ON INFORMATION COUPLING IN COOPERATIVE NETWORK SYNCHRONIZATION

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ABSTRACT

Wireless networks are growing in the value of application in many areas, in which accurate clock synchronization is required when tasks are performed in a collaborative fashion among nodes. Especially, cooperative synchronization techniques lead to significant performance improvement compared with traditional methods. However, the correlation among agents renders the performance analysis of cooperative network synchronization difficult. In this paper, we introduce the concept of information coupling intensity to the analysis of interaction between agents. Our approach enables us to derive closed-form asymptotic expressions under specific network topologies, and relate them to various network parameters.

Index Terms— Cooperative network synchronization, Cramér-Rao bound (CRB), information coupling intensity, random walk.

1. INTRODUCTION

Due to their great value of application in diverse areas such as geolocation [1–4], industrial control [5] and surveillance [6], wireless networks (WNs) are gaining interest like never before. To avoid the resource-consuming routing process, there is a recent trend of performing tasks in a collaborative and distributed fashion over the network [7–10], which requires all agents operate under a common clock over the network. However, the clocks in agents suffer from various imperfections caused by both internal and environmental issues, making clock synchronization procedures a fundamental building block in WNs.

Traditional methods are typically performed in a layer-by-layer fashion, taking advantage of the tree-like structures in the networks. Reference Broadcast Synchronization [11] (RBS) and Time synchronization Protocol for Sensor Network [12] (TPSN) are the most well-known ones. These methods are essentially non-cooperative, requiring high topology maintaining overhead and are sensitive to abrupt link failures. Recently, distributed variational Bayesian inference methods have been introduced to the cooperative clock synchronization problem, such as Belief Propagation [13] (BP) and Variational Message Passing [14] (VMP). These methods fully utilize the correlation between agents, and result in excellent performance compared with aforementioned methods.

Although numerous algorithms have been proposed for the cooperative synchronization problem, only a small number of works are devoted to the performance analysis of these algorithms. At the physical level of the problem, the performance limits of phase synchronization has been discussed in [15]. Nevertheless, the cooperation between agents introduces correlation not only pairwise, but over the entire network. This correlation is tightly associated with the network topology and is nontrivial to calculate. Existing works have provided some complicated expressions without closed form [16–18]. Understanding the performance limits of cooperative clock synchronization problem, especially the correlation among agents, can lead to better algorithm designs, and can also bring improvement to the network deployment techniques.

In this paper, we propose the concept of *information coupling intensity* (ICI) to characterize the strength of information coupling among agents, which is defined based on a general form of the performance limit for the cooperative network clock synchronization problem. We also propose a random walk interpretation of ICI which relates it to the properties of Markov chains. Furthermore, we analyze ICI in infinite lattice networks, and provide asymptotic expressions as the maximum communication range increases quantifying the relation between ICI and network topology.

2. SYSTEM MODEL

Consider a network with N_s agents constituting a set $\mathcal{S} = \{1, \cdots, N_\mathrm{s}\}$. Each of the agents has an unknown clock offset $\theta_i,\ i \in \mathcal{S}$. Additionally, there exists a set $\mathcal{R} = \{N_\mathrm{s}+1, \cdots, N_\mathrm{s}+N_\mathrm{r}\}$ of reference nodes without clock offset. The network is embedded in $\mathbb{R}^2$, in which node i locates at $\mathbf{p}_i = [p_{\mathrm{x}i}\ p_{\mathrm{y}i}]^\mathrm{T}$. Two nodes can only communicate with each other if the Euclidean distance between them is less than the maximum communication range $R_{\max}$. We denote the relation "node i and j can communicate with each other" as $i \sim j$, and the set of all nodes k satisfying $i \sim k$ as $\mathcal{N}_i$.

The first-order model of the clock synchronization problem is adopted here, which can be expressed as follows

$$c_i(t) = t + \theta_i,\ i \in \mathcal{S} \tag{1}$$

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where t is the reference time and $c_i(t)$ is the local clock time of the ith agent.

