THREE-USER MIMO BROADCAST CHANNEL WITH DELAYED CSIT: A HIGHER ACHIEVABLE DOF

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ABSTRACT

Degrees of freedom (DoF) of the three-user multiple-input multipleoutput (MIMO) broadcast channel (BC) with delayed CSIT was derived for most antenna configurations except for the case of 2N < M < 3N, where transmitter has M antennas and each receiver has N antennas. In this paper, for that problem, we propose an effective scheme for acquiring a higher achievable DoF than the value via existing methods. In the initial transmission phase, we transmit more data symbols than the amount that the receivers can instantaneously decode. Then, we generate auxiliary symbols for decoding the data symbols. Specifically, our scheme introduces an integrated design for the generation of auxiliary symbols. As a result, a higher achievable DoF, i.e., $\frac{12MN}{7M+2N}$, can be achieved for specific antenna configurations, where 2N < M < 2.5N.

Index Terms— Delayed CSIT, Degrees of freedom, MIMO broadcast channel, multi-phase transmission, retrospective interference alignment

1. INTRODUCTION

For K-user multiple-input multiple-output (MIMO) broadcast channel (BC), there are one transmitter with M antennas and K receivers, each with N antennas, where the transmitter has separate information to be delivered to each receiver. The degrees of freedom (DoF) is a first-order approximation of channel capacity and denotes the maximal number of interference-free channels in practice. The DoF of K-user MIMO BC was given in [1], where the channel feedback is timely enough to capture the channel variation, that means instantaneous channel state information at the transmitter (CSIT) is available.

When the channel is fast-varying with time and the feedback is not instantaneous, the CSIT will be delayed (outdated) and has different value from the current one. Maddah-Ali and Tse (MAT) first derived the DoF of K-user multiple-input single-output (MISO) BC with delayed CSIT for $M \ge K$ antenna configurations by uncoded transmission [2]. Then, the DoF region of two-user MIMO BC with delayed CSIT was derived in [3]. References [4–6] derived the DoF of three-user MIMO BC with delayed CSIT. For $M \ge 3N$, the DoF was achieved by uncoded transmission. For $M \le 2N$, the DoF was achieved by coded transmission. For $M \le 2N$, the DoF was achieved by coded transmission. Coded and uncoded transmissions are different transmission modes for data symbols. For coded transmission, two data symbols are added then transmitted rather than straightforward uncoded transmission. On the other hand, for 2N < M < 3N, the DoF remains unclear. The existing DoF achievable schemes described in [4] adopted either coded or uncoded



Fig. 1. Three-user MIMO BC and delayed CSIT

transmissions. That is, for 2N < M < 2.4N, the coded transmission outperforms the uncoded one, whereas for 2.4N < M < 3N, the uncoded transmission has an advantage. The reference [7] investigated the achievable DoF of more-than-three-user MIMO BC with delayed CSIT.

BC has one transmitter, hence it is a centralized network. Extended to distributed networks such as interference channel and X channel, DoF with instantaneous CSIT was derived in [8–13]. Then, DoF with delayed CSIT was investigated in [14–17]. Moreover, the interplay between delayed CSIT and instantaneous CSIT was investigated in [18–20].

In this paper, we focus on three-user MIMO BC with delayed CSIT for 2N < M < 3N antenna configurations. We propose a coded transmission-based scheme for achieving a higher achievable DoF. In particular, we introduce a new auxiliary symbol design, helping us decode data symbols. The number of auxiliary symbols is less than that in [4], because we design the generation of auxiliary symbols in an integrated way rather than sequentially. As a result, we obtain a higher achievable DoF, i.e., $\frac{12MN}{7M+2N}$, than that via existing methods [4] for 2N < M < 2.5N antenna configurations. We find that 2N < M < 3N can be divided into two regions, namely, 2N < M < 2.5N and 2.5N < M < 3N according to the advantages of coded and uncoded transmissions for achievable DoF.

