# FASTER-THAN-NYQUIST SIGNALING WITH DIFFERENTIAL ENCODING AND NONCOHERENT DETECTION

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# ABSTRACT

In this paper, we propose a novel differential faster-than-Nyquist (DFTN) signaling scheme, which allows us to dispense with any channel estimation at the receiver while benefiting from the rate boost of faster-than-Nyquist (FTN) signaling. At the transmitter, differentially modulated binary phase-shift keying (DBPSK) symbols are transmitted with the symbol interval that is smaller than that defined by the Nyquist criterion. The receiver noncoherently estimates the DBPSK symbols, suffering from the effects of FTN-specific inter-symbol interference (ISI), based on frequency-domain equalization. This is enabled, because FTN-specific ISI is deterministic, by assuming that the FTN's symbol packing ratio and the roll-off factor of a shaping filter are known in advance at the receiver.

*Index Terms*— Differential encoding, faster-than-Nyquist, frequency-domain equalization, inter-symbol interference, noncoherent detection

## **1. INTRODUCTION**

In the 1970's, faster-than-Nyquist (FTN) signaling was invented for increasing a transmission rate, without expanding bandwidth and power consumption [1, 2, 3, 4]. In the FTN signaling, the symbol interval T is set to lower than that defined by the Nyquist criterion  $T_0$ , where  $\alpha$  is a symbol packing ratio. The main limitations of FTN signaling are that the effects of inter-symbol interference (ISI) are imposed on the receiver. In order to combat the limitations of FTN-specific ISI, several efficient equalization algorithms have been developed in the time [5, 6] and frequency domains [7, 8, 9]. Furthermore, in [10] low-complexity symbol-by-symbol detection was proposed for FTN receiver under the idealistic assumption of the AWGN channel, by adjusting the symbol packing ratio  $\alpha$  and the roll-off factor  $\beta$  of a root raised cosine (RRC) shaping filter. Furthermore, in [11, 12], the recent time-domain index modulation concept [13, 14] was incorporated into the FTN signaling, in order to reduce the ISI effects.

Another unveiled issue of FTN signaling is channel estimation, while in most of the previous studies perfect knowledge of channel state information (CSI) is assumed to be available at the FTN receiver. Most recently, in [9], the use of an FTN pilot (FTNP) sequence is considered for reducing the pilot overhead, while developing an efficient FTNP-based frequency-domain channel estimation algorithm. Furthermore, in [9] semi-blind joint channel estimation and data detection is carried out, in order to further reduce the overhead of FTNP. However, this benefit is achieved at the cost of additional complexity, which is imposed by the iterative process at the receiver, while still relying on pilot overhead and channel estimation.

Differential encoding and noncoherent detection were developed for allowing a receiver to detect symbols, while dispensing with any channel estimation [15]. Note that differential detection typically imposes the error-doubling effects at the receiver, in comparison to its coherent counterpart. Note that, in general differential noncoherent detection is possible only for an ISI-free frequency-flat channel, since that for a dispersive channel is an open issue. The exception is constituted by the noncoherent detection assisted by interference rejection spreading code [16]. Since FTN signaling naturally introduces ISI on the received signals, even in a frequencyflat channel, no differential schemes have not been proposed for FTN signaling systems, to the best of authors' knowledge.

Against the above-mentioned backcloth, the novel contributions of this paper are as follows. We propose a differential FTN (DFTN) signaling architecture, in order to enable noncoherent detection at the receiver, while achieving the fundamental benefits of the conventional FTN signaling. More specifically, under the assumption of a frequency-flat Rayleigh fading channel, the noncoherent detection becomes realistic by exploiting the fact that FTN-specific ISI is deterministic, where the associated channel impulse response (CIR) is accurately acquired without CSI estimation, when the symbol packing ratio  $\alpha$  and the roll-off factor of a shaping filter are known at the receiver in advance of transmission. Hence, the low-complexity frequency-domain equalization (FDE) [7, 17] is carried out, in order to cancel the deterministic ISI, and then the DFTN symbols are detected with the aid of differential noncoherent detection.

The remainder of this paper is organized as follows. In Section 2, we present the system model of the proposed DFTN scheme. In Section 3 our performance results are provided, and in Section 4 the present paper is concluded.