The two-way timing procedure has been discussed extensively in the literature [16], which is illustrated in Fig. 1. The procedure is initiated by node i by first sending a message containing its clock reading $c_i\left(t_{i,\mathrm{T}}^{(1)}\right)$ at time $t_{i,\mathrm{T}}^{(1)}$. Node j receives this message at time $t_{j,\mathrm{R}}^{(1)}$, and replies with a message containing $c_j\left(t_{j,\mathrm{R}}^{(1)}\right)$ and $c_j\left(t_{j,\mathrm{T}}^{(1)}\right)$ at time $t_{j,\mathrm{T}}^{(1)}$, which will be received by node i at time $t_{i,\mathrm{R}}^{(1)}$. In next round, node i add its clock reading $c_i\left(t_{i,\mathrm{R}}^{(1)}\right)$ in the message. After N rounds, N observations are collected at each node $\left\{T_{ij}^{(n)}\right\}_{n=1}^{N}$ as

$$T_{ij}^{(n)} = c_j\left(t_{j,\mathrm{R}}^{(n)}\right) - c_i\left(t_{i,\mathrm{T}}^{(n)}\right) + c_j\left(t_{j,\mathrm{T}}^{(n)}\right) - c_i\left(t_{i,\mathrm{R}}^{(n)}\right). \quad (2)$$

The relation of the clock readings in (2) and signal propagation is modeled as

$$c_j\left(t_{j,\mathrm{R}}^{(n)}\right) - c_i\left(t_{i,\mathrm{T}}^{(n)}\right) = \theta_j - \theta_i + d_{ij} + \omega_n \quad (3a)$$

$$c_j\left(t_{j,\mathrm{T}}^{(n)}\right) - c_i\left(t_{i,\mathrm{R}}^{(n)}\right) = \theta_j - \theta_i - d_{ij} - \omega_n' \quad (3b)$$

where d_{ij} is the deterministic part of message delay and ω_n and ω_n' denote the stochastic counterpart. We assume that ω_n and ω_n' are independently, identically distributed (i.i.d.) Gaussian variables. Following these assumptions, the observations can be rewritten as

$$T_{ij}^{(n)} = 2(\theta_j - \theta_i) + \xi_n \quad (4)$$

where $\xi_n = \omega_n - \omega_n'$ is zero-mean Gaussian random variable with variance σ^2. The joint likelihood function can thus be obtained as

$$p(\mathbf{T}_{ij}|\theta_i, \theta_j) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left\{-\frac{1}{2\sigma^2}\sum_{n=1}^{N}\left[T_{ij}^{(n)} - 2(\theta_j - \theta_i)\right]^2\right\} \quad (5)$$

where $\mathbf{T}_{ij} \triangleq \left[T_{ij}^{(1)}, \cdots, T_{ij}^{(N)}\right]^{\mathrm{T}}$. For the simplicity of further derivation, we further stack $\mathbf{T}_{ij}$'s into a set $\mathcal{T} = \{\mathbf{T}_{ij} : i \sim j\}$.

Furthermore, we assume that each node has some *a priori* knowledge on their clock offsets, modeled as prior distributions $p_{\theta_i}(\theta_i)$. Based on (5), the joint distribution of $\mathcal{T}$ and $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_{N_\mathrm{s}}]^{\mathrm{T}}$ is given by

$$\begin{aligned} p(\mathcal{T}, \boldsymbol{\theta}) &= \prod_{i\in\mathcal{S}} p(\theta_i) \prod_{\substack{j\in\mathcal{S}\cup\mathcal{R}\\ i\sim j}} p(\mathbf{T}_{ij}|\theta_i, \theta_j) \\ &\propto \exp\left\{-\frac{1}{2\sigma^2}\sum_{i\in\mathcal{S}}\sum_{i\sim j}\sum_{n=1}^{N}\left[T_{ij}^{(n)} - 2(\theta_j - \theta_i)\right]^2\right\} \\ &\quad \times \prod_{i\in\mathcal{S}} p_{\theta_i}(\theta_i). \end{aligned} \quad (6)$$

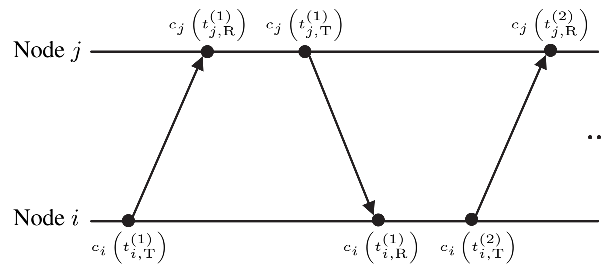

Fig. 1. Illustration of the two-way timing procedure.