2. SYSTEM MODEL

Consider a three-user MIMO BC with delayed CSIT, depicted in Fig. 1. At time slot *i*, the channel state matrix from the transmitter to the receiver *j* is denoted by $\mathbf{H}_j[i] \in \mathbb{C}^{N \times M}$, whose elements are i.i.d. across space and time, and drawn from a continuous distribution. The CSIT is delayed, i.e., $\mathbf{H}_j[i - \tau], \tau = 1, 2, \cdots$ is available at transmitter. The transmit signal and received signal at the receiver *j* are denoted by $\mathbf{x}[i]$ and $\mathbf{y}_i[i]$, respectively.

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Fig. 2. The existing sequential design of the generation of auxiliary symbols.



Fig. 3. The proposed integrated design of the generation of auxiliary symbols.

The DoF characterizes the first-order approximation of channel capacity C in the high SNR regime.

$$C = \text{DoF} \log(\text{SNR}) + o(\log(\text{SNR})) \text{ bps/Hz}$$
(1)

where

$$(\log(SNR)) = \lim_{SNR\to\infty} \frac{C}{\log(SNR)} = 0$$

In addition, DoF measures the number of interference-free channels when SNR increases to infinity, i.e., the maximal number of multiplexing data streams. In the following, we omit the impact of noise, due to the high SNR regime.

3. PROPOSED SCHEME

3.1. Transmission

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We divide the transmission into three phases, in which data symbols are transmitted in Phase I, order-2 auxiliary symbols are transmitted in Phase II, and order-3 auxiliary symbols are transmitted in Phase III. During Phase I, the transmitter has no CSIT, at the beginning of Phases II and III, the transmitter will obtain the channel state matrices of the previous phase. The delayed CSIT is needed to generate auxiliary symbols. Compared to existing design of generation auxiliary symbols, we generate both order-2 and order-3 symbols after Phase I rather than order-2 symbols only. The difference between proposed design and existing design is depicted in Fig. 2 and Fig. 3.

Phase I (Coded Data Transmission): 6N time slots and M antennas are used to transmit a total of 12MN symbols with 4MN symbols for each receiver. The symbols desired by receivers 1, 2, and 3 are denoted by $\mathbf{a}_1, \dots, \mathbf{a}_{4N} \in \mathbb{C}^M, \mathbf{b}_1, \dots, \mathbf{b}_{4N} \in \mathbb{C}^M$ and $\mathbf{c}_1, \dots, \mathbf{c}_{4N} \in \mathbb{C}^M$. The total time slots of Phase I are grouped into 2N groups. Each group contains 3 time slots. The transmission signal for the time slot $i, 1 \leq i \leq 6N$ is designed as the summation of two different symbols, namely coded data transmission,

$$\mathbf{x}[i] \triangleq \begin{cases} \mathbf{a}_{1+2k} + \mathbf{b}_{1+2k} \in \mathbb{C}^M, & \mod(i,3) = 1\\ \mathbf{b}_{2+2k} + \mathbf{c}_{1+2k} \in \mathbb{C}^M, & \mod(i,3) = 2\\ \mathbf{c}_{2+2k} + \mathbf{a}_{2+2k} \in \mathbb{C}^M, & \mod(i,3) = 0 \end{cases}$$

where $k = \lfloor \frac{i-1}{3} \rfloor$ denotes the ordinal of groups.

Auxiliary Symbol Design: In each time slot, we transmit 2M symbols, while the number of receive antennas is just N. In addition, the desired symbols are mixed with interference. This shows the inability to decode the data symbols. To facilitate the decoding of data symbols, auxiliary symbols should be designed and delivered to corresponding receivers. We generate two kinds of auxiliary symbols, namely, order-2 and order-3 symbols, which are requested by receivers 2 and 3, respectively. To decode data symbols, we design auxiliary symbols. The order-2 symbols for receivers 1 and 2 are equal to:

$$\mathbf{ab}^{\text{Ph1 Group 0}} \triangleq \begin{bmatrix} \mathbf{H}_2[1]\mathbf{a}_1 \\ \mathbf{H}_1[1]\mathbf{b}_1 \\ \{\mathbf{y}_1[2] + \mathbf{H}_3[1]\mathbf{a}_1\}_1^{M-2N} \end{bmatrix} \in \mathbb{C}^M$$

where $\{\bullet\}_1^{M-2N}$ denotes from the 1^{st} to the $M - 2N^{th}$ rows in a vector or matrix are extracted. The order-2 symbols for receivers 2 and 3 are equal to:

$$\mathbf{b}\mathbf{c}^{\mathsf{Ph1 Group 0}} \triangleq \begin{bmatrix} \mathbf{H}_3[2]\mathbf{b}_2 \\ \mathbf{H}_2[2]\mathbf{c}_1 \\ \{\mathbf{y}_2[3] + \mathbf{H}_1[2]\mathbf{b}_2\}_1^{M-2N} \end{bmatrix} \in \mathbb{C}^M$$

The order-2 symbols for receivers 3 and 1 are equal to:

$$\mathbf{ca}^{\mathsf{Ph1 Group 0}} \triangleq \begin{bmatrix} \mathbf{H}_1[3]\mathbf{c}_2 \\ \mathbf{H}_3[3]\mathbf{a}_2 \\ \{\mathbf{y}_3[1] + \mathbf{H}_2[3]\mathbf{c}_2\}_1^{M-2N} \end{bmatrix} \in \mathbb{C}^M$$

Aside from the order-2 symbols, we also need the following M-2N order-3 symbols, which are requested for 3 receivers.

 $\mathbf{abc}^{\mathsf{Ph1}\,\mathsf{Group}\,0} \triangleq \{\mathbf{y}_3[1] + \mathbf{y}_1[2] + \mathbf{y}_2[3]\}_1^{M-2N} \in \mathbb{C}^{M-2N}$

If receivers obtain their order-2 and order-3 symbols, we can decode the data symbols of the first 3 time slots at each receiver. To sum up, in each group, we generate 3M order-2 symbols and M-2N order-3 symbols. We have 2N groups. Thus, 6MN order-2 symbols and 2(M-2N)N order-3 symbols are generated in Phase I.

Toy Example: Take the decoding of \mathbf{a}_1 and \mathbf{a}_2 as an example, which are desired by receiver 1. The corresponding received signals are depicted in Fig. 4, in which the underlined received signals are used to design the auxiliary symbols. The decoding of \mathbf{a}_1 needs M linearly independent equations. Via order-2 symbols $\mathbf{H}_1[1]\mathbf{b}_1$, receiver 1 can obtain N linearly independent equations, namely, $\mathbf{H}_1[1]\mathbf{a}_1$, by the cancellation $\mathbf{y}_1[1] - \mathbf{H}_1[1]\mathbf{b}_1$. On the other hand, receiver 1 can also attain another N linearly independent equations via order-2 symbols $H_2[1]a_1$ directly. Now, receiver 1 still needs M - 2N linearly independent equations, which can be provided by $\{\mathbf{H}_3[1]\mathbf{a}_1\}_1^{M-2N} = \{\mathbf{y}_1[2] + \mathbf{H}_3[1]\mathbf{a}_1 - \mathbf{y}_1[2]\}_1^{M-2N}$. In the above process, we do not use any order-3 symbols, which are very useful in the decoding of \mathbf{a}_2 . To decode \mathbf{a}_2 , via order-2 symbols $\mathbf{H}_1[3]\mathbf{c}_2$, receiver 1 can obtain N linearly independent equations, namely, $\mathbf{H}_1[3]\mathbf{a}_2$, by the cancellation $\mathbf{y}_1[3] - \mathbf{H}_1[3]\mathbf{c}_2$. On the other hand, receiver 1 can attain other N linearly independent equations via order-2 symbols $\mathbf{H}_3[3]\mathbf{a}_2$ directly. Now, receiver 1 still needs M = 2N linearly independent equations, which can be provided by $\{\mathbf{H}_2[3]\mathbf{a}_2\}_1^{M-2N} = \mathbf{abc}^{\text{Ph1 Group 0}} - \{\mathbf{y}_3[1] + \mathbf{H}_2[3]\mathbf{c}_2\}_1^{M-2N} - \{\mathbf{y}_1[2]\}_1^M$