## 2. SYSTEM MODEL OF DFTN

In this section, we present the system model of our DFTN transmitter and receiver.

## 2.1. Transmitter Model

We consider a single-carrier block transmission of DFTN signaling. At the transmitter, N binary phase-shift keying (BPSK) symbols  $\mathbf{x} = [x_1, \cdots, x_N]^T \in \Re^N$  are modulated per block. In this paper, we consider BPSK modulation scheme for the sake of simplicity. However, a higher-order M-point phase-shift keying (PSK) modulation scheme is readily applicable in our DFTN signaling. The detailed investigations will be included in our future study. Then, the N BPSK-modulated symbols are differentially-encoded, in order to have the differential BPSK (DBPSK) symbol block  $\mathbf{s} = [s_0, \cdots, s_N]^T \in \Re^{N+1}$  as follows:  $s_i = x_i s_{i-1}$  $(1 \le i \le N)$ , where we consider the initial reference symbol of  $s_0 = 1$ . Furthermore, a  $2\nu$ -length cyclic prefix (CP) is added in each block. Here,  $\nu$  is designed sufficiently longer than the single-side tap length of FTN-induced ISI. The  $(N + 2\nu + 1)$ -length symbols are bandlimited with the aid of an RRC filter a(t), having a roll-off factor of  $\beta$ , in order to have the time-domain signals with an FTN symbol interval of  $T = \alpha T_0$  as follows: [7]

$$s(t) = \sum_{n} s_n a(t - nT).$$
<sup>(1)</sup>

Hence, the spectral efficiency of our CP-assisted DFTN signaling is formulated by

$$R = \frac{N}{N+2\nu+1} \frac{\log_2 \mathcal{M}}{\alpha(1+\beta)}.$$
 (2)

Note that the block length N is sufficiently higher than the CP length  $2\nu$ . Also, the coefficient  $1/\alpha$  in (2) represents the rate boost, owing to the FTN signaling.

#### 2.2. Receiver Model

Under the assumption of a frequency-flat Rayleigh fading, the received signals, which are matched-filtered by  $a^*(-t)$ , are expressed as

$$y(t) = h \sum_{n} s_n g(t - nT) + \eta(t), \qquad (3)$$

where we consider  $g(t) = \int a(\tau)a^*(\tau - t)d\tau$  and  $\eta(t) = \int n(\tau)a^*(\tau - t)d\tau$ , and n(t) is the complex-valued AWGN

with a zero mean and a noise variance of  $N_0$ . Moreover, h represents a channel coefficient, which obeys a complexvalued Gaussian distribution, having a zero mean and a unit variance. Assume that the channel coefficient h remains constant over each block transmission, while the packing ratio  $\alpha$ , the roll-off factor  $\beta$ , and the noise variance  $N_0$  are available at the receiver.

The *i*th sample is represented by

$$y_i = y(iT) \tag{4}$$

$$= h \sum_{n} s_n g((i-n)T) + \eta(iT), \tag{5}$$

where the noise components  $\eta(iT)$   $(i = 1, \dots, N + 2\nu + 1)$ in each block are correlated, while we have the relationship of  $\mathbb{E}[\eta(iT)\eta(jT)] = N_0g((i-j)T)$  [7] and  $\mathbb{E}[\cdot]$  is the expectation operation. After removing the first and last  $\nu$  samples from the  $(N + 2\nu + 1)$ -length received block of (5), we arrive at the tractable signal notation of

$$\mathbf{y} = h\mathbf{G}\mathbf{s} + \boldsymbol{\eta} \in \mathbb{C}^{N+1}, \tag{6}$$

where  $\mathbf{G} \in \Re^{(N+1) \times (N+1)}$  is the circulant matrix, composed of the vector of

$$\mathbf{g} = [g(-\nu T), \cdots, g(0), \cdots, g(\nu T)]^T, \tag{7}$$

and we have

$$\boldsymbol{\eta} = [\eta(0), \eta(T), \cdots, \eta(NT)]^T.$$
(8)

Furthermore, the eigenvalue decomposition of **G** is efficiently implemented with the aid of discrete Fourier transform (DFT) as  $\mathbf{G} = \mathbf{Q}^T \mathbf{\Lambda} \mathbf{Q}^*$ , where  $\mathbf{Q} \in \mathbb{C}^{(N+1) \times (N+1)}$  represents the normalized DFT matrix, whose *k*th-row and *l*th-column element is defined by

$$\frac{1}{\sqrt{N+1}} \exp\left[-2\pi j \frac{(k-1)(l-1)}{N+1}\right],$$
(9)

and  $\Lambda$  is a diagonal matrix, whose diagonal elements correspond to the DFT coefficients of G [7].