3. PERFORMANCE LIMITS AND INFORMATION COUPLING INTENSITY

It is well-known that the a lower bound on the variance of any unbiased estimators for deterministic unknown parameters is given by the Cramér-Rao bound (CRB) [19]. Bayesian Cramér-Rao bound (BCRB) can be used instead for stochastic parameters. The BCRB for the cooperative clock synchronization problem can be given using the Fisher information matrix (FIM) defined as

$$\mathbf{J}_{\boldsymbol{\theta}} = \mathbb{E}_{\boldsymbol{\theta},\mathcal{T}}\left\{[\nabla_{\boldsymbol{\theta}} \ln p(\mathcal{T}, \boldsymbol{\theta})]\,[\nabla_{\boldsymbol{\theta}} \ln p(\mathcal{T}, \boldsymbol{\theta})]^{\mathrm{T}}\right\}. \quad (7)$$

The following proposition characterizes the specific structure of the FIM.

Proposition 1 (Structure of the FIM) *The matrix $\mathbf{J}_{\boldsymbol{\theta}}$ takes the following form*

$$\mathbf{J}_{\boldsymbol{\theta}} = \frac{2N}{\sigma^2}(\mathbf{D}_{\boldsymbol{\theta}} + \mathbf{D}_{\boldsymbol{\theta}}^{\mathrm{R}} - \mathbf{A}_{\boldsymbol{\theta}}) + \mathbf{J}_{\boldsymbol{\theta}}^{\mathrm{P}} \quad (8)$$

where

$$\begin{aligned} [\mathbf{A}_{\boldsymbol{\theta}}]_{i,j} &= \begin{cases} 1, & i \sim j; \\ 0, & \textit{otherwise}, \end{cases} \\ \mathbf{D}_{\boldsymbol{\theta}} &= \mathrm{diag}\left(\deg(1), \ldots, \deg(N_\mathrm{s})\right) \\ \mathbf{D}_{\boldsymbol{\theta}}^{\mathrm{R}} &= \mathrm{diag}\left(\deg_\mathrm{R}(1), \ldots, \deg_\mathrm{R}(N_\mathrm{s})\right), \\ \mathbf{J}_{\boldsymbol{\theta}}^{\mathrm{P}} &= \mathrm{diag}\left(\xi_{\mathrm{P},1}, \ldots, \xi_{\mathrm{P},N_\mathrm{s}}\right) \end{aligned}$$

and $\deg(i) = |\mathcal{S} \cap \mathcal{N}_i|$ *is the number of agents in the neighborhood of node i,* $\deg_\mathrm{R}(i) = |\mathcal{R} \cap \mathcal{N}_i|$ *is the number of neighboring reference nodes of node i,* $\mathbf{J}_{\boldsymbol{\theta}}^{\mathrm{P}}$ *denotes the FIM from the* a priori *information of* $\boldsymbol{\theta}$.

With the FIM $\mathbf{J}_{\boldsymbol{\theta}}$ given in Proposition 1, we can bound the MSE of estimator $\hat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}$ using

$$\mathbb{E}_{\mathcal{T},\boldsymbol{\theta}}\left\{\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}\right)\left(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}\right)^{\mathrm{T}}\right\} \succeq \mathbf{J}_{\boldsymbol{\theta}}^{-1}.$$

The following theorem provides some intuition for the entries of $\mathbf{J}_{\boldsymbol{\theta}}^{-1}$.

Theorem 1 (Structure of Inverse FIM) *The (i,j)-th entry in $\mathbf{J}_{\boldsymbol{\theta}}^{-1}$ can be expressed as follows*

$$\left[\mathbf{J}_{\boldsymbol{\theta}}^{-1}\right]_{i,j} = \begin{cases} \frac{1+\Delta_{ii}}{\frac{2N}{\sigma^2}(\deg(i)+\deg_{\mathrm{R}}(i))+\xi_{\mathrm{P},i}}, & i=j; \\ \frac{\Delta_{ij}}{\frac{2N}{\sigma^2}(\deg(j)+\deg_{\mathrm{R}}(j))+\xi_{\mathrm{P},i}}, & i\neq j, \end{cases} \tag{9}$$

with $\Delta_{ij}\geq 0$ defined as

$$\Delta_{ij} \triangleq \sum_{n=1}^{\infty}\left[\left(\left(\mathbf{D}_{\boldsymbol{\theta}}^{\mathrm{C}}+\mathbf{D}_{\boldsymbol{\theta}}^{\mathrm{R}}+\frac{\sigma^2}{2N}\mathbf{J}_{\boldsymbol{\theta}}^{\mathrm{P}}\right)^{-1}\mathbf{A}_{\boldsymbol{\theta}}\right)^n\right]_{i,j}.$$