Phase II (Order-2 Symbol Transmission): All 6MN order-2 symbols are transmitted with 2N transmit antennas and 3M time

Time Slot 1	Time Slot 2	Time Slot 3
$\mathbf{y}_1 [1] = \mathbf{H}_1 [1] \mathbf{a}_1 + \mathbf{H}_1 [1] \mathbf{b}_1$	$\mathbf{y}_{1}[2] = \mathbf{H}_{1}[2]\mathbf{b}_{2} + \mathbf{H}_{1}[2]\mathbf{c}_{1}$	$\mathbf{y}_1[3] = \mathbf{H}_1[3]\mathbf{c}_2 + \mathbf{H}_1[3]\mathbf{a}_2$
$\mathbf{y}_{2}[1] = \mathbf{H}_{2}[1]\mathbf{a}_{1} + \mathbf{H}_{2}[1]\mathbf{b}_{1}$	$\mathbf{y}_{2}[2] = \mathbf{H}_{2}[2]\mathbf{b}_{2} + \mathbf{H}_{2}[2]\mathbf{c}_{1}$	$\mathbf{y}_{2}[3] = \mathbf{H}_{2}[3]\mathbf{c}_{2} + \mathbf{H}_{2}[3]\mathbf{a}_{2}$
$\mathbf{y}_{3}[1] = \mathbf{H}_{3}[1]\mathbf{a}_{1} + \mathbf{H}_{3}[1]\mathbf{b}_{1}$	$\mathbf{y}_{3}[2] = \mathbf{H}_{3}[2]\mathbf{b}_{2} + \mathbf{H}_{3}[2]\mathbf{c}_{1}$	$\mathbf{y}_{3}[3] = \mathbf{H}_{3}[3]\mathbf{c}_{2} + \mathbf{H}_{3}[3]\mathbf{a}_{2}$

Fig. 4. Phase I Group 0 (Phase I first 3 time slots) received signals for 3 receivers.

slots. In Phase II, the 3M time slots are also grouped into M groups with 3 time slots in each group. In this phase, 2N antennas are used and $\mathbf{ab}^{Ph1 \operatorname{Group 0}}$, $\mathbf{bc}^{Ph1 \operatorname{Group 0}}$, $\mathbf{ca}^{Ph1 \operatorname{Group 0}}$ have M dimensions. To match the maximal 2N linearly independent transmit symbols in each time slot, we recast all order-2 symbols as $\mathbf{ab}_1, \dots, \mathbf{ab}_M \in \mathbb{C}^{2N}$ (for receivers 1 and 2), $\mathbf{bc}_1, \dots, \mathbf{bc}_M \in \mathbb{C}^{2N}$ (for receivers 2 and 3), and $\mathbf{ca}_1, \dots, \mathbf{ca}_M \in \mathbb{C}^{2N}$ (for receivers 3 and 1). In each group, the order-2 symbols are transmitted in sequence.

$$\mathbf{x}[i] \triangleq \begin{cases} \mathbf{ab}_{1+k} \in \mathbb{C}^{2N}, & \mod(i,3) = 1\\ \mathbf{bc}_{1+k} \in \mathbb{C}^{2N}, & \mod(i,3) = 2\\ \mathbf{ca}_{1+k} \in \mathbb{C}^{2N}, & \mod(i,3) = 0 \end{cases}$$

for $6N + 1 \le i \le 6N + 3M$, where $k = \lfloor \frac{i-1}{3} \rfloor$ denotes the ordinal of groups.