By carrying out the inverse DFT (IDFT) in (6), we obtain the signals of

$$\mathbf{y}_f = \mathbf{Q}^* \mathbf{y} \tag{10}$$

$$= h\Lambda \underbrace{\mathbf{Q}^*_{\mathbf{s}}}_{\mathbf{s}_f} + \mathbf{Q}^* \boldsymbol{\eta} \in \mathbb{C}^{N+1}, \qquad (11)$$

where  $\mathbf{s}_f = \mathbf{Q}^* \mathbf{s}$  are the frequency-domain DFTN symbols.

Then, since the matrix **G** is also available at the receiver,<sup>1</sup> the estimates of  $hs_f$  are calculated by the noise-whitening minimum mean-square error (MMSE)-based FDE [9] as follows:

$$\mathbf{v}_f = \mathbf{W}\mathbf{y}_f \tag{12}$$

<sup>&</sup>lt;sup>1</sup>Note that the matrix **G** is determined by the FTN parameters  $(\alpha, \beta)$ .

Table 1. Basic System Parameters

Block-length	CP-length	Roll-off factor	Channel
N = 512	$2\nu = 20$	$\beta = 0.3$	Frequency-flat Rayleigh fading

where the noise-whitening MMSE weights  $\mathbf{W} \in \mathbb{C}^{(N+1) \times (N+1)}$ are given by [9]

$$\mathbf{W} = \mathbf{\Lambda}^{H} \left( \mathbf{\Lambda} \mathbf{\Lambda}^{H} + N_0 \mathbf{\Phi} \right)^{-1}, \qquad (13)$$

and  $\mathbf{\Phi} = \text{diag}[\Psi_0, \cdots, \Psi_N]$  is also a diagonal matrix, where the *i*th element of which is calculated by

$$\Phi_i = \frac{1}{N+1} \sum_{k=0}^{N} \sum_{l=0}^{N} g((k-l)T) \exp\left(2\pi j \frac{(k-l)i}{N+1}\right).$$
(14)

Note that since the weight matrix  $\mathbf{W}$  exhibits a diagonal structure, its inverse calculations imposes as low complexity as the order of N. Moreover, the estimates of hs are given by carrying out DFT in (12) as follows:

$$\mathbf{v} = [v_0, \cdots, v_N]^T \tag{15}$$

$$= \mathbf{Q}^T \mathbf{v}_f \tag{16}$$

Finally, the transmitted BPSK symbols  $x_i$   $(i = 1, \dots, N)$  are differentially demodulated without relying on any CSI estimation as follows:

$$\hat{x}_i = v_i v_{i-1}^* \text{ for } 1 \le i \le N.$$
 (17)

Hence, the demodulation of DFTN symbols are completed without carrying out any channel estimation.

## 3. PERFORMANCE RESULTS

In this section, we provide our performance results based on the Monte Carlo simulations, in order to characterize the proposed DFTN signaling. The basic system parameters are listed in Table 1, where the block length and the CP length were set to N = 512 and  $2\nu = 20$ , respectively, while the roll-off factor of the RRC filter was given by  $\beta = 0.3$ . Also, the frequency-flat Rayleigh fading channel was considered. The conventional coherent FTN signaling scheme was considered as a benchmark scheme.