Definition 1 (Cooperative Dilution Intensity (CDI)) *The term Δ_{ii} in* (9) *is defined as the Cooperative Dilution Intensity of node i.*

Definition 2 (Information Coupling Intensity (ICI)) *The quantity Δ_{ij} ($i\neq j$) is referred to as the ICI between agent i and agent j.*

Remark 1 *Note that for an estimator $\hat{\boldsymbol{\theta}}^{(o)}$ attaining the BCRB, the BCRB matrix $\mathbf{J}_{\boldsymbol{\theta}}^{-1}$ coincides with its covariance matrix, i.e., $[\mathbf{J}_{\boldsymbol{\theta}}^{-1}]_{i,j}=\mathbb{C}\mathrm{ov}\{\hat{\theta}_i^{(o)},\hat{\theta}_j^{(o)}\}$. The ICI, Δ_{ij}, is proportional to the covariance between two random variables, which quantifies the strength of the coupling between θ_i and θ_j.*

Now we consider a network with FIM denoted as $\mathbf{J}_{\boldsymbol{\theta}}$. If agent i is turned into a reference node, using Corollary 1 in [20], the BCRB under the constraint "$\theta_i=0$" can be written as

$$\left([\mathbf{J}_{\boldsymbol{\theta}}]_{\bar{i}}\right)^{-1}=\mathbf{J}_{\boldsymbol{\theta}}^{-1}-\mathbf{J}_{\boldsymbol{\theta}}^{-1}\mathbf{v}_i\left(\mathbf{v}_i^{\mathrm{T}}\mathbf{J}_{\boldsymbol{\theta}}^{-1}\mathbf{v}_i\right)^{-1}\mathbf{v}_i^{\mathrm{T}}\mathbf{J}_{\boldsymbol{\theta}}^{-1} \tag{10}$$

where $\mathbf{v}_i=\left[\mathbf{0}_{i-1}^{\mathrm{T}}\;1\;\mathbf{0}_{N_{\mathrm{a}}-i}^{\mathrm{T}}\right]^{\mathrm{T}}\in\mathbb{R}^{N_{\mathrm{a}}}$. Therefore the MSE lower bound of agent j is given by

$$\left[\left([\mathbf{J}_{\boldsymbol{\theta}}]_{\bar{i}}\right)^{-1}\right]_{j,j}=\left[\mathbf{J}_{\boldsymbol{\theta}}^{-1}\right]_{j,j}-\frac{\Delta_{ij}^2}{1+\Delta_{ii}} \times\frac{\frac{2N}{\sigma^2}\left(\deg_{\mathrm{A}}(i)+\deg_{\mathrm{R}}(i)\right)+\xi_{\mathrm{P},i}}{\left(\frac{2N}{\sigma^2}\left(\deg_{\mathrm{A}}(j)+\deg_{\mathrm{R}}(j)\right)+\xi_{\mathrm{P},j}\right)^2}. \tag{11}$$

It can be seen from (11) that turning an agent i into a reference node leads to a MSE reduction for any other agent j that is accessible from i. The amount of MSE lower bound reduction is characterized by the neighborhood structure of agent i and agent j, as well as the CDI Δ_{ii} and the ICI Δ_{ij}.

With Theorem 1, we can also give random walk interpretations of both CDI and ICI.

Theorem 2 (Random Walk Interpretation) *Δ_{ij} can be expressed as the following summation*

$$\Delta_{ij}=\sum_{n=1}^{\infty}p(X_n=j|X_0=i) \tag{12}$$

where $p(X_n=j|X_0=i)$ is the N-step transition probability of a Markov chain with following one-step transition probability

$$p(X_k=b|X_{k-1}=a)=\frac{\left[\mathbf{J}_{\tilde{\boldsymbol{\theta}}}\right]_{a,b}}{\left[\mathbf{J}_{\tilde{\boldsymbol{\theta}}}\right]_{a,a}}. \tag{13}$$

where $\mathbf{J}_{\tilde{\boldsymbol{\theta}}}$ is obtained by treating reference nodes as agents with infinite a priori *information [21]. Especially, a is an absorbing state of the Markov chain if $a\in\mathcal{R}$.*

Remark 2 *From Theorem 2, ICI Δ_{ij} can be interpreted as the sum of N-step transition probabilities of a Markov chain. Furthermore, once the chain reaches a reference node or a virtual reference node, its state will change no further, and thus the corresponding path will not contribute to ICI. Therefore, reference nodes can be recognized as the "coupling absorbers" in the network.*