Auxiliary Symbol Design: In each time slot, because each receiver needs to decode 2N order-2 symbols and the number of receiver antennas is N, we need another N equations. For group 6N + 1, we need extra 2N equations for each receiver. To provide enough equations for decoding, the following order-3 symbols are designed and desired by 3 receivers. Our design prevents the use of pre-stored random coefficients at receivers.

$$\mathbf{abc}^{\mathsf{Ph2 Group 0}} \triangleq \begin{bmatrix} \mathbf{y}_3 \left[6N+1 \right] + \mathbf{y}_1 \left[6N+2 \right] \\ \mathbf{y}_2 \left[6N+3 \right] + \mathbf{y}_1 \left[6N+2 \right] \end{bmatrix} \in \mathbb{C}^{2N}$$

In each group (3 time slots), 2N order-3 symbols are produced. At the end of Phase II, we generated a total of 2MN order-3 symbols.

Phase III (Order-3 Symbol Transmission): Only N transmit antennas are used to make transmitted signals decodable at receivers without any auxiliary symbols. In Phase I, we generated 2(M - 2N)N order-3 symbols and in Phase II, we produced 2MN order-3 symbols, so that 4(M - N) time slots are needed to transmit the total 4(M - N)N order-3 symbols.

3.2. Decoding

The decoding process is divided into three stages. Stages I and II are used to decode order-3 and order-2 symbols, respectively. The desired data symbols are decoded in the Stage-III.

Stage I (Order-3 Symbol Decoding): The order-3 symbols are decoded instantaneously, because the number of transmitted order-3 symbols is equal to that of receive antennas.

Stage II (Order-2 Symbol Decoding): For receiver 1, to decode \mathbf{ab}_1 and \mathbf{ca}_1 , we need 4N equations. The decoding equation is given

by

$$\begin{bmatrix} \mathbf{y}_{1}[6N+1] \\ \mathbf{y}_{1}[6N+3] \\ (\mathbf{y}_{3}[6N+1]+\mathbf{y}_{1}[6N+2])-\mathbf{y}_{1}[6N+2] \\ (\mathbf{y}_{2}[6N+3]+\mathbf{y}_{1}[6N+2])-\mathbf{y}_{1}[6N+2] \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{1}[6N+1] & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{1}[6N+3] \\ \mathbf{H}_{3}[6N+1] & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{2}[6N+3] \end{bmatrix} \begin{bmatrix} \mathbf{a}\mathbf{b}_{1} \\ \mathbf{c}\mathbf{a}_{1} \end{bmatrix}$$
(2)

Whereas for receiver 2, to decode \mathbf{bc}_1 and \mathbf{ab}_1 , we need 4N equations. The decoding equations can be derived as

$$\begin{bmatrix} \mathbf{y}_{2}[6N+2] \\ \mathbf{y}_{2}[6N+1] \\ (\mathbf{y}_{2}[6N+3] + \mathbf{y}_{1}[6N+2]) - \mathbf{y}_{2}[6N+3] \\ (\mathbf{y}_{3}[6N+1] + \mathbf{y}_{1}[6N+2]) - \mathbf{y}_{1}[6N+2] \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{2}[6N+1] \\ \mathbf{0} & \mathbf{H}_{2}[6N+1] \\ \mathbf{H}_{1}[6N+2] & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{3}[6N+1] \end{bmatrix} \begin{bmatrix} \mathbf{b}\mathbf{c}_{1} \\ \mathbf{a}\mathbf{b}_{1} \end{bmatrix}$$
(3)

Note that $\mathbf{y}_1[6N+2]$ can be obtained by $(\mathbf{y}_2[6N+3] + \mathbf{y}_1[6N+2]) - \mathbf{y}_2[6N+3]$ at the receiver 2. Finally, for receiver 3, to decode **b** \mathbf{c}_1 and **c** \mathbf{a}_1 , we also need 4N equations, and the decoding equation can be written as

$$\begin{bmatrix} \mathbf{y}_{3}[6N+2] \\ \mathbf{y}_{3}[6N+3] \\ (\mathbf{y}_{3}[6N+1] + \mathbf{y}_{1}[6N+2]) - \mathbf{y}_{3}[6N+1] \\ (\mathbf{y}_{2}[6N+3] + \mathbf{y}_{1}[6N+2]) - \mathbf{y}_{1}[6N+2] \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{3}[6N+2] & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{3}[6N+3] \\ \mathbf{H}_{1}[6N+2] & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{2}[6N+3] \end{bmatrix} \begin{bmatrix} \mathbf{b} \mathbf{c}_{1} \\ \mathbf{c} \mathbf{a}_{1} \end{bmatrix}$$
(4)