Firstly, Fig. 1 show the bit-error-rate (BER) performance of our DFTN scheme in the scenario of quasi-static Rayleigh fading. The DFTN scheme's packing ratio was set to  $\alpha =$ 0.9, 0.8 and 0.7, where the spectral efficiencies of each  $\alpha$ corresponded to 0.82, 0.92 and 1.06 [bps/Hz], respectively. Moreover, we also plotted the achievable BER performance of the conventional DBPSK scheme ( $\alpha = 1$ ), whose spectral efficiency was 0.77 [bps/Hz]. In Fig. 1, it was found that the proposed DFTN scheme is capable of correctly demodulating the DFTN signaling in the range of  $\alpha \geq 0.8$ . More



Fig. 1. The BER performance of the proposed DFTN scheme, where the packing ratio was set to  $\alpha = 1.0, 0.9, 0.8$  and 0.7.

specifically, the proposed DFTN scheme having the packing ratio of  $\alpha = 0.9$  achieved the same performance as that of the classic DBPSK scheme, based on the Nyquist criterion. Additionally, the DFTN scheme with  $\alpha = 0.8$  exhibited approximately 3-dB performance loss, in comparison to the DBPSK scheme ( $\alpha = 1$ ) as well as the DFTN scheme with  $\alpha = 0.9$ , while achieving 12% rate increase over them. However, upon decreasing the packing ratio to  $\alpha = 0.7$ , the DFTN scheme exhibited a severe error floor, which was caused by high ISI effects. The same detrimental effect is typically seen even in a coherent FTN counterpart, having a high  $\alpha$ . This error floor may be eliminated with the aid of powerful channel coding schemes, such as turbo and low-density parity-check codes [18], and the detailed investigations will be left for the future study.

Furthermore, in Fig. 2 we compared the BER performance of our DFTN scheme and the conventional coherent FTN counterpart [9], both employing MMSE-aided FDE at the receiver. Here, we assumed that the perfect CSI is available at the receiver of coherent FTN scheme, for the sake of simplicity. Observe in Fig. 2 that the 3-dB performance loss was seen in the DFTN scheme in comparison to the coherent FTN scheme for  $\alpha \leq 0.9$ . This penalty was caused due to the well-known noise-doubling effects imposed by the differential demodulation. However, in the high-ISI  $\alpha = 0.8$ scenario, the performance penalty imposed on the proposed DFTN scheme increased to 5 dB. Note that when considering a practical pilot-based channel estimation for the coherent FTN scheme, this performance gap may be reduced. Furthermore, for  $\alpha = 0.7$ , the BER curves of both the DFTN and coherent FTN schemes exhibited an error floor, similar to Fig. 1.

Finally, in Fig. 3 we investigated the effects of the timevarying channel on the achievable BER performance of the



Fig. 2. The BER comparison between the proposed DFTN scheme and the conventional coherent FTN scheme of [9]. Here, the packing ratio was set to  $\alpha = 1.0, 0.9, 0.8$  and 0.7.

DFTN and coherent FTN schemes. The received signals of (3) was modified to

$$y(t) = h(t) \sum_{n} s_n g(t - nT) + \eta(t),$$
 (18)

where h(t) represent the coefficient of a time-varying channel in each block, which was generated according to  $\mathbb{E}[h(t)h(t +$  $[\tau]^* = J_0(2\pi F_d T \tau)$ . Furthermore,  $F_d T$  denotes the normalized Doppler frequency, and  $J_0(\cdot)$  is the zero-order Bessel function of the first kind. The detection algorithm used in this scenario remained the same as that used in the quasistatic scenarios. Here, we assumed that in the coherent FTN scheme, the initial channel coefficient h(0) in each block was accurately acquired at the receiver. Moreover, the packing ratio was fixed to  $\alpha = 0.8$ . As shown in Fig. 3, the conventional coherent FTN scheme exhibited an error floor, upon introducing the effects of the time-varying channel, while the BER performance of the DFTN scheme with the normalized Doppler frequency of  $F_d T = 1.0 \times 10^{-6}$  remained unchanged from that of the time-invariant scenario ( $F_dT = 0$ ). More specifically, upon increasing  $F_dT$ , the performance advantage of the DFTN scheme became more explicit.

## 4. CONCLUSIONS

This paper first proposed the DFTN concept, which allows noncoherent detection, while attaining the explicit benefits of FTN signaling. The proposed DFTN receiver has the capability of correctly demodulating the ISI-induced DFTN symbols, which is achieved with the aid of low-complexity MMSEaided FDE and differential detection.



Fig. 3. The effects of the normalized Doppler frequency  $F_dT$  on the achievable BER performance of the DFTN and the coherent FTN schemes, where the packing ratio was fixed to  $\alpha = 0.8$ .

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