4. ANALYSIS ON INFINITE LATTICE NETWORKS

For a certain agent in a wireless network, ICI characterizes the strength of information coupling among agents. Unfortunately, its calculation is generally intractable since the N-step transition probabilities of Markov chains have no closed-form expressions in general. In this section, we develop an asymptotic expression of ICI under a specific network topology, namely, infinite lattice networks. In the derivation henceforth, if not otherwise stated, we make following assumptions:

- There is no explicit reference node in the network, i.e., $\deg_{\mathrm{R}}(i)=0,\ \forall i\in\mathcal{S}$. The *a priori* information of agents serve as virtual reference nodes.
- All agents have the same amount of *a priori* information, i.e., $\xi_{\mathrm{P},i}=\xi_{\mathrm{P}},\ \forall i\in\mathcal{S}$.

In infinite lattice networks, there are infinite number of agents, whose positions cover all lattice points (points with integer coordinates) in the space $\mathbb{R}^2$. Under previous assumptions, Δ_{ij} can be expressed as

$$\Delta_{ij}=\sum_{n=1}^{\infty}\frac{\left[\mathbf{A}_{\boldsymbol{\theta},\mathrm{IG}}^n\right]_{i,j}}{\left(\bar{d}+N_{\mathrm{p}}\right)^n}. \tag{14}$$

where $\bar{d}$ is the number of neighboring agents of a agent, which is identical for all agents. According to the random walk interpretation in Theorem 2, Δ_{ij} can now be expressed as

$$\Delta_{ij}=\sum_{n=1}^{\infty}p_{ij}^{(n)}\left(\frac{\bar{d}}{\bar{d}+N_{\mathrm{p}}}\right)^n \tag{15}$$

where $p_{ij}^{(n)}\triangleq p(X_n=j|X_0=i)$ is the N-step transition probability of the a Markov chain with following one-step transition probability

$$p(X_k=j|X_{k-1}=i)=\frac{\mathbb{I}(i\in\mathcal{N}_j)}{\bar{d}}. \tag{16}$$

With the help of previous results, we can derive an asymptotic expression of ICI. Thanks to the properties of infinite lattice networks, instead of the Markov chain defined by (16), we can obtain $p_{ij}^{(n)}$ by considering the following stochastic process

$$\mathbf{x}_k=\mathbf{x}_{k-1}+\mathbf{y}_k$$

where $\mathbf{x}_k$ is the state (location) at time k, $\mathbf{y}_k$ is a random variable with following distribution

$$p_{\mathbf{y}_k}(\mathbf{y}) = \frac{1}{\bar{d}} \cdot \mathbb{I}(\|\mathbf{y}\| \in (0, R_{\max}])\mathbb{I}(\mathbf{y} \in \mathbb{Z}^2) \tag{17}$$

and $\{\mathbf{y}_k\}$ are i.i.d. random variables.

Now note that $\mathbf{y}_k$'s are i.i.d. random variables. According to the local central limit theorem [22], we can approximate the corresponding probability mass function as Gaussian so that

$$\begin{aligned} p_{\mathbf{x}_k|\mathbf{x}_0}(\mathbf{x}) = & \frac{1}{2\pi k \sigma_{\mathrm{R}}^2} \exp\left\{ -\frac{1}{k\sigma_{\mathrm{R}}^2} \|\mathbf{x} - \mathbf{x}_0\|^2 \right\} \\ & + \frac{1}{k} E_1(k, \|\mathbf{x} - \mathbf{x}_0\|) \end{aligned} \tag{18}$$

where σ_{R}^2 is chosen such that $\mathbb{C}\mathrm{ov}(p_{\mathbf{x}_1|\mathbf{x}_0}(\mathbf{x})) = \sigma_{\mathrm{R}}^2 \mathbf{I}_2$, and $E_1(k, \|\mathbf{x} - \mathbf{x}_0\|)$ is an error term tends to zero as $k \to \infty$ for all $\mathbf{x}$. From (18) we have the following estimate on the order of the ICI Δ_{ij}.