Note that $\mathbf{y}_1[6N+2]$ can be obtained by $(\mathbf{y}_3[6N+1] + \mathbf{y}_1[6N+2]) - \mathbf{y}_3[6N+1]$ at the receiver 3.

We have showed that 6N order-2 symbols can be decoded with 2N order-3 symbols. By the same way, 6MN order-2 symbols can be decoded with 2MN order-3 symbols.

Stage III (Data Symbol Decoding): For receiver 1, to decode $\mathbf{a}_1, \mathbf{a}_2$, we need 2M equations. The decoding equation is given by the eqn. (5). For receiver 2, to decode $\mathbf{b}_1, \mathbf{b}_2$, we need 2M equations, and the eqn. (6) gives the decoding process. Finally, for receiver 3, to decode $\mathbf{c}_1, \mathbf{c}_2$, we again need 2M equations, and the decoding is facilitated by the eqn. (7).

We use 3M order-2 symbols and 2(M - N) order-3 symbols to decode the 6M symbols transmitted in Phase I Group 0. In Stages I



Table 1. DoF comparison for specific antenna configurations in 2N < M < 2.5N.

(M, N)	Existing	Proposed	Upper Bound
(13 , 6)	9.0435	9.0874	9.1765
(14, 6)	9.0811	9.1636	9.3333
(19, 8)	12.1196	12.2416	12.4932
(19, 9)	13.5446	13.5894	13.6800

and II, we obtain 6MN order-2 and 4(M - N)N order-3 symbols so that 12MN data symbols can be decoded.

3.3. Achievable DoF

The Phase I, II, and III cost 6N, 3M, and 4(M - N) time slots and use M, 2N, and N transmit antennas, respectively. The total time slots used for transmission is 7M + 2N. The total number of desired data symbols is 12MN (4MN for each receiver). Therefore, the proposed scheme can achieve a DoF of $\frac{12MN}{7M+2N}$ for 2N < M < 3N antenna configurations.

4. DISCUSSION & CONCLUSION

As a comparison, for 2N < M < 3N antenna configurations, the state-of-the-art highest achievable DoF is obtained by Scheme I and Scheme II in [4], i.e., $\max\left\{\frac{24MN}{15M+2N}, \frac{12MN}{5M+7N}\right\}$. Moreover, the DoF upper bound is $\frac{6MN}{2N+2N}$ and given in [4].

DoF upper bound is $\frac{6MN}{3M+2N}$ and given in [4]. Fig. 5 shows that the proposed coded transmission-based scheme achieves a better DoF, i.e., $\frac{12MN}{7M+2N}$, which outperforms that of



Fig. 5. DoF upper bound and achievable DoF comparison.

Schemes I and II for 2N < M < 2.5N antenna configurations. It is intriguing to note that M = 2.5N is a turning point for coded and uncoded transmission, dividing the focused region equally.

Table 1 illustrates that our proposed achievable DoF is higher than that of existing methods and near the DoF upper bound. In addition, it shows that if N is fixed, achievable DoF increases with M, and vice versa.

As a conclusion, in this work, we proposed a new scheme for acquiring a higher achievable DoF, i.e., $\frac{12MN}{7M+2N}$, for 2N < M < 2.5N antenna configurations. In particular, an integrated design in generating auxiliary symbols to effect the decoding process is introduced. Via this result, we show that the coded and uncoded transmissions have an equivalent importance for the problematic antenna configuration region.

5. REFERENCES

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