text{opt}}$, can be calculated as $\mathbf{A}_i^{\text{opt}} = \mathbf{H}_i^* \mathbf{C}_i^{\text{opt}} \mathbf{H}_i^H$.

Although not proven here, one can prove that when M is very large, the minimum total transmit power decreases as M^{-1}.

4. NUMERICAL RESULTS

In our numerical examples, the small-scale channel coefficients are modeled as complex Gaussian random variables with zero mean and unit variance. We adopt the combined path-loss and shadowing model introduced in [14], as the large-scale fading model. Here, the path-loss exponent is 3.8, and the standard deviation of the shadowing effect is 8 dB, whereas the noise variance is $\sigma^2 = -130$ dBm. In Fig. 1, we show the cumulative distribution function (CDF) of the achievable signal-to-interference-plus-noise-ratios (SINRs) versus required SNR threshold for networks with different numbers of antennas per relay (i.e., $M = 50, 100, 200, 1000$). It is seen that for SNR threshold equal to 0 dB, increasing the number of antennas

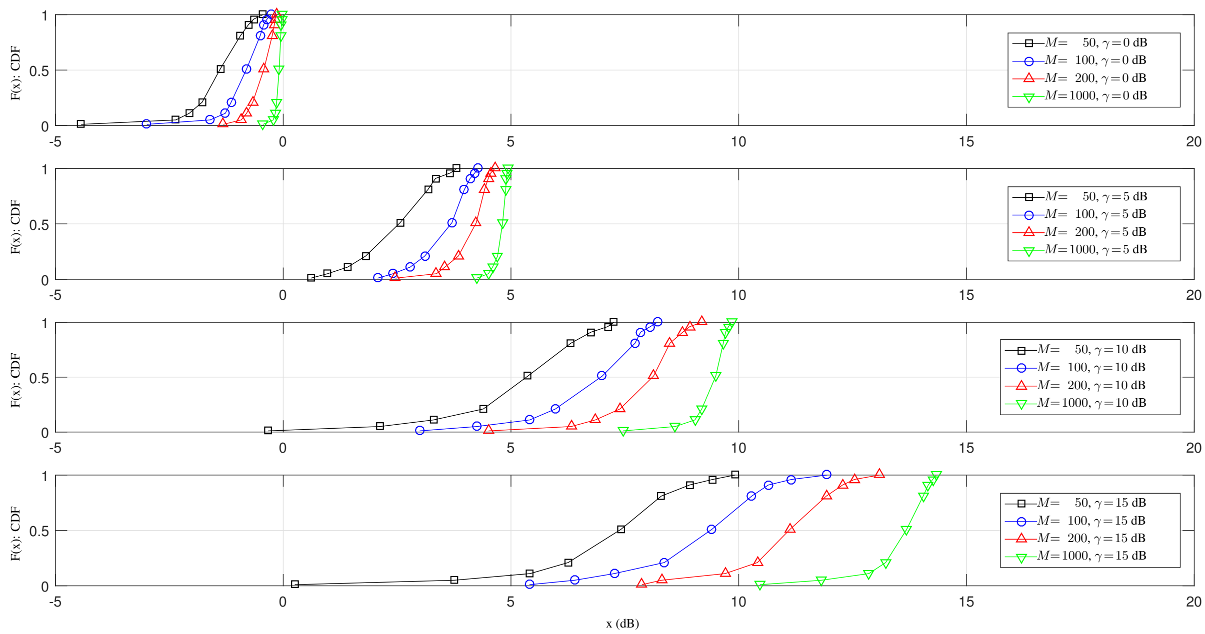

Fig. 1. The CDF of the achievable SINR values versus SNR thresholds, for $K = 4$, $n_r = 4$, and $M = 50, 100, 200, 1000$.

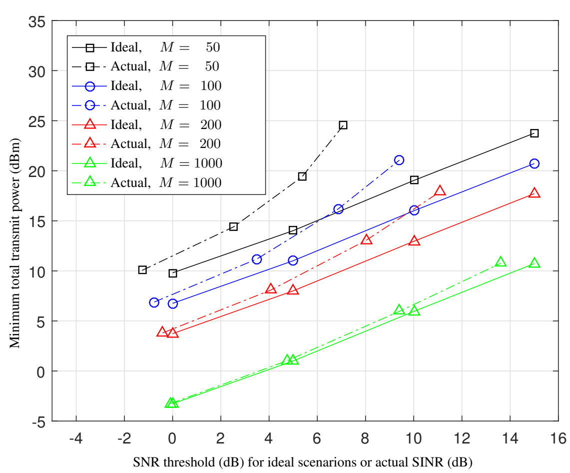

Fig. 2. The minimum total transmit power vs. the minimum required SNR at the transceivers for ideal scenario and the actual minimum total transmit power vs. actual SINR, for $K = 4$ and $n_r = 4$.

per relay from $M = 50$ to $M = 1000$, leads to an increase in the CDF slope and a shrink in the SINR range (from 2.54 dB to 0.29 dB). Moreover, when the required SINR threshold is increased from 0 dB to 15 dB, the CDF slope is increased whereas the range of the achievable SINRs is extended.

In Fig. 2, the performance of the network is evaluated once under the assumption that channel vectors are orthogonal as in (6) (i.e., the *ideal condition*), and once under no such assumption (i.e., the *actual condition*). This figure shows that the average total transmit power required for a network to achieve a certain SNR threshold in the actual condition is higher than that in the ideal condition. For a very large number of relay antennas the performance gap between ideal and actual conditions is significantly small. For instance, to achieve an SNR threshold equal to 15 dB, increasing the number of relay antennas from $M = 50$ to $M = 1000$, reduces the gap between the the SNR threshold and the actual SINR from 7.80 dB to 1.45 dB, and at the same time, the actual power is reduced by 14 dB.

5. CONCLUSION

We studied a two-way network of multiple multi-antennas relays which enable multiple pairs of transceivers to establish pairwise communications. Each relay is equipped with a very large number of antennas leading to the transceiver-relay channel vectors being approximately orthogonal. As a result, intra- and inter-pair interference will be negligible. Aiming to maintain the signal-to–noise ratio (SNR) at receiver front-end of each transceiver above a certain threshold, we obtained the relay beamforming matrices and the transceiver powers such that the total transmit power consumed in the entire network is minimized. To do so, we assume that the channel vectors between each relay and different transceivers are asymptotically orthogonal. For such power minimization problem, we derive computationally efficient solutions. Our simulation results show that as the number of relay antennas is increased, the gap between signal-to-interference-plus-noise ratio (SINR) and SNR threshold as well as the minimum total transmit power are reduced.

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