Proposition 2 (Asymptotic ICI) *For a given $R_{\max}$, the ICI Δ_{ij} $(i \neq j)$ of infinite lattice networks has the following asymptotic behavior*

$$\begin{aligned} \Delta_{ij} \sim & \frac{\left(2 \ln \frac{\bar{d}+N_{\mathrm{p}}}{\bar{d}}\right)^{-\frac{1}{4}}}{\sqrt{2\pi}\sigma_{\mathrm{R}}^2} \sqrt{\frac{\sigma_{\mathrm{R}}}{\|\mathbf{p}_i - \mathbf{p}_j\|}} \\ & \times \exp\left(-\sqrt{2 \ln \frac{\bar{d}+N_{\mathrm{p}}}{\bar{d}}} \cdot \frac{\|\mathbf{p}_i - \mathbf{p}_j\|}{\sigma_{\mathrm{R}}} \right) \end{aligned} \tag{19}$$

as $\|\boldsymbol{p}_i - \boldsymbol{p}_j\| \to +\infty$.

Remark 3 (Exponential Decay of ICI) *Proposition 2 indicates that, for a given $R_{\max}$, when agent i and agent j are sufficiently distant from each other, the ICI Δ_{ij} $(i \neq j)$ decreases with the distance between them at a rate slightly faster than exponential decreasing. Furthermore, the rate of ICI decay grows with the amount of a priori information, i.e., ξ_{P}.*

5. NUMERICAL RESULTS

In this section, our previous analytical results are illustrated and validated using numerical examples. Without loss of generality, we consider networks with lattice size of $1\mathrm{m}^2$.

We first consider the behavior of ICI as a function of the distance between two agents in infinite lattice networks. The maximum communication range is set as $R_{\max} = 10$m. Figure 2 shows the numerical result with $N_{\mathrm{p}} = 10^{-2}$, 1, and 5. It can be seen that the asymptotic values and the numerical results agrees well even for small distances. All curves drop exponentially for large distances as Proposition 2 states.

Next we consider the MSE lower bound reduction for a given agent j, when another agent i is turned into a reference node. N_{p} is set as $N_{\mathrm{p}} = 0.5$, 0.1, and 0.01. As can be seen from 3, the MSE lower bound reduction of agent j is determined by the distance between agent i and j, and drops approximately exponentially as the distance increases. This can be understood by observing (11), which indicates that the MSE lower bound reduction is proportional to ICI - a quantity that decays exponentially as the distance grows.

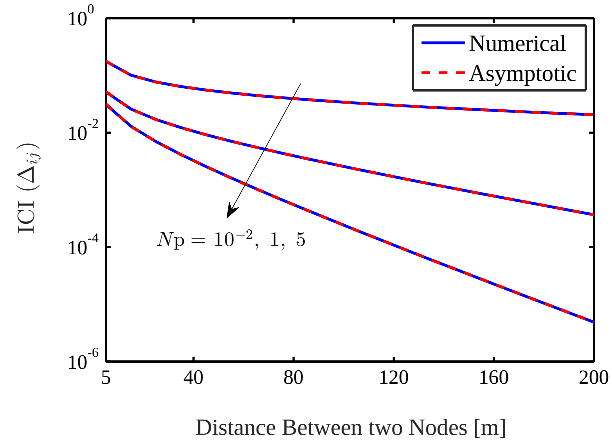

Fig. 2. ICI as a function of the distance between two agents in infinite lattice networks, with $R_{\max} = 10$m and different number of equivalent observations for prior distributions N_{p}.

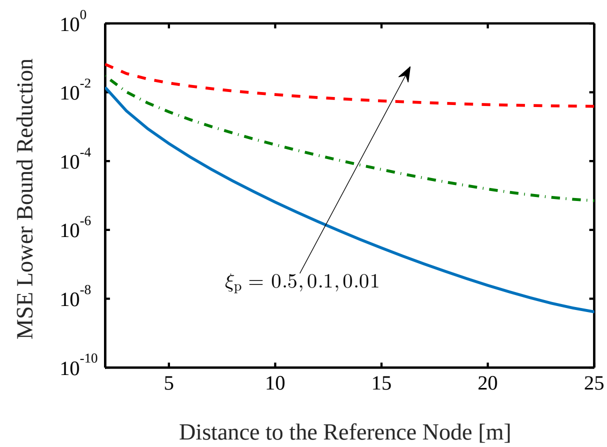

Fig. 3. The MSE lower bound reduction for agent j caused by turning an agent i into a reference node, as a function of distance between agent i and j.

6. CONCLUSIONS

In this paper, we have proposed the general expression of the inverse FIM, based on which the concept of ICI is introduced. We also provide a random walk interpretation of ICI. To illustrate our framework, we have derived asymptotic expressions of ICI in infinite lattice networks, which reflect the relation between ICI and network topology. Our analysis provides new insights into the network synchronization problem from a network-level point of view, and can be useful in algorithm design as well as network optimization.

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